

5.3 Answers

A-Level Pure Mathematics : Year 1 Exponentials and Logarithms

Answer 1

$$\begin{aligned}2^x &= 3 \\ \ln 2^x &= \ln 3 \\ x \ln 2 &= \ln 3 \\ x &= \frac{\ln 3}{\ln 2} \\ &= 1.585\end{aligned}$$

[2 marks]

Answer 2

(a)

$$\begin{aligned}5^x &= 7 \\ \ln 5^x &= \ln 7 \\ x \ln 5 &= \ln 7 \\ x &= \frac{\ln 7}{\ln 5} \\ &= 1.21\end{aligned}$$

[2 marks]

(b)

$$\begin{aligned}5^{2x} - 12(5^x) + 35 &= 0 \\ (5^x)^2 - 12(5^x) + 35 &= 0 \\ z^2 - 12z + 35 &= 0 && \text{by letting } z = 5^x \\ (z - 5)(z - 7) &= 0 \\ \therefore \text{ Either } z &= 5 && \text{or } z = 7 \\ 5^x &= 5 && \text{or } 5^x = 7 \\ x &= 1 && \text{or } \ln 5^x = \ln 7 \\ &&& x = 1.21 \text{ (from part(a))}\end{aligned}$$

[3 marks]

Answer 3**(a)**

$$25^x - 5^{x+1} + 4 = 0$$

$$(5^2)^x - 5^1 5^x + 4 = 0$$

$$(5^x)^2 - 5(5^x) + 4 = 0$$

$$z^2 - 5z + 4 = 0$$

$$(z - 4)(z - 1) = 0$$

$$(5^x - 4)(5^x - 1) = 0$$

□

[3 marks]**(b)**

$$\text{Either } 5^x = 4 \quad \text{or } 5^x = 1$$

$$\ln 5^x = \ln 4 \quad \text{or } x = 0$$

$$x \ln 5 = \ln 4$$

$$x = \frac{\ln 4}{\ln 5}$$

$$x = 0.861$$

[2 marks]**Answer 4**

$$4^{2x+2} = 8^{x-5}$$

$$(2^2)^{2x+2} = (2^3)^{x-5}$$

$$2^{4x+4} = 2^{3x-15}$$

$$4x + 4 = 3x - 15$$

$$x = -19$$

[3 marks]**Answer 5**

$$9^x = 27^{2-x}$$

$$(3^2)^x = (3^3)^{2-x}$$

$$3^{2x} = 3^{6-3x}$$

$$2x = 6 - 3x$$

$$5x = 6$$

$$x = \frac{6}{5}$$

[3 marks]

Answer 6

$$\begin{aligned}
9^x - 3^{x+2} + 20 &= 0 \\
(3^2)^x - 3^x 3^2 + 20 &= 0 \\
(3^x)^2 - 9(3^x) + 20 &= 0 \\
z^2 - 9z + 20 &= 0 \quad \text{by letting } z = 3^x \\
(z - 4)(z - 5) &= 0 \\
\therefore \text{ Either } z = 4 &\quad \text{or } z = 5 \\
3^x = 4 &\quad \text{or } 3^x = 5 \\
\ln 3^x = \ln 4 &\quad \ln 3^x = \ln 5 \\
x \ln 3 = \ln 4 &\quad x \ln 3 = \ln 5 \\
x = \frac{\ln 4}{\ln 3} &\quad x = \frac{\ln 5}{\ln 3} \\
x = 1.262 &\quad \text{or } x = 1.465
\end{aligned}$$

[5 marks]**Answer 7**

$$\begin{aligned}
3^x \times 3^{2x+1} &= 9^x \\
3^{3x+1} &= (3^2)^x \\
3^{3x+1} &= 3^{2x} \\
3x + 1 &= 2x \\
x &= -1
\end{aligned}$$

[3 marks]**Answer 8**

$$\begin{aligned}
16^x \times 8^{4x+3} &= 4^x \\
(2^4)^x \times (2^3)^{4x+3} &= (2^2)^x \\
2^{4x} \times 2^{12x+9} &= 2^{2x} \\
2^{16x+9} &= 2^{2x} \\
16x + 9 &= 2x \\
14x &= -9 \\
x &= -\frac{9}{14}
\end{aligned}$$

[3 marks]

Answer 9

$$\begin{aligned}5^x &= 17 \\ \ln 5^x &= \ln 17 \\ x \ln 5 &= \ln 17 \\ x &= \frac{\ln 17}{\ln 5} \\ &= 1.76\end{aligned}$$

[2 marks]

Answer 10

$$\begin{aligned}9^x - 2 \times 3^{x+1} + 8 &= 0 \\ (3^2)^x - 2 \times 3^x 3^1 + 8 &= 0 \\ (3^x)^2 - 6(3^x) + 8 &= 0 \\ z^2 - 6z + 8 &= 0 \quad \text{by letting } z = 3^x \\ (z - 2)(z - 4) &= 0 \\ \therefore \text{ Either } z = 2 &\quad \text{or} \quad z = 4 \\ 3^x = 2 &\quad \text{or} \quad 3^x = 4 \\ \ln 3^x = \ln 2 &\quad \ln 3^x = \ln 4 \\ x \ln 3 = \ln 2 &\quad x \ln 3 = \ln 4 \\ x = \frac{\ln 2}{\ln 3} &\quad x = \frac{\ln 4}{\ln 3} \\ x = 0.631 &\quad \text{or} \quad x = 1.262\end{aligned}$$

[5 marks]

Answer 11

$$\begin{aligned}4^x - 6(2^x) + 5 &= 0 \\ (2^2)^x - 6(2^x) + 5 &= 0 \\ (2^x)^2 - 6(2^x) + 5 &= 0 \\ z^2 - 6z + 5 &= 0 \quad \text{by letting } z = 2^x \\ (z - 1)(z - 5) &= 0 \\ \therefore \text{ Either } z = 1 &\quad \text{or} \quad z = 5 \\ 2^x = 1 &\quad \text{or} \quad 2^x = 5 \\ x = 0 &\quad x = \frac{\ln 5}{\ln 2} \\ &\quad x = 2.322\end{aligned}$$

[5 marks]

Answer 12

$$(a) \quad 4^x = (2^x)^2 = y^2$$

[1 mark]

$$(b) \quad 8(4^x) - 9(2^x) + 1 = 0$$

$$8y^2 - 9y + 1 = 0 \quad \text{using the part (i) suggestion}$$

$$(8y - 1)(y - 1) = 0$$

$$\therefore \text{ Either } y = \frac{1}{8} \quad \text{or} \quad y = 1$$

$$2^x = \frac{1}{8} \quad \text{or} \quad 2^x = 1$$

$$x = -3 \quad \text{or} \quad x = 0$$

[4 marks]**Answer 13**

$$8^x - 2^{x+6} = 0$$

$$(2^3)^x - 2^x 2^6 = 0$$

$$(2^x)^3 - 64(2^x) = 0$$

$$2^x (2^{2x} - 64) = 0$$

Either $2^x = 0$ which has no solutions

or $2^{2x} = 64$ which gives $x = 3$

[4 marks]