

4.3 Answers

A-Level Pure Mathematics : Year 1 Progress Test Revision

4.3.1 Solution to 4.1 Example

(i) $5^{4x} = (5^x)^4 = y^4$

[1 mark]

(ii) $5^{x-2} = 5^x \times 5^{-2} = \frac{y}{25}$

[1 mark]

(iii) $5^{-\frac{x}{2}} = \left(\frac{5}{1}\right)^{-\frac{x}{2}} = \left(\frac{1}{5}\right)^{\frac{x}{2}} = \left(\frac{1^x}{5^x}\right)^{\frac{1}{2}} = \frac{1}{\sqrt{y}}$

[2 marks]

(iv) $125^x = (5^3)^x = (5^x)^3 = y^3$

[1 mark]

(v) $\frac{5^3}{25^{x+2}} = \frac{5^3}{(5^2)^x \times (5^2)^2} = \frac{5^3}{(5^x)^2 \times 5^4} = \frac{1}{5y^2}$

[3 marks]

4.3.2 Solutions to 4.2 Exercise

Answer 1

(i) $3^{0.5x} = (3^x)^{0.5} = \sqrt{y}$

[1 mark]

(ii) $3^{1+2x} = 3^1 \times (3^x)^2 = 3y^2$

[1 mark]

(iii) $3^{-3x} = \left(\frac{3}{1}\right)^{-3x} = \left(\frac{1}{3}\right)^{3x} = \left(\frac{1^x}{3^x}\right)^3 = \frac{1}{y^3}$

[2 marks]

(iv) $\frac{1}{9^{x+1}} = \frac{1}{9^x \times 9^1} = \frac{1}{(3^2)^x \times 9} = \frac{1}{(3^x)^2 \times 9} = \frac{1}{9y^2}$

[2 marks]

(v) $\frac{1^x}{27^{2x}} = \frac{1}{(3^3)^{2x}} = \frac{1}{(3^x)^6} = \frac{1}{y^6}$

[2 marks]

Answer 2**(i)**

$$\begin{aligned}
 f(x) &= x^2 + 6x - 41 \\
 &= [x^2 + 6x] - 41 \\
 &= [(x + 3)^2 - 9] - 41 \\
 &= (x + 3)^2 - 50 \\
 a &= 3, \quad b = -50
 \end{aligned}$$

[2 marks]**(ii)**

$$\begin{aligned}
 x^2 + 6x - 41 &= 0 \\
 (x + 3)^2 - 50 &= 0 \quad \text{from part (i)} \\
 (x + 3)^2 &= 50 \\
 x + 3 &= \pm\sqrt{50} \\
 x &= -3 \pm \sqrt{25 \times 2} \\
 &= -3 \pm 5\sqrt{2} \\
 p &= -3, \quad q = 5, \quad r = 2
 \end{aligned}$$

[3 marks]**Answer 3**

$$\begin{aligned}
 \frac{3x^2 + 12x + 9}{108x - 12x^3} &= \frac{3(x^2 + 4x + 3)}{12x(9 - x^2)} \\
 &= \frac{(x + 3)(x + 1)}{4x(3 - x)(3 + x)} \\
 &= \frac{x + 1}{-4x(x - 3)}
 \end{aligned}$$

$$a = 1, \quad b = -4 \text{ and } c = -3$$

[3 marks]**Answer 4**By the factor theorem, $f(-2) = 0$

$$\begin{aligned}
 (-2 + 4)((-2)^2 - 3(-2) + k) - 42 &= 0 \\
 2(4 + 6 + k) &= 42 \\
 k + 10 &= 21 \\
 k &= 11
 \end{aligned}$$

[3 marks]

Answer 5

$$\frac{9^{x-1}}{3^{y+2}} = 81$$

$$\frac{(3^2)^{x-1}}{3^{y+2}} = 3^4$$

$$3^{2x-2-y-2} = 3^4$$

$$2x - 2 - y - 2 = 4$$

$$2x - y = 8$$

$$y = 2x - 8$$

[3 marks]**Answer 6**

$$(i) \quad 16a^2 = 2\sqrt{a} \quad \Rightarrow \quad \dots \text{alternatively } \dots$$

$$8a^2 = \sqrt{a} \quad 256a^4 = 4a$$

$$8a^2 - \sqrt{a} = 0 \quad 256a^4 - 4a = 0$$

$$\sqrt{a}(8a^{\frac{3}{2}} - 1) = 0 \quad 4a(64a^3 - 1) = 0$$

$$\text{Either } a = 0 \text{ or } a^{\frac{3}{2}} = \frac{1}{8} \quad a = 0 \text{ or } 64a^3 = 1$$

$$a^3 = \frac{1}{64} \quad \Leftarrow$$

$$a = \frac{1}{4}$$

[4 marks]

$$(ii) \quad b^4 + 7b^2 - 18 = 0$$

Let $z = b^2$ in which case,

$$z^2 + 7z - 18 = 0$$

$$(z + 9)(z - 2) = 0$$

$$\text{Either } z = -9 \text{ or } z = 2$$

$$b^2 = -9 \text{ or } b^2 = 2$$

$$b = \pm\sqrt{2} \text{ are the only real solutions}$$

[4 marks]

Answer 7

$$3 \times 2^x = 28 - 2^{x-1}$$

$$3 \times 2^x = 28 - \frac{2^x}{2}$$

Multiply through by 2

$$6 \times 2^x = 56 - 2^x$$

$$6 \times 2^x + 2^x = 56$$

$$7 \times 2^x = 56$$

$$2^x = 8$$

$$x = 3$$

[4 marks]