4.4 Answers

4.4.1 Solutions to 4.3 Exercise

Answer 1

$$\begin{aligned} 2^{x+5} &= 3^x \\ ln\, 2^{x+5} &= ln\, 3^x \\ (x+5)\, ln\, 2 &= x\, ln\, 3 \\ x\, ln\, 2 + 5\, ln\, 2 &= x\, ln\, 3 \\ x\, ln\, 3 - x\, ln\, 2 &= 5\, ln\, 2 \\ x(ln\, 3 - ln\, 2) &= 5\, ln\, 2 \\ x &= \frac{5\, ln\, 2}{ln\, 3 - ln\, 2} \\ x &= 8.547 \end{aligned}$$

[4 marks]

Answer 2

$$\begin{aligned} 3^{4x} \times 3^{5-x} &= 2^x \\ 3^{3x+5} &= 2^x \\ ln\, 3^{3x+5} &= ln\, 2^x \\ (3x+5)\, ln\, 3 &= x\, ln\, 2 \\ 3x\, ln\, 3 + 5\, ln\, 3 &= x\, ln\, 2 \\ x\, ln\, 2 - 3x\, ln\, 3 &= 5\, ln\, 3 \\ x(ln\, 2 - 3\, ln\, 3) &= 5\, ln\, 3 \\ x &= \frac{5\, ln\, 3}{ln\, 2 - 3\, ln\, 3} \\ x &= -2.11 \end{aligned}$$

[4 marks]

Answer 3

$$\frac{1}{3^{2x-1}} = 4^x$$
$$3^{1-2x} = 4^x$$
$$ln\, 3^{1-2x} = ln\, 4^x$$
$$(1 - 2x)\, ln\, 3 = x\, ln\, 4$$
$$ln\, 3 - 2x\, ln\, 3 = x\, ln\, 4$$
$$x\, ln\, 4 + 2x\, ln\, 3 = ln\, 3$$
$$x(ln\, 4 + 2\, ln\, 3) = ln\, 3$$
$$x = \frac{ln\, 3}{ln\, 4 + 2\, ln\, 3}$$
$$x = 0.307$$

[4 marks]

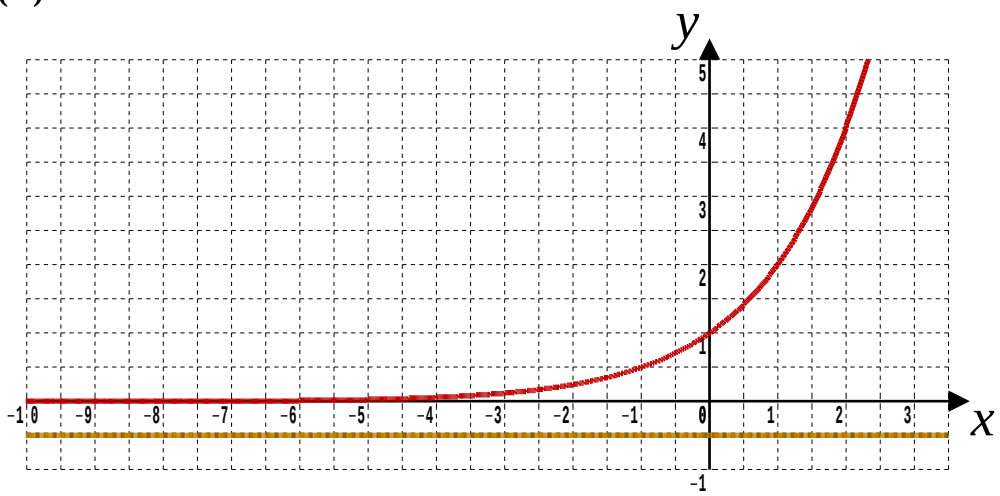
Answer 4

$$3^x - 5 = 0$$
$$3^x = 5$$
$$ln\, 3^x = ln\, 5$$
$$x\, ln\, 3 = ln\, 5$$
$$x = \frac{ln\, 5}{ln\, 3}$$
$$= 1.465$$

[3 marks]

Answer 5

(i)



Red: $y = 2^x$ Gold: $y = -0.5$

(ii) As the two graphs never intersect the equation $2^x = -0.5$ has no solutions.

[2, 2 marks]

Answer 6

$$\begin{aligned} 5 \times 3^x &= 4^{3x} \\ ln\left(5 \times 3^x\right) &= ln\, 4^{3x} \\ ln\, 5 + ln\, 3^x &= ln\, 4^{3x} \\ ln\, 5 + x\, ln\, 3 &= 3x\, ln\, 4 \\ 3x\, ln\, 4 - x\, ln\, 3 &= ln\, 5 \\ x\left(3\, ln\, 4 - ln\, 3\right) &= ln\, 5 \\ x &= \frac{ln\, 5}{3\, ln\, 4 - ln\, 3} \\ &= 0.526 \end{aligned}$$

[4 marks]

Answer 7

$$\begin{aligned} 3 \times 5^{x+2} &= 8^x \\ ln\left(3 \times 5^{x+2}\right) &= ln\, 8^x \\ ln\, 3 + ln\, 5^{x+2} &= ln\, 8^x \\ ln\, 3 + (x+2)\, ln\, 5 &= x\, ln\, 8 \\ ln\, 3 + x\, ln\, 5 + 2\, ln\, 5 &= x\, ln\, 8 \\ x\, ln\, 8 - x\, ln\, 5 &= ln\, 3 + 2\, ln\, 5 \\ x(\, ln\, 8 - ln\, 5) &= ln\, 3 + 2\, ln\, 5 \\ x &= \frac{ln\, 3 + 2\, ln\, 5}{ln\, 8 - ln\, 5} \\ &= 9.186 \end{aligned}$$

[4 marks]

Answer 8

(i) $log_2 x = 6 \Leftrightarrow x = 2^6 \Leftrightarrow x = 64$

[1 mark]

(ii) $log_5 x = 0.5 \Leftrightarrow x = 5^{0.5} \Leftrightarrow x = 2.236$

[1 mark]

(iii) $log_2 x = -4 \Leftrightarrow x = 2^{-4} \Leftrightarrow x = \frac{1}{16}$

[1 mark]

Answer 9

$$\begin{aligned} 3^{2x+5} &= 4^{x-2} \\ \ln 3^{2x+5} &= \ln 4^{x-2} \\ (2x+5)\ln 3 &= (x-2)\ln 4 \\ 2x\ln 3 + 5\ln 3 &= x\ln 4 - 2\ln 4 \\ x\ln 4 - 2x\ln 3 &= 5\ln 3 + 2\ln 4 \\ x(\ln 4 - 2\ln 3) &= 5\ln 3 + 2\ln 4 \\ x &= \frac{5\ln 3 + 2\ln 4}{\ln 4 - 2\ln 3} \\ &= -10.193 \end{aligned}$$

[5 marks]

Answer 10

$$\begin{aligned} 4 \times 3^{2x+1} &= 5^x \\ \ln\left(4 \times 3^{2x+1}\right) &= \ln 5^x \\ \ln 4 + \ln 3^{2x+1} &= \ln 5^x \\ \ln 4 + (2x+1)\ln 3 &= x\ln 5 \\ \ln 4 + 2x\ln 3 + \ln 3 &= x\ln 5 \\ x\ln 5 - 2x\ln 3 &= \ln 4 + \ln 3 \\ x(\ln 5 - 2\ln 3) &= \ln 4 + \ln 3 \\ x &= \frac{\ln 4 + \ln 3}{\ln 5 - 2\ln 3} \\ &= -4.228 \end{aligned}$$

[5 marks]