Lesson 12

A-Level Pure Mathematics : Year 1

Exponentials and Logarithms

12.1 TEST

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available: 50

Question 1

Solve the equation

$$3^x = 65$$

Give your answer correct to 3 significant figures.

[2 marks]

Question 2

By first using a Law of Indices, or otherwise, solve the equation,

$$7^{3x} \times 7^{2x+3} = 43$$

Give your answer to three decimal places.

[3 marks]

Question 3

Solve the following equation, giving the exact answer.

$$\log_2(5x + 3) - \log_2 x = \log_2 12$$

[3 marks]

Question 4

Solve the following equation

$$2\log_7(x - 4) - \log_7 x = \log_7 2$$

[5 marks]

Question 5

Solve the equation

$$5^{2x} - 13\left(5^{x}\right) + 22 = 0$$

giving your answers correct to three decimal places.

[4 marks]

Question 6

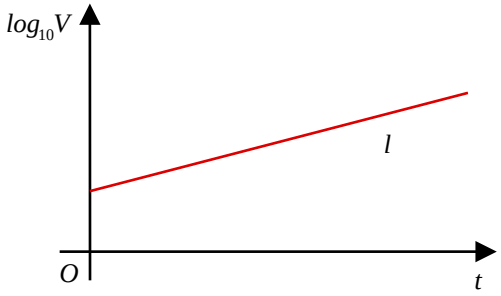
Solve the equation

$$log_2(3x + 1) + log_2(2x) = 3$$

[4 marks]

Question 7

As-Level Examination Question from May 2018, Paper 1, Q13 (Edexcel)



The value of a rare painting, £V, is modelled by the equation $V = p\,q^t$, where p and q are constants and t is the number of years since the value of the painting was first recorded on 1st January 1980.

The line l shown on the graph illustrates the linear relationship between t and $log_{10}V$ since 1st January 1980.

The equation of line l is $log_{10}V = 0.05t + 4.8$

(a) Find, to 4 significant figures, the value of p and the value of q.

[4 marks]

(b) With reference to the model interpret

(i) the value of the constant p

[1 mark]

(ii) the value of the constant q

[1 mark]

(c) Find the value of the painting, as predicted by the model, on 1st January 2010.
Give your answer to the nearest thousand pounds.

[2 marks]

Question 8

A-Level Sample Assessment Materials from 2017, Paper 1, Q12 (Edexcel)

In a controlled experiment, the number of microbes, N, present in a culture T days after the start of the experiment were counted.

N and T are expected to satisfy a relationship of the form,

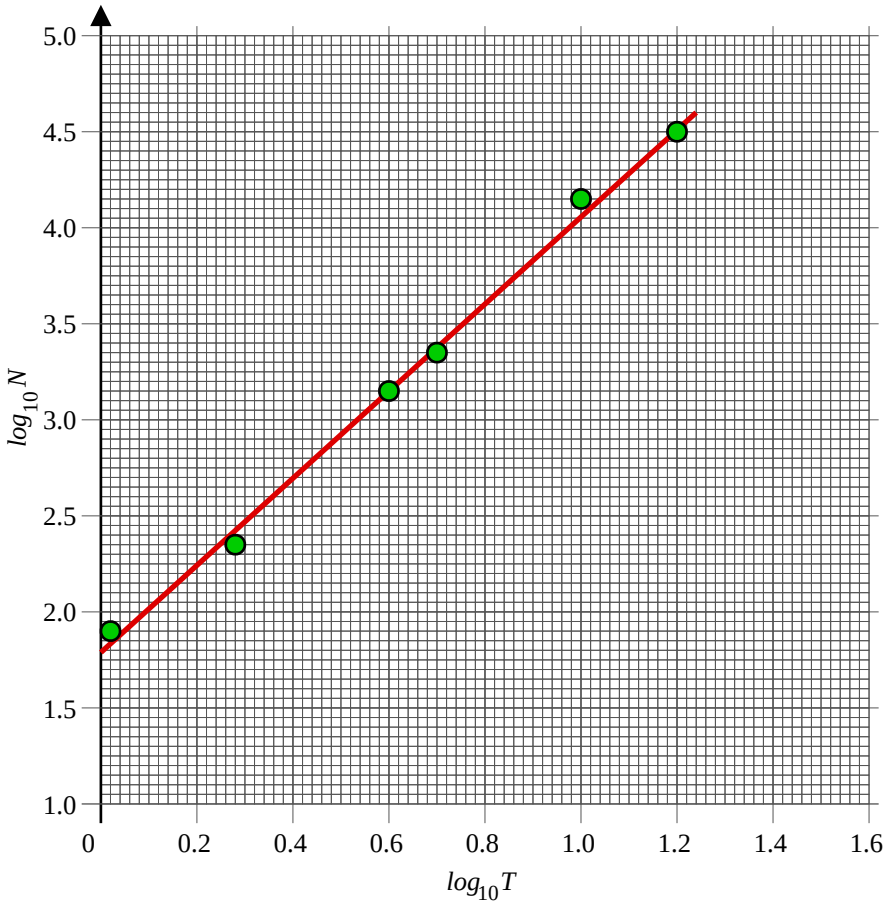
$$N = a\,T^{b}, \qquad \text{where } a \text{ and } b \text{ are constants}$$

(a) Show that this relationship can be expressed in the form

$$\log_{10} N = m \log_{10} T + c$$

giving m and c in terms of the constants a and/or b.

[2 marks]



The graph shows the line of best fit for values of $log_{10} N$ against values of $log_{10} T$

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

[4 marks]

(c) Explain why the information provided could not reliably be used to estimate the day when the number of microbes in the culture first exceeds 1000000.

[2 marks]

(d) With reference to the model, interpret the value of the constant *a*

[1 mark]

Question 9

The density of a pesticide in a section of field, P mg/m^2, can be modelled by the equation,

$$P = 160\, e^{-0.063t}$$

where t is the time in days since the pesticide was first applied.

(a) What was the initial density of the pesticide immediately after it had been applied to the field ?

[1 mark]

(b) By solving an appropriate mathematical equation, find the number of days that must pass before the density of the pesticide has fallen to 80 mg/m^2

[3 marks]

(c) The farmer wishes to harvest the crop of carrots that are being grown in another field in twenty days time but has a problem with Carrot Rust Flies and needs to spray the field.
The field has not been sprayed in the past.
When he harvests the crop, health and safety regulations state that the density of pesticide must be below 36 mg/m^2
What is the maximum density of pesticide that he can immediately apply such that he can still harvest the crop as planned ?

[3 marks]

Question 10

(i) Given that

$$9^x = 3^a$$

express a in terms of x

[1 mark]

(ii) Hence, or otherwise, solve the following equation

$$9^x - 2 \times 3^{x+1} + 8 = 0$$

Give your answers to 3 decimal places.

[4 marks]