10.7 Answers

A-Level Pure Mathematics, Year 1
Additional Mathematics
Coordinate Geometry

10.7.1 Solution to 10.4 Making Use of The Key Idea (Teaching Video)

$$y = \frac{x^3}{9} - x + 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x^2}{9} - 1$$

$$= \frac{x^2}{3} - 1$$

$$\therefore \left.\frac{dy}{dx}\right|_{x=3} = \frac{3^2}{3} - 1$$

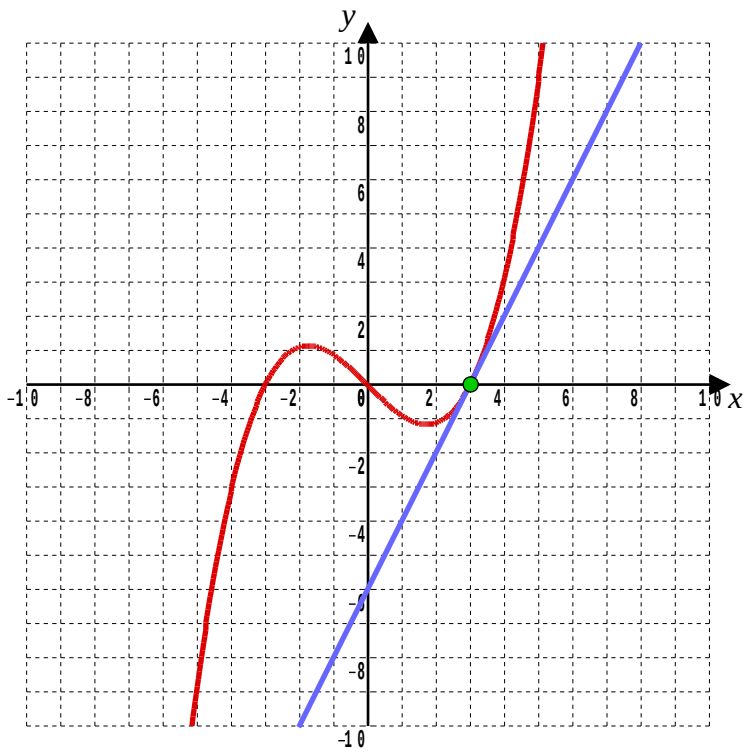
$$= 2$$

$\therefore$ The tangent is the straight line with gradient 2 through $(3,\ 0)$

$$y = mx + c$$

$$0 = 2 \times 3 + c \Rightarrow c = -6$$

The equation of the tangent is, $y = 2x - 6$



[6 marks]

10.7.2 Solutions to 10.6 Exercise

Answer 1

$$y = 4x^8 - 3x^5 \qquad \Rightarrow \qquad \frac{dy}{dx} = 32x^7 - 15x^4$$

[2 marks]

Answer 2

$$y = 16x^{0.4} \qquad \Rightarrow \qquad \frac{dy}{dx} = 6x^{-0.6}$$

[2 marks]

Answer 3

$$y = 7 + \frac{1}{2}x^6 \qquad \Rightarrow \qquad \frac{dy}{dx} = 3x^5$$

[2 marks]

Answer 4

$$y = 0.2x^9 + 0.1x \qquad \Rightarrow \qquad \frac{dy}{dx} = 1.8x^8 + 0.1$$

[2 marks]

Answer 5

$$y = x^{-5} \qquad \Rightarrow \frac{dy}{dx} = -5x^{-6}$$

[2 marks]

Answer 6

$$y = (3x + 8)(2x + 1)$$

$$= 6x^2 + 19x + 8 \qquad \Rightarrow \qquad \frac{dy}{dx} = 12x + 19$$

[3 marks]

Answer 7

$$y = \frac{x^3}{9} + 4 \qquad \Rightarrow \qquad \frac{dy}{dx} = \frac{x^2}{3}$$

[2 marks]

Answer 8

$$y = \frac{12}{x^2}$$

$$= 12x^{-2} \qquad \Rightarrow \qquad \frac{dy}{dx} = -24x^{-3}$$

$$= -\frac{24}{x^3}$$

[2 marks]

Answer 9

$$y = x^3 + 4 \Rightarrow \frac{dy}{dx} = 3x^2 \text{ and so when } x = 1, \frac{dy}{dx} = 3$$

[3 marks]

Answer 10

$$y = 0.1x^5 \Rightarrow \frac{dy}{dx} = 0.5x^4 \text{ and so when } x = 2, \frac{dy}{dx} = 8$$

[2 marks]

Answer 11

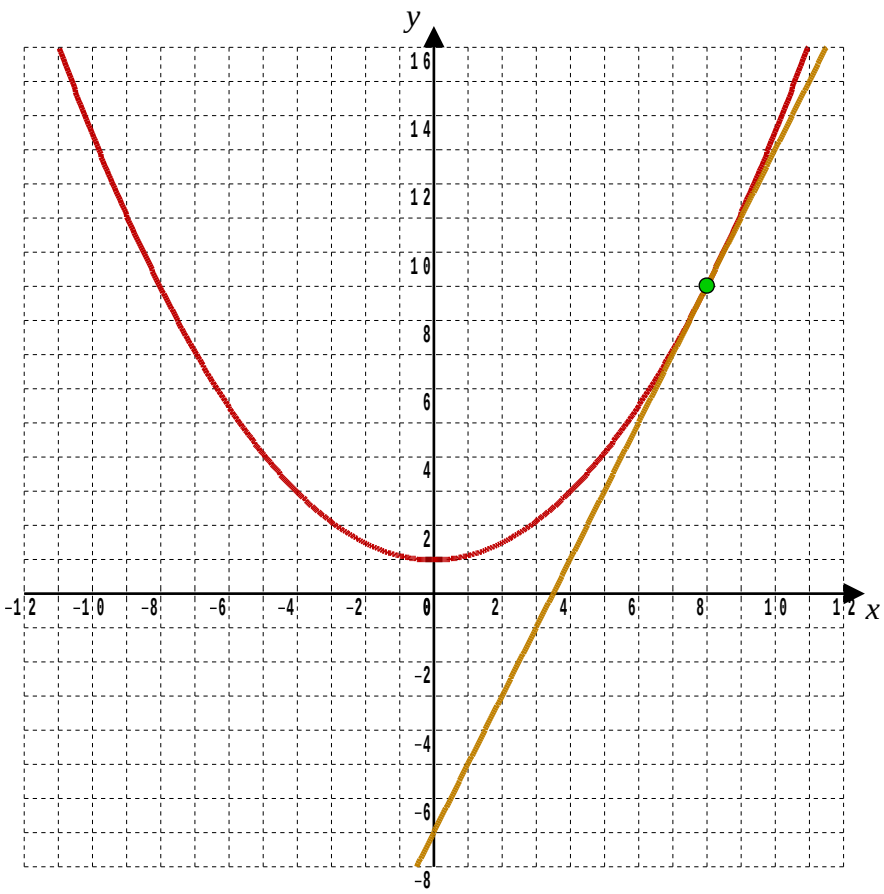
$$y = \frac{x^2}{8} + 1 \Rightarrow \frac{dy}{dx} = \frac{x}{4} \text{ and so when } x = 8, \frac{dy}{dx} = 2$$

$$y = mx + c$$

$$9 = 2 \times 8 + c \Rightarrow c = -7$$

$\therefore$ The equation of the tangent is $y = 2x - 7$

[4 marks]



[2 marks]

Answer 12

$$y = x^3 - x^2 - 2x - 3 \Rightarrow \frac{dy}{dx} = 3x^2 - 2x - 2$$

$$\text{when } x = 3,\ \frac{dy}{dx} = 27 - 6 - 2$$

$$= 19$$

$$y = mx + c$$

$$9 = 19 \times 3 + c \Rightarrow c = -48$$

Equation of the tangent is $y = 19x - 48$

[5 marks]

Answer 13

$$y = x^3 - 3x + 4 \Rightarrow \frac{dy}{dx} = 3x^2 - 3$$

$$\text{when } x = 2,\ \frac{dy}{dx} = 9$$

$$y = mx + c$$

$$6 = 9 \times 2 + c \Rightarrow c = -12$$

Equation of the tangent is $y = 9x - 12$

[4 marks]

Answer 14

(i)

$$f(x) = \frac{x^3 - x + 8}{16x}, \qquad x > 0$$

$$= \frac{x^3}{16x} - \frac{x}{16x} + \frac{8}{16x}$$

$$= \frac{x^2}{16} - \frac{1}{16} + \frac{x^{-1}}{2}$$

$$\therefore A = \frac{1}{16},\ B = -\frac{1}{16} \text{ and } C = \frac{1}{2}$$

[4 marks]

(ii)

$$f'(x) = \frac{2x}{16} - \frac{x^{-2}}{2}$$

$$= \frac{x}{8} - \frac{1}{2x^2}$$

[2 marks]

(iii) When $x = 8$, $f(8) = \dfrac{8^3 - 8 + 8}{16 \times 8} = \dfrac{8 \times 8 \times 8}{2 \times 8 \times 8} = 4$

$\therefore$ $(8,\ 4)$ is the point at which the tangent to the curve is wanted

When $x = 8$, $f'(8) = \dfrac{8}{8} - \dfrac{1}{2 \times 8 \times 8}$

$= 1 - \dfrac{1}{128} = \dfrac{127}{128}$

$y = mx + c$

$4 = \dfrac{127}{128} \times 8 + c$

$4 = \dfrac{127}{16} + c$

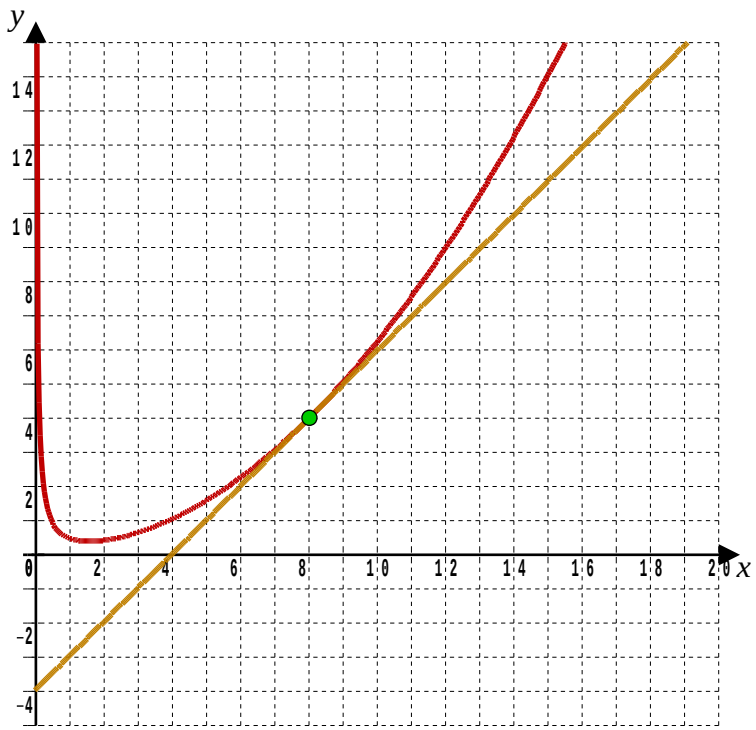
$c = 4 - \dfrac{127}{16} = -\dfrac{63}{16}$

Equation of tangent is $y = \dfrac{127}{128}x - \dfrac{63}{16}$

(This is very close to being $y = x - 4$)

[4 marks]

(iv)



[3 marks]