

9.6 Answers

A-Level Pure Mathematics : Year 1 Exponentials and Logarithms

9.6.1 Solution to 9.4 Example

- (i) This is the coding to test for an exponential relationship of the form

$$y = a b^x$$

The coded curve tells us that this is NOT the form of a suitable equation with which to model the data.

[1 mark]

- (ii) This is the coding to test for a power function relationship of the form

$$y = a x^b$$

The coded straight line tells us that this is the form of a suitable equation with which to model the data.

[2 marks]

- (iii) Notice that the data has been coded using a *log* to the base 10

$$\begin{aligned} a &= 10^{1.2} & b &= 0.4 \\ &= 15.8 \\ \therefore y &= 15.8 \times x^{0.4} \end{aligned}$$

[3 marks]

9.6.2 Solution to 9.5 Exercise

Answer 1

- (i) Notice that the data has been coded using a *log* to the base 10

$$\begin{aligned} a &= 10^{-0.947} & b &= 2.84 \\ &= 0.113 \\ \therefore y &= 0.113 \times x^{2.84} \end{aligned}$$

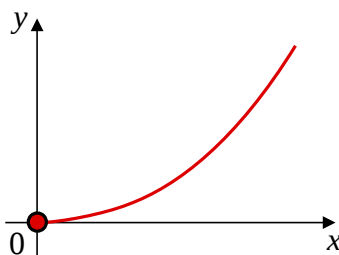
[2 marks]

- (ii)

<i>x</i> metres	2	4	6	8
<i>y</i> millimetres	0.81	5.79	18.32	41.5

[2 marks]

- (iii)



Like all power function curves, this passes through the origin.

[2 marks]

Answer 2

- (i) Notice that the data has been coded using a *log* to the base 10

$$a = 10^{0.3} \quad b = 0.5$$

$$= 2$$

$$\therefore T = 2\sqrt{L}$$

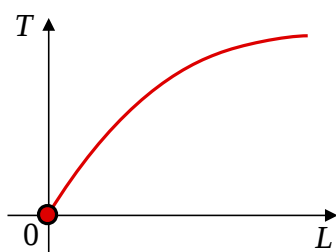
[2 marks]

- (ii)

<i>L</i> metres	0.1	0.2	0.5	1
<i>T</i> seconds	0.63	0.89	1.41	2

[2 marks]

- (iii)



Like all power function curves, this passes through the origin.

[2 marks]

Answer 3

- (i) $m = \frac{\Delta y}{\Delta x} = \frac{5.1 - 2.4}{6} = 0.45 \quad \therefore \text{Straight line is } Y = 2.4 + 0.45X$

[2 marks]

- (ii) The coding is for an exponential curve and not for a power function !

$$N = a b^t$$

$$a = 10^{2.4} \quad b = 10^{0.45}$$

$$= 251 \quad = 2.82$$

$$\therefore N = 251 \times 2.82^t$$

[3 marks]

- (iii) (a) 708 (b) 15 873 (c) 1 003 845

[3 marks]

- (iv) 8.075×10^{15}

After 30 days some people may have recovered so the disease may stop to spread as quickly as the immune proportion of the population increases. The give away is the mention of 'end points' in the question which reveals that the known data was gathered over 6 days and so 30 days is well beyond the data domain that gave rise to the exponential curve being used to model the disease.

[2 marks]

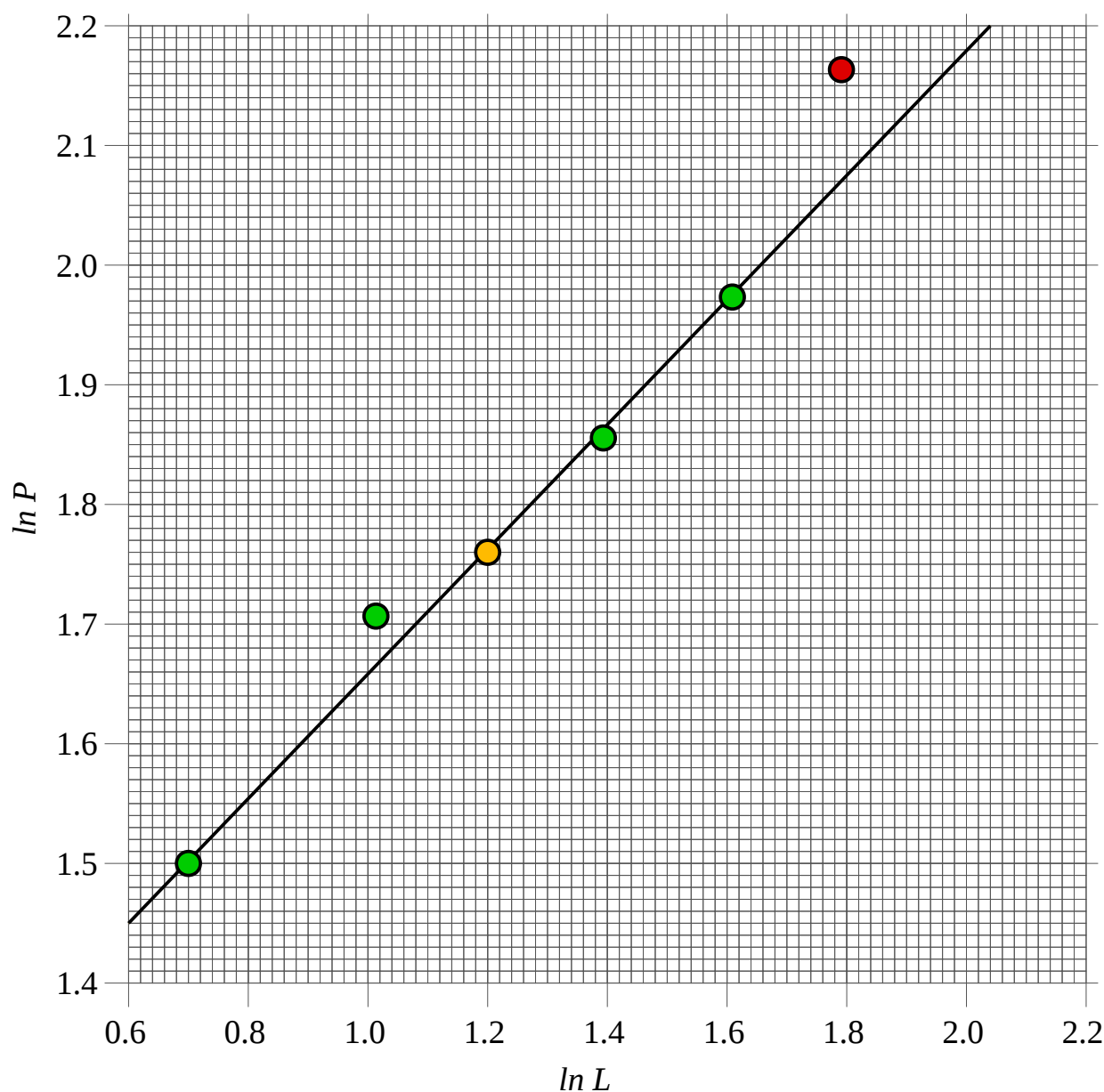
Answer 4

(i)

$\ln L$	0.693	1.099	1.386	1.609	1.792
$\ln P$	1.504	1.705	1.856	1.974	2.163

[2 marks]

(ii) The points need to be plotted accurately to avoid rejecting the wrong point !



[2 marks]

(iii) For the four green points,

$$(\text{Mean } \ln L, \text{Mean } \ln P) = (1.196, 1.760)$$

This point has been added in orange.

The line of best fit should go through this point.

[2 marks]

(iv) The exact answer is,

$$Y = 1.146 + 0.513 X$$

For 'by eye' solutions, some variation on this should be allowed.

[3 marks]

(v)

$$P = a L^b$$

$$a = e^{1.146} \quad b = 0.513$$

$$= 3.15$$

$$P = 3.15 L^{0.513}$$

[3 marks]

(vi)

$$P = 3.15 \times 6^{0.513}$$

$$= 7.9$$

$\therefore$ The 8.7 should have been 7.9

[1 mark]