11.5 Answers

A-Level Pure Mathematics, Year 1
Additional Mathematics
Coordinate Geometry

11.5.1 Solution to 11.3 Example (Teaching Video)

(i)

$$y = \sqrt{x} = x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2} \times \frac{1}{x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$$

When $x = 4$, $y = 2$ and $\left.\frac{dy}{dx}\right|_{x=4} = \frac{1}{2\sqrt{4}}$

$$= \frac{1}{4}$$

If after the tangent, this would be the tangent's gradient

For the normal, the sign changed reciprocal gradient is -4

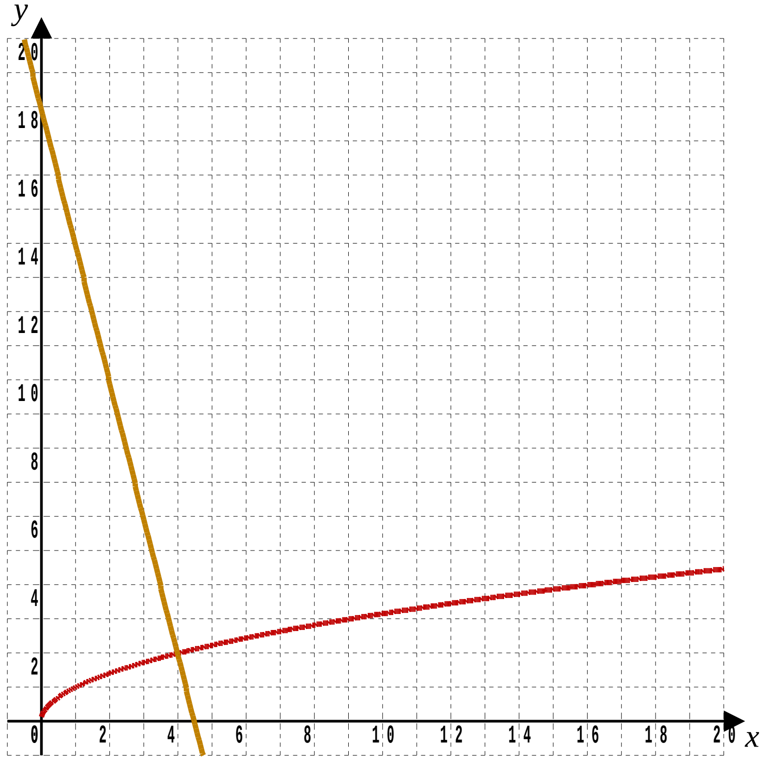
So the normal sought is a straight line through $(4, 2)$ with gradient -4

$$y = mx + c$$

$$2 = -4 \times 4 + c \Rightarrow c = 18$$

The normal has equation $y = -4x + 18$

(ii)



[5 marks]

11.5.2 Solutions to 11.4 Exercise

Answer 1

(i)

$$y = x^2 - 4x$$

$$\frac{dy}{dx} = 2x - 4$$

$$\text{When } x = 4,\ y = 0 \text{ and } \frac{dy}{dx} = 4$$

If after the tangent, this would be the tangent's gradient

For the normal, the sign changed reciprocal gradient is $-\dfrac{1}{4}$

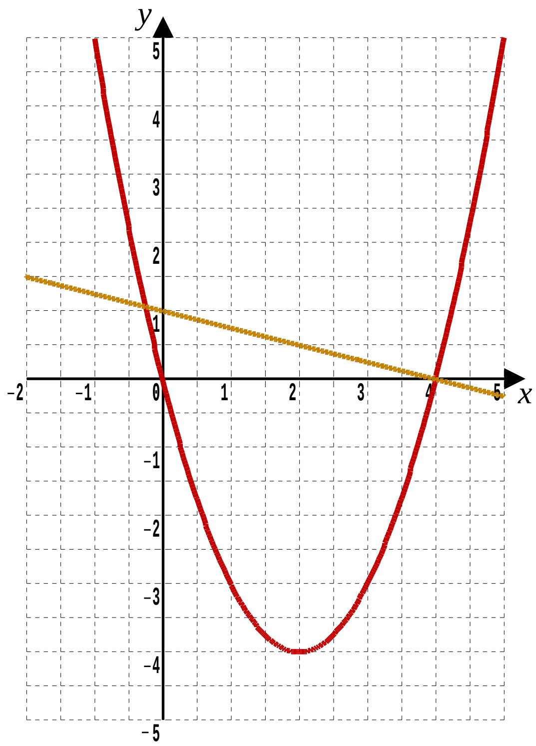
So the normal sought is a straight line through $(4,\ 0)$ with gradient $-\dfrac{1}{4}$

$$y = mx + c$$

$$0 = -\frac{1}{4} \times 4 + c \Rightarrow c = 1$$

The normal has equation $y = -\dfrac{1}{4}x + 1$

(ii)



[5 marks]

Answer 2

$$y = x^3 + 5x - 7$$

$$\frac{dy}{dx} = 3x^2 + 5$$

When $x = 1$, $\frac{dy}{dx} = 8$

$\therefore$ Normal has gradient $-\frac{1}{8}$ and passes through the point $(1, -1)$

$$y = mx + c$$

$$-1 = -\frac{1}{8} \times 1 + c \Rightarrow -\frac{7}{8}$$

Equation of the normal is $y = -\frac{1}{8}x - \frac{7}{8}$

Alternatively : $x + 8y + 7 = 0$

[5 marks]

Answer 3

$$y = x^3 - 2x^2 + 2x + 4$$

$$\frac{dy}{dx} = 3x^2 - 4x + 2$$

When $x = 2$, $\frac{dy}{dx} = 12 - 8 + 2$

$$= 6$$

$\therefore$ Normal has gradient $-\frac{1}{6}$ and passes through the point $(2, 8)$

$$y = mx + c$$

$$8 = -\frac{1}{6} \times 2 + c \Rightarrow \frac{25}{3}$$

Equation of the normal is $y = -\frac{1}{6}x + \frac{25}{3}$

Alternatively : $x + 6y - 50 = 0$

[6 marks]

Answer 4

(i)

$$x^2 + 2x - 5 = 7x - 9$$

$$x^2 - 5x + 4 = 0$$

$$(x - 1)(x - 4) = 0$$

$$\text{Either } x = 1 \text{ or } x = 4$$

$\therefore$ Points of intersection are $(1, -2)$, $(4, 19)$

[4 marks]

(ii) The line has a gradient of 7 which, if it is a normal will correspond to a gradient on the curve being $-\frac{1}{7}$

$$y = x^2 + 2x - 5$$

$$\frac{dy}{dx} = 2x + 2$$

At $(1, -2)$, $\frac{dy}{dx} = 4$ so not the line is not normal to the curve here

At $(4, 19)$, $\frac{dy}{dx} = 10$ so the line is not normal to the curve here

[3 marks]

Answer 5

(i)

$$y = 2x^2 + x - 5$$

$$\frac{dy}{dx} = 4x + 1$$

The gradient is 5 when $4x + 1 = 5$

$$4x = 4$$

$$x = 1 \qquad \Rightarrow P(1, -2)$$

[3 marks]

(ii) The normal will have gradient $-\frac{1}{5}$ and pass through $P(1, -2)$

$$y = mx + c$$

$$-2 = -\frac{1}{5} \times 1 + c \Rightarrow c = -\frac{9}{5}$$

Equation of the normal is $y = -\frac{1}{5}x - \frac{9}{5}$

Alternatively : $x + 5y + 9 = 0$

[3 marks]

Answer 6

(i)

$$y = \frac{2}{3}x^2 - 2x + 10$$

$$\frac{dy}{dx} = \frac{4}{3}x - 2$$

When $x = 3$, $\frac{dy}{dx} = 2$

$\therefore$ Tangent has gradient 2 through $A(3, \ 10)$

$$y = mx + c$$

$$10 = 2 \times 3 + c \Rightarrow c = 4$$

Equation of tangent is $y = 2x + 4$

[4 marks]

(ii)

When $x = 0$, $\frac{dy}{dx} = -2$

$\therefore$ Normal has gradient $\frac{1}{2}$ through $B(0, \ 10)$

$$y = mx + c$$

$$10 = -2 \times 0 + c \Rightarrow c = 10$$

Equation of normal is $y = \frac{1}{2}x + 10$

That is, $2y - 1 = 20$ $\square$

[3 marks]

(iii)

$$2x + 4 = \frac{1}{2}x + 10$$

multiply through by 2

$$4x + 8 = x + 20$$

$$3x = 12$$

$$x = 4 \quad \Rightarrow C(4, \ 12)$$

[3 marks]

(iv)

$$|BC| = \sqrt{(4 - 0)^2 + (12 - 10)^2}$$

$$= \sqrt{4^2 + 2^2}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5}$$

[2 marks]

Answer 7

(a)

$$y = 20 - 4x - \frac{18}{x}$$

$$\text{When } x = 2, \ y = 20 - 4 \times 2 - \frac{18}{2}$$

$$= 3 \qquad \therefore A(2, 3)$$

$$y = 20 - 4x - 18x^{-1}$$

$$\frac{dy}{dx} = -4 + 18x^{-2}$$

$$= -4 + \frac{18}{x^2}$$

$$\text{When } x = 2, \ \frac{dy}{dx} = \frac{1}{2}$$

$\therefore$ Normal has gradient -2 through $A(2, 3)$

$$y = mx + c$$

$$3 = -2 \times 2 + c \Rightarrow c = 7$$

Equation of normal is $y = -2x + 7$ ☐

[6 marks]

(b)

$$-2x + 7 = 20 - 4x - \frac{18}{x}$$

$$2x - 13 + \frac{18}{x} = 0$$

multiply through by x

$$2x^2 - 13x + 18 = 0$$

$$(x - 2)(2x - 9) = 0$$

$$\text{Either } x = 2 \text{ or } x = \frac{9}{2}$$

$$A(2, 3), \ B\left(\frac{9}{2}, -2\right)$$

[5 marks]