13.5 Answers

13.5.1 Answer to Exercise 13.4

Answer 1

$$3^x(3^x + 4) = 5$$

$$(3^x)^2 + 4(3^x) - 5 = 0$$

Let $z = 3^x$ in which case,

$$z^2 + 4z - 5 = 0$$

$$(z - 1)(z + 5) = 0$$

Either $z = 1$ or $z = -5$

That is, either $3^x = 1$ giving $x = 0$

or $3^x = -5$ which has no solutions.

[4 marks]

Answer 2

$$3^x = 65$$

Take the natural log of both sides

$$ln\, 3^x = ln\, 65$$

$$x\, ln\, 3 = ln\, 65$$

$$x = \frac{ln\, 65}{ln\, 3}$$

$$= 3.80 \text{ (Must be to 3 sf)}$$

[3 marks]

Answer 3

$$7^{3x} \times 7^{2x+3} = 43$$

$$7^{5x+3} = 43$$

Take the natural log of both sides

$$ln\, 7^{5x+3} = ln\, 43$$

$$5x + 3 = \frac{ln\, 43}{ln\, 7}$$

$$5x + 3 = 1.93287$$

$$5x = -1.06713$$

$$x = -0.213$$

[4 marks]

Answer 4

$$\log_2(5x+3) - \log_2 x = \log_2 12$$

$$\log_2(5x+3) = \log_2 12 + \log_2 x$$

$$\log_2\left(\frac{5x+3}{x}\right) = \log_2 12$$

$$5x + 3 = 12x$$

$$7x = 3$$

$$x = \frac{3}{7}$$

[3 marks]

Answer 5

$$2\log_7(x-4) - \log_7 x = \log_7 2$$

$$\log_7(x-4)^2 = \log_7 2 + \log_7 x$$

$$\log_7(x-4)^2 = \log_7 2x$$

$$(x-4)^2 = 2x$$

$$x^2 - 8x + 16 = 2x$$

$$x^2 - 10x + 16 = 0$$

$$(x-2)(x-8) = 0$$

Either $x = 2$ or $x = 8$

However $x = 2$ is an extraneous solution !

The only solution is $x = 8$

[5 marks]

Answer 6

$$5^{2x} - 13(5^x) + 22 = 0$$

Let $z = 5^x$ in which case,

$$z^2 - 13z + 22 = 0$$

$$(z-2)(z-11) = 0$$

Either $z = 2$ or $z = 11$

$$5^x = 2 \qquad 5^x = 11$$

$$x = \frac{\ln 2}{\ln 5} \qquad x = \frac{\ln 11}{\ln 5}$$

$$x = 0.431 \text{ or } x = 1.490$$

[5 marks]

Answer 7

$$\log_2(3x+1) + \log_2(2x) = 3$$

$$\log_2\left((3x+1)(2x)\right) = 3$$

$$\log_2\left(6x^2 + 2x\right) = 3$$

$$2^3 = 6x^2 + 2x \qquad \text{Jump out of logs rule}$$

$$6x^2 + 2x - 8 = 0$$

Divide through by 2

$$3x^2 + x - 4 = 0$$

$$(3x+4)(x-1) = 0$$

$$\text{Either } x = -\frac{4}{3} \text{ or } x = 1$$

However $x = -\frac{4}{3}$ is an extraneous solution !

The only solution is $x = 1$

[6 marks]

Answer 8

(i)

$$9^x = 3^a$$

$$\left(3^2\right)^x = 3^a$$

$$a = 2x$$

[1 mark]

(ii)

$$9^x - 2 \times 3^{x+1} + 8 = 0$$

$$\left(3^x\right)^2 - 2\left(3^x\right)3 + 8 = 0$$

Let $z = 3^x$ in which case,

$$z^2 - 6z + 8 = 0$$

$$(z-4)(z-2) = 0$$

$$\text{Either } z = 4 \text{ or } z = 2$$

$$3^x = 4 \qquad 3^x = 2$$

$$x = \frac{\ln 4}{\ln 3} \qquad x = \frac{\ln 2}{\ln 3}$$

$$x = 1.262 \text{ or } x = 0.631$$

[5 marks]