6.4 Answers

6.4.1 Solution to 6.2 Example

$$\log_6(x+5) + \log_6(x-4) = 2$$
$$\log_6\big((x+5)(x-4)\big) = 2$$
$$(x+5)(x-4) = 6^2$$
$$x^2 + x - 20 = 36$$
$$x^2 + x - 56 = 0$$
$$(x-7)(x+8) = 0$$

Either $x = 7$ or $x = -8$

However, $x = -8$ is an extraneous solution

$\therefore$ The only valid solution is $x = 7$

[4 marks]

It can be seen that it was on the very first manipulation that the extraneous solution was generated for,

$$(x+5)(x-4)$$

with $x = -8$ becomes $(-3) \times (-12)$

and with $x = 7$ becomes $(12) \times (3)$

Only one of these products has terms that can have their logarithms taken.

6.4.2 Solutions to 6.3 Exercise

In these solutions when it is required to take a logarithm of both sides the natural logarithm, *ln*, has been used simply because on the Casio ClassWiz calculator this involves a single button press rather than two or three.

Answer 1

$$5^x = 17$$

Take the *ln* of both sides

$$\ln 5^x = \ln 17$$
$$x \ln 5 = \ln 17$$
$$x = \frac{\ln 17}{\ln 5}$$
$$= 1.76 \qquad \text{(Three significant figures asked for)}$$

[3 marks]

Answer 2

(a)

$$5^x = 8$$

Take the *ln* of both sides

$$ln\, 5^x = ln\, 8$$

$$x\, ln\, 5 = ln\, 8$$

$$x = \frac{ln\, 8}{ln\, 5}$$

$$= 1.29$$ (Three significant figures asked for)

[3 marks]

(b)

$$log_2 (x + 1) - log_2 x = log_2 7$$

$$log_2 \left(\frac{x + 1}{x} \right) = log_2 7$$

$$\therefore \frac{x + 1}{x} = 7$$

$$x + 1 = 7x$$

$$x = \frac{1}{6}$$ (Which is a valid solution)

[3 marks]

Answer 3

(a)

$$5^x = 10$$

Take the log_{10} of both sides

$$log_{10} 5^x = log_{10} 10$$

$$x\, log_{10} 5 = 1$$

$$x = \frac{1}{log_{10} 5}$$

$$= 1.43$$ (Three significant figures asked for)

[2 marks]

(b)

$$log_3 (x - 2) = -1$$

$$3^{-1} = x - 2$$

$$x = 2 + \frac{1}{3}$$

$$x = 2.33$$ (to three significant figures)

[2 marks]

Answer 4
(a)

$$3^x = 5$$

Take the ln of both sides

$$ln\, 3^x = ln\, 5$$

$$x\, ln\, 3 = ln\, 5$$

$$x = \frac{ln\, 5}{ln\, 3}$$

$$= 1.46$$ (Three significant figures asked for)

[3 marks]

(b)

$$log_2 (2x + 1) - log_2 x = 2$$

$$log_2 \left(\frac{2x + 1}{x} \right) = 2$$

$$2^2 = \frac{2x + 1}{x}$$

$$2x = 1$$

$$x = 0.5$$ (Which is a valid solution)

[2 marks]

Answer 5
(a)

$$8^x = 0.8$$

Take the ln of both sides

$$ln\, 8^x = ln\, 0.8$$

$$x\, ln\, 8 = ln\, 0.8$$

$$x = \frac{ln\, 0.8}{ln\, 8}$$

$$= -0.107$$ (Three significant figures asked for)

[2 marks]

(b)

$$2\, log_3 x - log_3 7x = 1$$

$$log_3 x^2 - log_3 7x = 1$$

$$log_3 \left(\frac{x^2}{7x} \right) = 1$$ (As $x = 0$ is clearly not a solution)

$$3^1 = \frac{x}{7}$$

$$x = 21$$

[4 marks]

Answer 6

(a)

$$\frac{x^2 + 4x + 3}{x^2 + x} = \frac{(x + 3)(x + 1)}{x(x + 1)}$$

$$= \frac{x + 3}{x}$$

[2 marks]

(b) $$log_2\left(x^2 + 4x + 3\right) - log_2\left(x^2 + x\right) = 4$$

$$log_2\left(\frac{x^2 + 4x + 3}{x^2 + x}\right) = 4$$

(As $x = 0$ or $x = -1$ are clearly not solutions)

$$log_2\left(\frac{x + 3}{x}\right) = 4$$

$$2^4 = \frac{x + 3}{x}$$

$$16x = x + 3$$

$x = 0.2$ (Which is a valid solution)

[4 marks]

Answer 7

(a) $$log_x 64 = 2$$

$$x^2 = 64$$

$\therefore\ x = 8$ (The base must be a positive real number not equal to 1)

[2 marks]

(b)

$$log_2(11 - 6x) = 2\,log_2(x - 1) + 3$$

$$log_2(11 - 6x) = log_2(x - 1)^2 + 3$$

$$log_2(11 - 6x) - log_2(x - 1)^2 = 3$$

$$log_2\left(\frac{11 - 6x}{(x - 1)^2}\right) = 3$$

$$2^3 = \left(\frac{11 - 6x}{(x - 1)^2}\right)$$

$$8\left(x^2 - 2x + 1\right) = 11 - 6x$$

$$8x^2 - 16x + 8 = 11 - 6x$$

$$8x^2 - 10x - 3 = 0$$

$$(4x + 1)(2x - 3) = 0$$

Either $x = -0.25$ or $x = 1.5$

Only $x = 1.5$ is a valid solution

[6 marks]

Answer 8

$$log_5 (4 - x) - 2 \, log_5 x = 1$$

$$log_5 (4 - x) - log_5 x^2 = 1$$

$$log_5 \left(\frac{4 - x}{x^2} \right) = 1$$

$$5^1 = \left(\frac{4 - x}{x^2} \right)$$

$$5 x^2 + x - 4 = 0$$

$$(5x - 4)(x + 1) = 0$$

$$\text{Either } x = \frac{4}{5} \text{ or } x = -1$$

$$\text{Only } x = \frac{4}{5} \text{ is a valid solution}$$

[6 marks]

Answer 9

(a)

$$5^x = 7$$

Take the *ln* of both sides

$$ln \, 5^x = ln \, 7$$

$$x \, ln \, 5 = ln \, 7$$

$$x = \frac{ln \, 7}{ln \, 5}$$

$$= 1.21$$ (Three significant figures asked for)

[2 marks]

(b)

$$5^{2x} - 12 (5^x) + 35 = 0$$

$$\left(5^x \right)^2 - 12 \left(5^x \right) + 35 = 0$$

$$\left(5^x - 5 \right)\left(5^x - 7 \right) = 0$$

$$\text{Either } 5^x = 5 \text{ or } 5^x = 7$$

$$x = 1 \text{ or } x = 1.21$$

[4 marks]

Answer 10

(a)

$$\begin{aligned} \text{LHS} &= \log_3 y \\ &= \log_3 \left(3x^2 \right) \\ &= \log_3 3 + \log_3 x^2 \\ &= 1 + 2\log_3 x \\ &= \text{RHS} \qquad \square \end{aligned}$$

[3 marks]

(b)

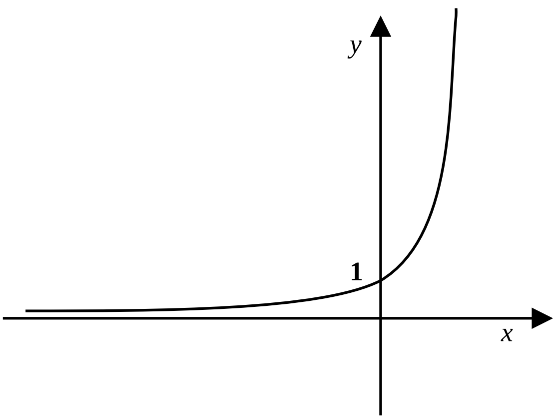
$$\begin{aligned} 1 + 2\log_3 x &= \log_3 (28x - 9) \\ \log_3 \left(3x^2 \right) &= \log_3 (28x - 9) \\ 3x^2 &= 28x - 9 \\ 3x^2 - 28x + 9 &= 0 \\ (3x - 1)(x - 9) &= 0 \end{aligned}$$

Either $x = \dfrac{1}{3}$ or $x = 9$ (Both valid solutions)

[3 marks]

Answer 11

(a)



[2 marks]

(b)

$$\begin{aligned} 7^{2x} - 4\left(7^x \right) + 3 &= 0 \\ \left(7^x \right)^2 - 4\left(7^x \right) + 3 &= 0 \\ \left(7^x - 3 \right)\left(7^x - 1 \right) &= 0 \end{aligned}$$

Either $7^x = 3$ or $7^x = 1$

$\ln 7^x = \ln 3$ or $x = 0$

$$x = \frac{\ln 3}{\ln 7}$$

$$x = 0.565$$

[6 marks]

Answer 12

(a)

$$2\,log_3(x-5) - log_3(2x-13) = 1$$

$$log_3(x-5)^2 - log_3(2x-13) = 1$$

$$log_3\left(\frac{(x-5)^2}{(2x-13)}\right) = 1$$

$$3^1 = \left(\frac{(x-5)^2}{(2x-13)}\right)$$

$$6x - 39 = x^2 - 10x + 25$$

$$x^2 - 16x + 64 = 0 \quad \square$$

[5 marks]

(b)

$$(x-8)^2 = 0$$

$$x - 8 = 0$$

$x = 8$ (Which is a valid solution)

[2 marks]

Answer 13

(a)

$$\begin{aligned} \text{LHS} &= 3 + 2\,log_2 x \\ &= 3 \times 1 + log_2 x^2 \\ &= 3 \times log_2 2 + log_2 x^2 \\ &= log_2 2^3 + log_2 x^2 \\ &= log_2 8 + log_2 x^2 \\ &= log_2 8x^2 \\ &= log_2 y \quad \text{where } y = 8x^2 \quad \square \end{aligned}$$

[3 marks]

(b)

$$3 + 2\,log_2 x = log_2(14x-3)$$

$$log_2 8x^2 = log_2(14x-3)$$

$$8x^2 = 14x - 3$$

$$8x^2 - 14x + 3 = 0$$

$$(4x-1)(2x-3) = 0$$

Either $x = 0.25$ or $x = 1.5$ (Both valid solutions)

$\therefore\ \alpha = 0.25$ and $\beta = 1.5$

[3 marks]

(c)

$$\begin{aligned}
\text{LHS} &= log_2\, \alpha \\
&= log_2\, 0.25 \\
&= log_2\, \frac{1}{4} \\
&= log_2\, \frac{1}{2^2} \\
&= log_2\, 2^{-2} \\
&= (-2)\ log_2\, 2 \\
&= (-2) \times 1 \\
&= -2 \\
&= \text{RHS} \qquad \square
\end{aligned}$$

[1 mark]

(d)

$$\begin{aligned}
\text{Let } x &= log_2\, \beta \\
x &= log_2\, 1.5 \\
\therefore\ 2^x &= 1.5
\end{aligned}$$

Take the *ln* of both sides

$$\begin{aligned}
ln\, 2^x &= ln\, 1.5 \\
x &= \frac{ln\, 1.5}{ln\, 2} \\
&= 0.585 \quad \text{(Three significant figures requested)}
\end{aligned}$$

[3 marks]

Answer 14

(a)

$$\begin{aligned} \log_2 y &= -3 \\ 2^{-3} &= y \\ y &= \frac{1}{8} \quad \text{(Which is a valid solution)} \end{aligned}$$

[2 marks]

(b)

$$\begin{aligned} \frac{\log_2 32 + \log_2 16}{\log_2 x} &= \log_2 x \\ \log_2 2^5 + \log_2 2^4 &= (\log_2 x)^2 \\ 9 \log_2 2 &= (\log_2 x)^2 \\ 9 \times 1 &= (\log_2 x)^2 \\ \log_2 x &= \pm 3 \\ \text{Either } x &= 2^3 \text{ or } x = 2^{-3} \\ &= 8 \quad \text{or} \quad = \frac{1}{8} \qquad \text{(Both valid solutions)} \end{aligned}$$

[5 marks]