2.4 Answers

2.4.1 Solutions to 2.4 Exercise

Answer 1

Statement: $log_2 3 + 2\,log_2 5 = log_2 75$

Proof

$$\begin{aligned} \text{LHS} &= log_2 3 + 2\,log_2 5 \\ &= log_2 3 + log_2 5^2 \\ &= log_2 \left(3 \times 5^2\right) \\ &= log_2 75 \\ &= \text{RHS} \qquad \square \end{aligned}$$

[3 marks]

Answer 2

Statement: $log_7 45 - 2\,log_7 3 = log_7 5$

Proof

$$\begin{aligned} \text{LHS} &= log_7 45 - 2\,log_7 3 \\ &= log_7 45 - log_7 3^2 \\ &= log_7 \left(\frac{45}{3^2} \right) \\ &= log_7 5 \\ &= \text{RHS} \qquad \square \end{aligned}$$

[3 marks]

Answer 3

Statement: $3\,log_5 3 - 2\,log_5 6 + 2\,log_5 2 = log_5 3$

Proof

$$\begin{aligned} \text{LHS} &= 3\,log_5 3 - 2\,log_5 6 + 2\,log_5 2 \\ &= log_5 3^3 - log_5 6^2 + log_5 2^2 \\ &= log_5 \left(\frac{3^3 \times 2^2}{6^2} \right) \\ &= log_5 3 \\ &= \text{RHS} \qquad \square \end{aligned}$$

[4 marks]

Answer 4

(i) 2

[1 mark]

(ii)
$$\begin{aligned} 2\log_a 3 + \log_a 11 &= \log_a 3^2 + \log_a 11 \\ &= \log_a\left(3^2 \times 11\right) \\ &= \log_a 99 \end{aligned}$$

[3 marks]

Answer 5

$$\begin{aligned} \log_3 a + \log_3 b &= 2 \\ \log_3(3b) + \log_3 b &= 2 \qquad \text{as } a = 3b \\ \log_3\left(3b \times b\right) &= 2 \\ \log_3\left(3b^2\right) &= 2 \\ 3^2 &= 3b^2 \qquad \text{Jump out of } \textit{logs} \text{ rule} \\ 3b^2 &= 9 \\ b^2 &= 3 \\ b &= \pm\sqrt{3} \\ b &= \sqrt{3} \qquad \text{as told } b \text{ is positive} \end{aligned}$$

$\therefore\ a = 3\sqrt{3}$ and $b = \sqrt{3}$

[6 marks]

Answer 6

(i) Statement: $3\log_3 x - \log_3 7x + \log_3 28 = 2\log_3 2x$

Proof

$$\begin{aligned}
\text{LHS} &= 3\log_3 x - \log_3 7x + \log_3 28 \\
&= \log_3 x^3 - \log_3(7x) + \log_3 28 \\
&= \log_3\left(\frac{28x^3}{7x}\right) \\
&= \log_3\left(4x^2\right) \\
&= \log_3(2x)^2 \\
&= 2\log_3 2x \\
&= \text{RHS} \qquad \square
\end{aligned}$$

[3 marks]

(ii) Statement: $3\log_2 x - \log_2 7x + \log_2 28 = 2(1 + \log_2 x)$

Proof

$$\begin{aligned}
\text{LHS} &= 3\log_2 x - \log_2 7x + \log_2 28 \\
&= 2\log_2 2x \qquad \text{from part (i)} \\
&= 2\left(\log_2 2x\right) \\
&= 2\left(\log_2 2 + \log_2 x\right) \\
&= 2\left(1 + \log_2 x\right) \\
&= \text{RHS} \qquad \square
\end{aligned}$$

[2 marks]