11.3 Answers

Answer 1

$$7^x = 5$$

Take the natural log of both sides

$$\ln 7^x = \ln 5$$

$$x \ln 7 = \ln 5$$

$$x = \frac{\ln 5}{\ln 7}$$

$$= 0.827$$

[2 marks]

Answer 2

$$5^x \times 5^{2x+1} = 10$$

$$5^{3x+1} = 10$$

Take the natural log of both sides

$$\ln 5^{3x+1} = \ln 10$$

$$(3x + 1) \ln 5 = \ln 10$$

$$3x + 1 = \frac{\ln 10}{\ln 5}$$

$$3x = 1.4307 - 1$$

$$x = \frac{0.4307}{3}$$

$$= 0.144$$

[3 marks]

Answer 3

$$\log_5 (2x + 3) - \log_5 x = \log_5 8$$

$$\log_5 \left(\frac{2x + 3}{x} \right) = \log_5 8$$

$$\frac{2x + 3}{x} = 8$$

$$2x + 3 = 8x$$

$$6x = 3$$

$$x = \frac{1}{2}$$

[3 marks]

Answer 4

$$2\,log_7(\,x - 6\,) - log_7 x = log_7 3$$

$$log_7\,(\,x - 6\,)^2 - log_7 x = log_7 3$$

$$log_7\left(\frac{(\,x - 6\,)^2}{x}\right) = log_7 3$$

$$\frac{(\,x - 6\,)^2}{x} = 3$$

$$x^2 - 12x + 36 = 3x$$

$$x^2 - 15x + 36 = 0$$

$$(\,x - 3\,)(\,x - 12\,) = 0$$

Either $x = 3$ or $x = 12$

Reject $x = 3$ as it gives rise to $log_7(\,3 - 6\,)$ which is not a valid entity.

$\therefore$ The only solution is $x = 12$

[5 marks]

Answer 5

$$2^{2x+1} - 2^x = 15$$

$$2^{2x} \times 2^1 - 2^x - 15 = 0$$

$$2\left(\,2^x\,\right)^2 - \left(\,2^x\,\right) - 15 = 0$$

Let $w = 2^x$,

$$2\,w^2 - w - 15 = 0$$

$$(\,2w + 5\,)(\,w - 3\,) = 0$$

Either $w = -\dfrac{5}{2}$ which has no solutions

or $w = 3$

$$2^x = 3$$

$$ln\,2^x = ln\,3$$

$$x\,ln\,2 = ln\,3$$

$$x = \frac{ln\,3}{ln\,2}$$

$$= 1.585$$

[5 marks]

Answer 6

(a) The rightmost graph is a straight line and so has an equation of the form,

$$y = mx + c$$

The y-axis intercept is given as, $c = -1.77$ and so,

$$y = mx - 1.77$$

$$log_{10} d = m\, log_{10} V - 1.77$$

$$log_{10} d = log_{10} V^m - 1.77$$

$$log_{10} V^m - log_{10} d = 1.77$$

$$log_{10}\left(\frac{V^m}{d} \right) = 1.77$$

$$10^{1.77} = \frac{V^m}{d}$$

$$d = \frac{V^m}{10^{1.77}}$$

$$\therefore \ d = 0.017\, V^m$$

which is of the form $d = k\, V^n$ with $k \approx 0.017$ □

[3 marks]

(b) From the leftmost graph it is known that $d = 20$ when $V = 30$

$$\therefore \ 20 = 0.017 \times 30^n$$

$$\left(\frac{20}{0.017} \right) = 30^n$$

Take the natural log of both sides

$$ln\left(\frac{20}{0.017} \right) = ln\, 30^n$$

$$n\, ln\, 30 = ln(\ 1176.471\ )$$

$$n = \frac{ln\, 1176.471}{ln\, 30}$$

$$= 2.08$$

$$\therefore \ d = 0.017 \times V^{2.08}$$

[3 marks]

(c)

$$d = 0.017 \times 60^{2.08}$$

$= 84.92$ metres is the stopping distance.

$$Reaction\ Distance = speed \times time = \frac{60 \times 1000}{60 \times 60} \times 0.8 = 13.33 \text{ metres}$$

As $84.92 + 13.33 = 98.25 < 100$ Sean is able to stop in time.

[3 marks]

Answer 7

(a)

$$m = \frac{\Delta y}{\Delta x} = \frac{3.7 - 5.2}{20 - 0} = -\frac{3}{40}$$

$$y = mx + c \Rightarrow y = -\frac{3}{40}x + 5.2$$

$$log_4 P = -\frac{3}{40}t + 5.2 \qquad \left(-\frac{3}{40} = -0.075\right)$$

[2 marks]

(b) When $t = 0$,

$$log_4 P = 5.2$$

$$4^{5.2} = P$$

$$P = 1350 \quad \text{(3 significant figures asked for)}$$

[1 mark]

(c)

$$log_4 P = -\frac{3}{40}t + 5.2$$

$$P = 4^{-0.075t + 5.2}$$

$$P = 4^{5.2} \times 4^{-0.075t}$$

$$P = 1350 \times \left(4^{-0.075}\right)^t$$

$$P = 1350 \times 0.901^t$$

$$\therefore \ a = 1350 \ \text{and} \ b = 0.901$$

[2 marks]

(d)

$$P = 1350 \times 0.901^{30}$$

$$= 59 \text{ people}$$

[1 mark]

Answer 8

(i)

$$log_3 (3b + 1) - log_3 (a - 2) = -1$$

$$log_3 \left(\frac{3b + 1}{a - 2} \right) = -1$$

$$3^{-1} = \frac{3b + 1}{a - 2}$$

$$3b + 1 = \frac{a - 2}{3}$$

$$3b = \frac{a - 2}{3} - \frac{3}{3}$$

$$3b = \frac{a - 5}{3}$$

$$b = \frac{a - 5}{9}$$

[3 marks]

(ii)

$$2^{2x + 5} - 7 (2^x) = 0$$

$$2^{2x} \times 2^5 - 7 (2^x) = 0$$

$$32 (2^x)^2 - 7 (2^x) = 0$$

$$(2^x)(32 \times 2^x - 7) = 0$$

Either $2^x = 0$ which has no solutions

Or $2^x = \frac{7}{32}$

$$ln\, 2^x = ln \left(\frac{7}{32} \right)$$

$$x\, ln\, 2 = ln \left(\frac{7}{32} \right)$$

$$x = \frac{ln \left(\frac{7}{32} \right)}{ln\, 2}$$

$$= -2.19$$

[4 marks]

This document is a part of a **Mathematics Community Outreach Project** initiated by Shrewsbury School
It may be freely duplicated and distributed, unaltered, for non-profit educational use
In October 2020, Shrewsbury School was voted "**Independent School of the Year 2020**"
© 2023 Number Wonder
Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk