

12.2 Answers

A-Level Pure Mathematics, Year 1 Additional Mathematics Coordinate Geometry

Answer 1

Centre $(-7, 3)$ Radius 9

[2 marks]

Answer 2

Working out the distance between the centre and the origin, which will be the radius...

$$\begin{aligned} R &= \sqrt{(5 - 0)^2 + (12 - 0)^2} \\ &= \sqrt{5^2 + 12^2} \\ &= 13 \\ \therefore (x - 5)^2 + (y - 12)^2 &= 13^2 \end{aligned}$$

[2 marks]

Answer 3

$$\begin{aligned} x^2 + y^2 + 10x - 6y + 18 &= 0 \\ [x^2 + 10x] + [y^2 - 6y] + 18 &= 0 \\ [(x + 5)^2 - 25] + [(y - 3)^2 - 9] + 18 &= 0 \\ (x + 5)^2 + (y - 3)^2 &= 16 \\ \therefore \text{Centre } (-5, 3) \text{ Radius } 4 \end{aligned}$$

[5 marks]

Answer 4

$$\begin{aligned} \text{(i)} \quad y &= 7x^{11} - 5x^3 & \Rightarrow & \quad \frac{dy}{dx} = 77x^{10} - 15x^2 \\ \text{(ii)} \quad y &= 24x^{0.25} & \Rightarrow & \quad \frac{dy}{dx} = 6x^{-0.75} \\ \text{(iii)} \quad y &= \frac{4}{5}x^{-5} & \Rightarrow & \quad \frac{dy}{dx} = -4x^{-6} \end{aligned}$$

[2, 2, 2 marks]

Answer 5

$$\begin{aligned} \text{(i)} \quad y &= \frac{x^3}{24} - x \\ \frac{dy}{dx} &= \frac{x^2}{8} - 1 \end{aligned}$$

[2 marks]

$$\text{(ii)} \quad \text{When } x = 6, \frac{dy}{dx} = \frac{7}{2}$$

[1 mark]

(iii)

$$y = mx + c$$

$$3 = \frac{7}{2} \times 6 + c \Rightarrow c = -18$$

$$\text{Equation of the tangent is } y = \frac{7}{2}x - 18$$

$$\text{Alternatively } 7x - 2y - 36 = 0$$

[2 marks]

(iv) Normal's "sign changed reciprocal" gradient will be $-\frac{2}{7}$

[1 mark]

(v)

$$y = mx + c$$

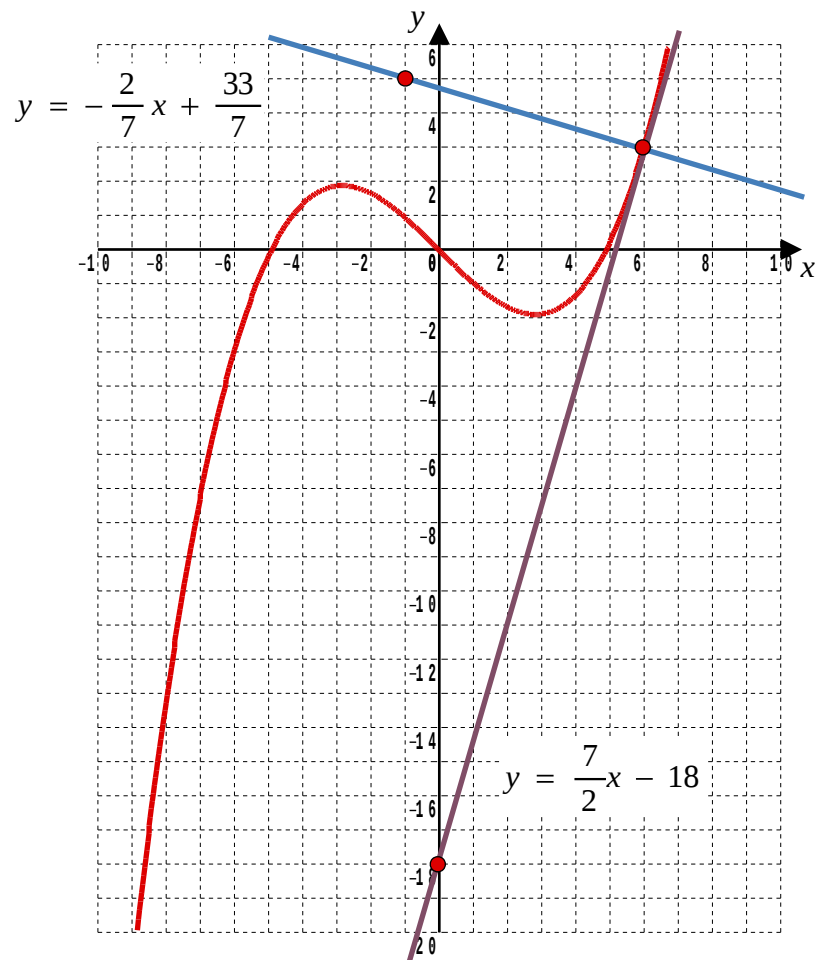
$$3 = -\frac{2}{7} \times 6 + c \Rightarrow c = \frac{33}{7}$$

$$\text{Equation of the normal is } y = -\frac{2}{7}x + \frac{33}{7}$$

$$\text{Alternatively } 7y + 2x - 33 = 0$$

[2 marks]

(vi)



[2 marks]

Answer 6

$$\begin{aligned}
\text{LHS} &= (x + 1)^2 + y^2 \\
&= (5 + 1)^2 + 7^2 \quad \text{if the circle passes through } (5, 7) \\
&= 6^2 + 7^2 \\
&= 36 + 49 \\
&= 85 \\
&> \text{RHS} \\
\therefore (5, 7) &\text{ is outside the circle} \quad \square
\end{aligned}$$

[3 marks]**Answer 7**

$$\begin{aligned}
(x - 3)^2 + (y + 2)^2 &= 13 \quad \text{and} \quad y = 3x - 2 \\
(x - 3)^2 + (3x - 2 + 2)^2 &= 13 \\
x^2 - 6x + 9 + 9x^2 &= 13 \\
10x^2 - 6x - 4 &= 0 \\
\text{divide through by 2} \\
5x^2 - 3x - 2 &= 0 \\
(5x + 2)(x - 1) &= 0 \\
\text{Either } x &= -\frac{2}{5} \text{ or } x = 1 \\
\text{Points of intersection are } &\left(-\frac{2}{5}, -\frac{16}{5}\right), (1, 1)
\end{aligned}$$

[6 marks]**Answer 8**

The centre of the circle is (3, 2) and the radius is 3

$$\therefore (x - 3)^2 + (y - 2)^2 = 9$$

[3 marks]**Answer 9**

The tangent to the curve at the same point must have a gradient of -8

$$\begin{aligned}
y &= (4x - 5)^2 \\
&= 16x^2 - 40x + 25 \\
\frac{dy}{dx} &= 32x - 40 \\
\therefore 32x - 40 &= -8 \\
x = 1 &\Rightarrow \text{The point is } (1, 1)
\end{aligned}$$

[4 marks]

Answer 10**(i)**

$$\text{Midpoint} = \left(\frac{5+3}{2}, \frac{-2+2}{2} \right) = (4, 0)$$

[1 mark]**(ii)**

$$m = \frac{\Delta Y}{\Delta X} = \frac{-2-2}{5-3} = -\frac{4}{2} = -2$$

[1 mark]**(iii)**

Perpendicular bisector will have gradient $\frac{1}{2}$

$$y = mx + c$$

$$0 = \frac{1}{2} \times 4 + c \Rightarrow c = -2$$

Perpendicular bisector has equation $y = \frac{1}{2}x - 2$

[2 marks]**(iv)**

$$\text{Midpoint} = \left(\frac{2+9}{2}, \frac{4+5}{2} \right) = \left(\frac{11}{2}, \frac{9}{2} \right)$$

$$m = \frac{\Delta Y}{\Delta X} = \frac{4-5}{2-9} = \frac{1}{7}$$

Perpendicular bisector will have gradient -7

$$y = mx + c$$

$$\frac{9}{2} = -7 \times \frac{11}{2} + c \Rightarrow c = 43$$

Perpendicular bisector has equation $y = -7x + 43$

[4 marks]**(v)**

$$\frac{1}{2}x - 2 = -7x + 43$$

multiply through by 2

$$x - 4 = -14x + 86$$

$$15x = 90$$

$$x = 6$$

Centre of the rotation is (6, 1)

[4 marks]

Answer 11

(i) $P(5, 1)$

[1 mark]

(ii) $V(2, 1), P(5, 1)$

$$\text{Midpoint } VP = \left(\frac{5+2}{2}, \frac{1+1}{2} \right) = M\left(\frac{7}{2}, 1 \right)$$

[2 marks]

(iii) $T(5, 10)$

$$y = x^2 - 4x + 5$$

$$\frac{dy}{dx} = 2x - 4$$

$$\text{When } x = 5, \frac{dy}{dx} = 2 \times 5 - 4$$

$$= 6$$

$$y = mx + c$$

$$10 = 6 \times 5 + c \Rightarrow c = -20$$

$$\text{Equation of the tangent at } T \text{ is } y = 6x - 20$$

[4 marks]

(iv)

$$y = 6x - 20 \quad M\left(\frac{7}{2}, 1 \right)$$

$$\text{Does } y = 1 \text{ when } x = \frac{7}{2} ?$$

$$\text{RHS} = 6x - 20$$

$$= 6 \times \frac{7}{2} - 20$$

$$= 1$$

$$= \text{LHS}$$

- (v)**
- Draw a line, L , parallel to the x -axis through V
 - Drop a perpendicular line from T to the line, L , calling the intersection P
 - Calculate the midpoint, M , of VP
 - Draw a line through T and M which will be the tangent at T

[3 marks]**Out Of Interest**

I have a proof that this will always work.

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