10.2 Answers

A-Level Pure Mathematics : Year 1
Exponentials and Logarithms

Answer 1

$$\log_5(3t + 8) = 2.1$$
$$5^{2.1} = 3t + 8 \quad \text{Using } \log_c a = b \Leftrightarrow c^b = a$$
$$29.36 = 3t + 8$$
$$t = 7.12$$

[3 marks]

Answer 2

$$r = 0.1\,e^{0.5t + 1}$$
$$10r = e^{0.5t + 1}$$
$$\ln(10r) = 0.5t + 1$$
$$\ln(10r) - 1 = 0.5t$$
$$t = 2\ln(10r) - 2$$

[3 marks]

Answer 3

(a) Accept "Time cannot go backwards"
The model has only been set up for positive values of t

[1 mark]

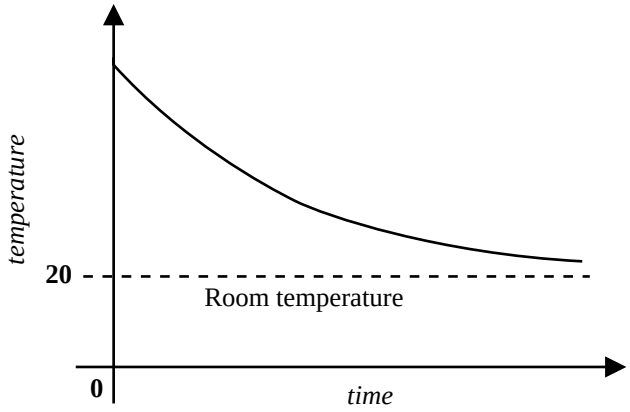
(b) When $t = 0$, $T = 75°C$

[1 mark]

(c)
$$50 = 55\,e^{-0.125t} + 20$$
$$30 = 55\,e^{-0.125t}$$
$$\frac{30}{55} = e^{-0.125t}$$
$$\ln\left(\frac{30}{55}\right) = -0.125t$$
$$t = \frac{\ln\left(\frac{30}{55}\right)}{-0.125}$$
$$= 4.849086$$
$$= 4 \text{ minutes } 51 \text{ seconds}$$

[3 marks]

(d)



The tea cannot become colder than the surrounding temperature.

[2 marks]

Answer 4

(a)

$$\begin{aligned} x &= D\,e^{-\,0.2t} \\ &= 15\,e^{-0.2\times 4} \\ &= 6.740 \text{ mg} \end{aligned}$$

[2 marks]

(b)

$$\begin{aligned} amount &= 15\,e^{-0.2\times 7} + 15\,e^{-0.2\times 2} \\ &= 13.754 \text{ mg} \end{aligned}$$

[2 marks]

Answer 5

$$\begin{aligned} y &= 20\,x^{n} \\ 590 &= 20 \times 15^{n} \\ 29.5 &= 15^{n} \\ n &= \frac{ln\,29.5}{ln\,15} \\ &= 1.25 \end{aligned}$$

[5 marks]

Answer 6

(a)

$$log_{10} P = 2 + 0.01\,t$$

[3 marks]

(b)

$$P = a\,b^t$$

$$a = 10^2$$

$$= 100$$

$\therefore$ The initial population is 100

[3 marks]

(c)

$$b = 10^{0.01}$$

$$= 1.023$$

[2 marks]

(d) Thus,

$$P = 100 \times 1.023^t$$

$$= 100 \times 1.023^{30}$$

$$= 198$$

(Accept answers between 195 and 200)

The more accurate answer is 199.526...

[1 mark]

Answer 7

(a) For 1400, $t = 0$

$$y = A\,e^{kt}$$

$$375 = A\,e^0$$

$$A = 375 \quad \text{as expected}$$

[1 mark]

(b)

$$y = 375\,e^{kt}$$

$$945 = 375\,e^{4k}$$

$$e^{4k} = 2.52$$

$$4k = ln\,2.52$$

$$4k = 0.9243$$

$$k = 0.231$$

[4 marks]

(c)

$$y = 375\,e^{0.231t}$$
$$= 375\,e^{0.231 \times 2}$$
$$= 595$$

It's an overestimate of the actual population which was 540 million
The percentage error on the actual measured population is,

$$\frac{594 - 540}{540} \times 100 = 10\,\%$$

This can be justified as being reasonable or not depending on your expectations of the mathematics and how accurate you think the figures in the table were in the first place.

[2 marks]

Answer 8

$$2^{2x} + 2^3 = 2^{x+3}$$
$$\left(2^x\right)^2 + 8 = 8 \times 2^x$$
$$\left(2^x\right)^2 - 8 \times \left(2^x\right) + 8 = 0$$
$$2^x = \frac{8 \pm \sqrt{8^2 - 4 \times 1 \times 8}}{2 \times 1}$$
$$2^x = 4 \pm 2\sqrt{2}$$
$$x = \frac{ln\left(4 \pm 2\sqrt{2}\right)}{ln\,2}$$
$$= 2.772 \text{ or } 0.228$$

[4 marks]