12.2 Answers

Answer 1

$$3^x = 65$$

Take the natural log of both sides

$$\ln 3^x = \ln 65$$
$$x \ln 3 = \ln 65$$
$$x = \frac{\ln 65}{\ln 3}$$
$$= 3.80$$

[2 marks]

Answer 2

$$7^{3x} \times 7^{2x+3} = 43$$
$$7^{5x+3} = 43$$

Take the natural log of both sides

$$\ln 7^{5x+3} = \ln 43$$
$$(5x + 3) \ln 7 = \ln 43$$
$$5x + 3 = \frac{\ln 43}{\ln 7}$$
$$5x = 1.9337 - 3$$
$$x = -\frac{1.0671}{5}$$
$$= -0.213$$

[3 marks]

Answer 3

$$\log_2(5x + 3) - \log_2 x = \log_2 12$$
$$\log_2\left(\frac{5x + 3}{x}\right) = \log_2 12$$
$$\frac{5x + 3}{x} = 12$$
$$5x + 3 = 12x$$
$$7x = 3$$
$$x = \frac{3}{7}$$

[3 marks]

Answer 4

$$2\log_7(x-4) - \log_7 x = \log_7 2$$

$$\log_7(x-4)^2 - \log_7 x = \log_7 2$$

$$\log_7\left(\frac{(x-4)^2}{x}\right) = \log_7 2$$

$$\frac{(x-4)^2}{x} = 2$$

$$x^2 - 8x + 16 = 2x$$

$$x^2 - 10x + 16 = 0$$

$$(x-2)(x-8) = 0$$

Either $x = 2$ or $x = 8$

Reject $x = 2$ as it gives rise to $\log_7(2-6)$ which is not a valid entity.

$\therefore$ The only solution is $x = 8$

[5 marks]

Answer 5

$$5^{2x} - 13\left(5^x\right) + 22 = 0$$

$$\left(5^x\right)^2 - 13\left(5^x\right) + 22 = 0$$

Let $w = 5^x$,

$$w^2 - 13w + 22 = 0$$

$$(w-2)(w-11) = 0$$

Either $w = 2$ or $w = 11$

$5^x = 2$	$5^x = 11$
$\ln 5^x = \ln 2$	$\ln 5^x = \ln 11$
$x \ln 5 = \ln 2$	$x \ln 5 = \ln 11$
$x = \frac{\ln 2}{\ln 5}$	$x = \frac{\ln 11}{\ln 5}$
$= 0.431$	$x = 1.490$

[4 marks]

Answer 6

$$\log_2(3x + 1) + \log_2(2x) = 3$$
$$\log_2\big((3x + 1)(2x)\big) = 3$$
$$6x^2 + 2x = 2^3$$
$$3x^2 + x = 4$$
$$3x^2 + x - 4 = 0$$
$$(3x + 4)(x - 1) = 0$$

Either $x = -\frac{4}{3}$ which is rejected as it does not satisfy the original equation.

or $x = 1$

[4 marks]

Answer 7

(a)
$$\log_{10}V = 0.05t + 4.8$$
$$V = 10^{0.05t + 4.8}$$
$$= 10^{0.05t} \times 10^{4.8}$$
$$= 63100 \times \left(10^{0.05}\right)^t$$
$$= 63100 \times 1.122^t$$
$$\therefore\ p = 63100 \text{ and } q = 1.122$$

[4 marks]

(b) **(i)** p is the value of the painting on 1st January 1980.

[1 mark]

(ii) q is the annual rate at which the value of the painting is increasing.
1.122 corresponds to an annual growth rate of 12.2 %

[1 mark]

(c) As $t = 0$ corresponded to 1st January 1980,
$t = 30$ will correspond to 1st January 2010.

$$\therefore\ V = 63100 \times 1.122^{30}$$
$$= £\,1\,994\,000$$

[2 marks]

Answer 8

(a) $N = a T^b$

Take the log to the base 10 of both sides

$$log_{10} N = log_{10} \left(a T^b \right)$$

$$log_{10} N = log_{10} a + log_{10} T^b$$

$$log_{10} N = b \, log_{10} T + log_{10} a$$

which is in the form requested with $m = b$ and $c = log_{10} a$

[2 marks]

(b) From the graph, the y-axis intercept is 1.8

$$log_{10} a = 1.8$$

$$a = 10^{1.8}$$

$$= 63.1$$

From the graph, picking two points on the line, far apart,

$$(0,\ 1.8) \text{ and } (1.2,\ 4.5) \Rightarrow m = \frac{4.5 - 1.8}{1.2 - 0} = \frac{9}{4} = 2.25 \ (= b)$$

$$N = a T^b$$

$$= 63.1 \times 3^{2.25}$$

$$= 747 \text{ microbes}$$

[4 marks]

(c) From the graph, the largest value looked at was,

$$log_{10} N = 4.5$$

$$N = 10^{4.5}$$

$$= 31600 \text{ microbes}$$

As 1000000 microbes is well outside of the known range of data, no reliable estimate can be made.

[2 marks]

(d) The constant a is the number of microbes, when $T = 1$.
That is, about 63 microbes, one day after the experiment began.

[1 mark]

Answer 9

(a) Let $t = 0$ in the equation $P = 160\,e^{-0.063t}$ to get $P_0 = 160$ mg/m^2

[1 mark]

(b) Let $P = 80$,

$$80 = 160\,e^{-0.063t}$$

Divide both sides by 160

$$0.5 = e^{-0.063t}$$

Take the natural log of both sides

$$ln\,0.5 = -0.063\,t$$

$$t = \frac{ln\,0.5}{-0.063}$$

$$= 11 \text{ days}$$

[3 marks]

(c)

$$36 = a\,e^{-0.063\times 20}$$

$$a = \frac{36}{e^{-1.26}}$$

$$= 127 \text{ mg/m}^2$$

[3 marks]

Answer 10

(i) $a = 2x$

[1 mark]

(ii)

$$9^x - 2 \times 3^{x+1} + 8 = 0$$

$$\left(3^x\right)^2 - 6\left(3^x\right) + 8 = 0$$

Let $w = 3^x$

$$w^2 - 6w + 8 = 0$$

$$(w - 2)(w - 4) = 0$$

Either $w = 2$ or $w = 4$

$3^x = 2$ $\qquad$ $3^x = 4$

$x = \frac{ln\,2}{ln\,3}$ $\qquad$ $x = \frac{ln\,4}{ln\,3}$

$= 0.631$ $\qquad$ $= 1.262$

[4 marks]