

4.6 Answers

A-Level Pure Mathematics : Year 1 Exponentials and Logarithms

4.6.1 Solutions to 4.5 Exercise

Answer 1

$$\begin{aligned}3^{x-7} &= 2^x \\ \ln 3^{x-7} &= \ln 2^x \\ (x-7) \ln 3 &= x \ln 2 \\ x \ln 3 - 7 \ln 3 &= x \ln 2 \\ x \ln 3 - x \ln 2 &= 7 \ln 3 \\ x (\ln 3 - \ln 2) &= 7 \ln 3 \\ x &= \frac{7 \ln 3}{\ln 3 - \ln 2} \\ &= 18.967\end{aligned}$$

[4 marks]

Answer 2

$$\begin{aligned}2^{5x} \times 2^{2x-3} &= 3^x \\ 2^{7x-3} &= 3^x \\ \ln 2^{7x-3} &= \ln 3^x \\ (7x-3) \ln 2 &= x \ln 3 \\ 7x \ln 2 - 3 \ln 2 &= x \ln 3 \\ 7x \ln 2 - x \ln 3 &= 3 \ln 2 \\ x (7 \ln 2 - \ln 3) &= 3 \ln 2 \\ x &= \frac{3 \ln 2}{7 \ln 2 - \ln 3} \\ &= 0.554\end{aligned}$$

[4 marks]

Answer 3

$$\begin{aligned}\frac{1}{5^{3x-1}} &= 7^x \quad \Leftrightarrow \quad 5^{1-3x} = 7^x \\ \ln 5^{1-3x} &= \ln 7^x \\ (1-3x) \ln 5 &= x \ln 7 \\ \ln 5 - 3x \ln 5 &= x \ln 7 \\ x \ln 7 + 3x \ln 5 &= \ln 5 \\ x(\ln 7 + 3 \ln 5) &= \ln 5 \\ x &= \frac{\ln 5}{\ln 7 + 3 \ln 5} \\ &= 0.238\end{aligned}$$

[4 marks]

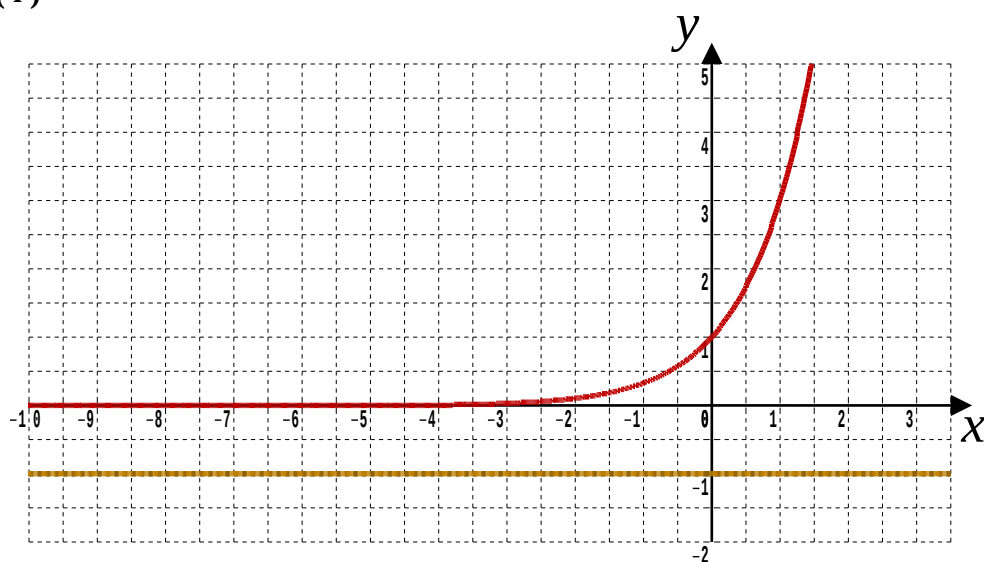
Answer 4

$$\begin{aligned}5^x - 3 &= 0 \quad \Leftrightarrow \quad 5^x = 3 \\ \ln 5^x &= \ln 3 \\ x \ln 5 &= \ln 3 \\ x &= \frac{\ln 3}{\ln 5} \\ &= 0.683\end{aligned}$$

[3 marks]

Answer 5

(i)



Red: $y = 3^x$ Gold: $y = -1$

(ii) As the two graphs never intersect the equation $3^x = -1$ has no solutions.

[2, 2 marks]

Answer 6

$$\begin{aligned}
4 \times 3^x &= 5^{3x} \\
\ln(4 \times 3^x) &= \ln 5^{3x} \\
\ln 4 + \ln 3^x &= \ln 5^{3x} \\
\ln 4 + x \ln 3 &= 3x \ln 5 \\
3x \ln 5 - x \ln 3 &= \ln 4 \\
x(3 \ln 5 - \ln 3) &= \ln 4 \\
x &= \frac{\ln 4}{3 \ln 5 - \ln 3} \\
&= 0.372
\end{aligned}$$

[4 marks]**Answer 7**

$$\begin{aligned}
3 \times 7^{x+2} &= 4^x \\
\ln(3 \times 7^{x+2}) &= \ln 4^x \\
\ln 3 + \ln 7^{x+2} &= \ln 4^x \\
\ln 3 + (x + 2) \ln 7 &= x \ln 4 \\
\ln 3 + x \ln 7 + 2 \ln 7 &= x \ln 4 \\
x \ln 4 - x \ln 7 &= \ln 3 + 2 \ln 7 \\
x(\ln 4 - \ln 7) &= \ln 3 + 2 \ln 7 \\
x &= \frac{\ln 3 + 2 \ln 7}{\ln 4 - \ln 7} \\
&= -8.918
\end{aligned}$$

[4 marks]**Answer 8**

(i) $\log_3 x = 4 \Leftrightarrow x = 3^4 \Leftrightarrow x = 81$

[1 mark]

(ii) $\log_7 x = 0.25 \Leftrightarrow x = 7^{0.25} \Leftrightarrow x = 1.627$

[1 mark]

(iii) $\log_2 x = -5 \Leftrightarrow x = 2^{-5} \Leftrightarrow x = \frac{1}{32}$

[1 mark]

Answer 9

$$\begin{aligned}
5^{4x+7} &= 2^{x-3} \\
\ln 5^{4x+7} &= \ln 2^{x-3} \\
(4x+7) \ln 5 &= (x-3) \ln 2 \\
4x \ln 5 + 7 \ln 5 &= x \ln 2 - 3 \ln 2 \\
x \ln 2 - 4x \ln 5 &= 7 \ln 5 + 3 \ln 2 \\
x(\ln 2 - 4 \ln 5) &= 7 \ln 5 + 3 \ln 2 \\
x &= \frac{7 \ln 5 + 3 \ln 2}{\ln 2 - 4 \ln 5} \\
&= -2.323
\end{aligned}$$

[5 marks]**Answer 10**

$$\begin{aligned}
6 \times 5^{2x+1} &= 7^x \\
\ln(6 \times 5^{2x+1}) &= \ln 7^x \\
\ln 6 + \ln 5^{2x+1} &= \ln 7^x \\
\ln 6 + (2x+1) \ln 5 &= x \ln 7 \\
\ln 6 + 2x \ln 5 + \ln 5 &= x \ln 7 \\
x \ln 7 - 2x \ln 5 &= \ln 6 + \ln 5 \\
x(\ln 7 - 2 \ln 5) &= \ln 6 + \ln 5 \\
x &= \frac{\ln 6 + \ln 5}{\ln 7 - 2 \ln 5} \\
&= -2.672
\end{aligned}$$

[5 marks]