## 3.2 Answers



### 3.2.1 Solutions ( 3.1 Exercise )

**Answer 1**

**( i )** 3 **( ii )** $-3$
**( iii )** 0.5 **( iv )** 0
**( v )** 6 **( vi )** $-\frac{2}{3}$

**[ 6 marks ]**

**Answer 2**

**( i )** $\frac{1}{25}$ **( ii )** 4
**( iii )** 1 **( iv )** $\frac{1}{27}$
**( v )** 36 **( vi )** 100

**[ 6 marks ]**

**Answer 3**

**( i )** 3 **( ii )** $-4$
**( iii )** $\frac{3}{2}$ **( iv )** 1
**( v )** 0 **( vi )** $\frac{1}{2}$

**[ 6 marks ]**

**Answer 4**

**( i )** 2 **( ii )** 49
**( iii )** 9 **( iv )** 10
**( v )** 125 **( vi )** 3

**[ 6 marks ]**

**Answer 5**

**( i )** 32 **( ii )** $\frac{1}{5}$
**( iii )** 32 **( iv )** $\frac{1}{27}$
**( v )** $\frac{1}{16}$ **( vi )** $\frac{1}{100}$

**[ 6 marks ]**

**Answer 6**

**( i )** $3\,log_x 8 - 2\,log_x 4 = log_x 32$

METHOD 1 :

$$
\begin{aligned}
\text{LHS} &= 3\,log_x 8 - 2\,log_x 4 \\
&= log_x 8^3 - log_x 4^2 \\
&= log_x\left(\frac{8^3}{4^2}\right) \\
&= log_x\left(\frac{2^9}{2^4}\right) \\
&= log_x 2^5 \\
&= log_x 32 \\
&= \text{RHS} \qquad \square
\end{aligned}
$$

METHOD 2 :

$$
\begin{aligned}
\text{LHS} &= 3\,log_x 8 - 2\,log_x 4 \\
&= 3\,log_x 2^3 - 2\,log_x 2^2 \\
&= 9\,log_x 2 - 4\,log_x 2 \\
&= 5\,log_x 2 \\
&= log_x 2^5 \\
&= log_x 32 \\
&= \text{RHS} \qquad \square
\end{aligned}
$$

**[ 3 marks ]**

**( ii )** $3\,log_2 8 - 2\,log_2 4 = log_2 32$ using the part (i) result

$$\Leftrightarrow\ 2^x = 32$$

$$\therefore\ x = 5$$

**[ 2 marks ]**

**Answer 7**

$2\,log_7 2 + 3\,log_7 5 = log_7 500$

$$
\begin{aligned}
\text{LHS} &= 2\,log_7 2 + 3\,log_7 5 \\
&= log_7 2^2 + log_7 5^3 \\
&= log_7 (4 \times 125) \\
&= log_7 500 \\
&= \text{RHS} \qquad \square
\end{aligned}
$$

**[ 4 marks ]**

**Answer 8**

Substitute $a = 8b$ into $log_2 a + 2\, log_2 b = 4$

$$\therefore \; log_2 (8b) + 2\, log_2 b = 4$$

$$log_2 8 + log_2 b + 2\, log_2 b = 4$$

$$3 + 3\, log_2 b = 4$$

$$log_2 b = \frac{1}{3}$$

$$\Leftrightarrow \; 2^{\frac{1}{3}} = b$$

That is, $b = \sqrt[3]{2}$ and $a = 8\sqrt[3]{2}$

**[ 6 marks ]**

**Answer 9**

**( a )** Whilst it is true that $log\, A + log\, B = log(AB)$,

it does not follow that $2\, log\, A + log\, B = 2\, log(AB)$ which is the incorrect deduction made by the student in the first step in their solution.

Whilst it is true that $log_a b = c \Leftrightarrow a^c = b$ the student has become muddled and claimed incorrectly that $log_a b = c \Leftrightarrow c^a = b$ in the last step in their solution.

**( b )**

$$2\, log_2 x - log_2 \sqrt{x} = 3$$

$$log_2 x^2 - log_2 \sqrt{x} = 3$$

$$log_2 \left( \frac{x^2}{x^{\frac{1}{2}}} \right) = 3$$

$$log_2 x^{\frac{3}{2}} = 3$$

$$\frac{3}{2}\, log_2 x = 3$$

$$log_2 x = 2$$

$$x = 2^2 = 4$$

**[ 3 marks ]**

**Answer 10**

**( i )** 0.5

**[ 1 mark ]**

**( ii )**

$$3\, log_a 2 + log_a 13 = log_a 2^3 + log_a 13$$

$$= log_a (8 \times 13)$$

$$= log_a 104$$

**[ 3 marks ]**

**Answer 11**

$$f(x) = x^3 + ax^2 - ax + 48$$

**( a )** **( i )**

$$f(-6) = 0$$

$$(-6)^3 + a(-6)^2 - a(-6) + 48 = 0$$

$$-216 + 36a + 6a + 48 = 0$$

$$42a = 168$$

$$a = 4 \qquad \square$$

**[ 2 marks ]**

**( ii )**

$$\begin{array}{r|rrrrrrr}
 & & & x^2 & - & 2x & + & 8 \\ \cline{2-8}
x + 6 & x^3 & + & 4x^2 & - & 4x & + & 48 \\
 & x^3 & + & 6x^2 & & & & \\ \cline{2-4}
 & & - & 2x^2 & - & 4x & & \\
 & & - & 2x^2 & - & 12x & & \\ \cline{3-6}
 & & & & & 8x & + & 48 \\
 & & & & & 8x & + & 48 \\ \cline{6-8}
 & & & & & & & 0
\end{array} \quad \leftarrow \text{ As expected !}$$

$$x^3 + 4x^2 - 4x + 48 = (x + 6)(x^2 - 2x + 8)$$

**[ 2 marks ]**

**( b )**

$$2\,log_2(x + 2) + log_2 x - log_2(x - 6) = 3$$

$$log_2(x + 2)^2 + log_2 x - log_2(x - 6) = 3$$

$$log_2\left(\frac{(x + 2)^2 x}{(x - 6)}\right) = 3$$

$$\frac{(x + 2)^2 x}{x - 6} = 2^3$$

$$x(x + 2)^2 = 8(x - 6)$$

$$x^3 + 4x^2 + 4x - 8x + 48 = 0$$

$$x^3 + 4x^2 - 4x + 48 = 0 \qquad \square$$

**[ 4 marks ]**

**( c )** From parts (a) and (b),

$$2\,log_2\,(\,x + 2\,) + log_2\,x - log_2\,(\,x - 6\,) = 3$$

$$\Leftrightarrow\; x^3 + 4\,x^2 - 4x + 48 = 0$$

$$\Leftrightarrow\; (\,x + 6\,)\left(x^2 - 2x + 8\right) = 0$$

As the discriminant of $x^2 - 2x + 8$ is negative...

$$\ldots\; (-2)^2 - 4 \times 1 \times 8 = -28 \ldots$$

... the only possible root is $x = -6$ but this is immediately rejected because the logarithms of negative numbers do not exist.

That is, $2\,log_2\,(\,x + 2\,) + log_2\,x - log_2\,(\,x - 6\,) = 3$ has no real roots.

$\square$

**[ 2 marks ]**

**Answer 12**

$$5\,log_3\,x - 2\,log_3\,3x + log_3\,72 = 3\,log_3\,2x$$

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**( i )**

$$\begin{aligned}
\text{LHS} &= 5\,log_3\,x - 2\,log_3\,3x + log_3\,72 \\
&= log_3\,x^5 - log_3\,(3x)^2 + log_3\,(\,9 \times 8\,) \\
&= log_3\left(\frac{x^5 \times 9 \times 8}{3^2\,x^2}\right) \\
&= log_3\left(8x^3\right) \\
&= log_3\,(2x)^3 \\
&= 3\,log_3\,2x \\
&= \text{RHS} \qquad \square
\end{aligned}$$

**[ 4 marks ]**

**( ii )** From part (i),

$$\begin{aligned}
5\,log_2\,x - 2\,log_2\,3x + log_2\,72 &= 3\,log_2\,2x \\
&= 3\,\left(\,log_2\,2 + log_2\,x\,\right) \\
&= 3\,\left(\,1 + log_2\,x\,\right)
\end{aligned}$$

**[ 2 marks ]**