12.2 Answers

A-Level Pure Mathematics, Year 1
Additional Mathematics
Coordinate Geometry

Answer 1

Centre $(-7,\ 3)$ Radius 9

[2 marks]

Answer 2

Working out the distance between the centre and the origin, which will be the radius...

$$R = \sqrt{(5-0)^2 + (12-0)^2}$$
$$= \sqrt{5^2 + 12^2}$$
$$= 13$$
$$\therefore\ (x-5)^2 + (y-12)^2 = 13^2$$

[2 marks]

Answer 3

$$x^2 + y^2 + 10x - 6y + 18 = 0$$
$$[x^2 + 10x] + [y^2 - 6y] + 18 = 0$$
$$[(x+5)^2 - 25] + [(y-3)^2 - 9] + 18 = 0$$
$$(x+5)^2 + (y-3)^2 = 16$$
$$\therefore \text{ Centre } (-5,\ 3) \text{ Radius } 4$$

[5 marks]

Answer 4

(i) $y = 7x^{11} - 5x^3 \quad \Rightarrow \quad \frac{dy}{dx} = 77x^{10} - 15x^2$

(ii) $y = 24x^{0.25} \quad \Rightarrow \quad \frac{dy}{dx} = 6x^{-0.75}$

(iii) $y = \frac{4}{5}x^{-5} \quad \Rightarrow \quad \frac{dy}{dx} = -4x^{-6}$

[2, 2, 2 marks]

Answer 5

(i)

$$y = \frac{x^3}{24} - x$$
$$\frac{dy}{dx} = \frac{x^2}{8} - 1$$

[2 marks]

(ii) When $x = 6,\ \frac{dy}{dx} = \frac{7}{2}$

[1 mark]

(iii) $y = mx + c$

$$3 = \frac{7}{2} \times 6 + c \Rightarrow c = -18$$

Equation of the tangent is $y = \frac{7}{2}x - 18$

Alternatively $7x - 2y - 36 = 0$

[2 marks]

(iv) Normal's "sign changed reciprocal" gradient will be $-\frac{2}{7}$

[1 mark]

(v) $y = mx + c$

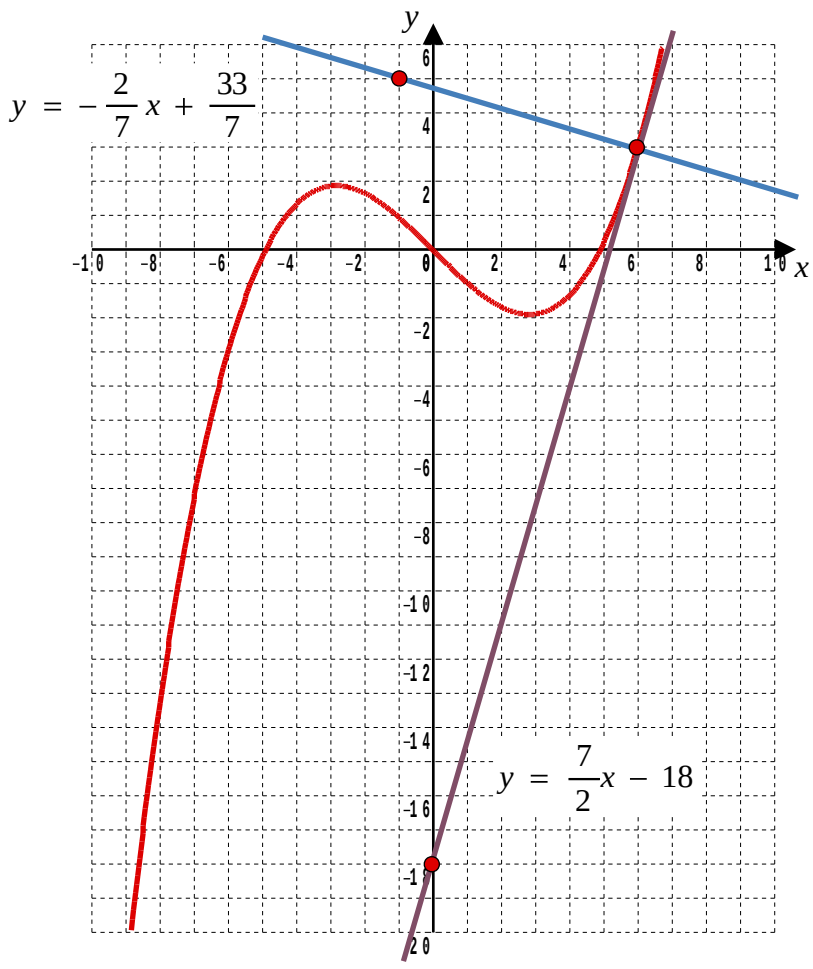
$$3 = -\frac{2}{7} \times 6 + c \Rightarrow c = \frac{33}{7}$$

Equation of the normal is $y = -\frac{2}{7}x + \frac{33}{7}$

Alternatively $7y + 2x - 33 = 0$

[2 marks]

(vi)



[2 marks]

Answer 6

$$\text{LHS} = (x+1)^2 + y^2$$
$$= (5+1)^2 + 7^2 \qquad \text{if the circle passes through } (5,\ 7)$$
$$= 6^2 + 7^2$$
$$= 36 + 49$$
$$= 85$$
$$> \text{RHS}$$
$$\therefore\ (5,\ 7) \text{ is outside the circle} \qquad \square$$

[3 marks]

Answer 7

$$(x-3)^2 + (y+2)^2 = 13 \ \text{ and } \ y = 3x - 2$$
$$(x-3)^2 + (3x-2+2)^2 = 13$$
$$x^2 - 6x + 9 + 9x^2 = 13$$
$$10x^2 - 6x - 4 = 0$$
$$\text{divide through by 2}$$
$$5x^2 - 3x - 2 = 0$$
$$(5x+2)(x-1) = 0$$
$$\text{Either } x = -\frac{2}{5} \text{ or } x = 1$$
$$\text{Points of intersection are } \left(-\frac{2}{5}, -\frac{16}{5}\right), (1,\ 1)$$

[6 marks]

Answer 8

$$\text{The centre of the circle is } (3,\ 2) \text{ and the radius is } 3$$
$$\therefore\ (x-3)^2 + (y-2)^2 = 9$$

[3 marks]

Answer 9

The tangent to the curve at the same point must have a gradient of -8

$$y = (4x-5)^2$$
$$= 16x^2 - 40x + 25$$
$$\frac{dy}{dx} = 32x - 40$$
$$\therefore\ 32x - 40 = -8$$
$$x = 1 \implies \text{The point is } (1,\ 1)$$

[4 marks]

Answer 10

(i)

$$Midpoint = \left(\frac{5+3}{2}, \frac{-2+2}{2} \right) = (4, 0)$$

[1 mark]

(ii)

$$m = \frac{\Delta Y}{\Delta X} = \frac{-2-2}{5-3} = -\frac{4}{2} = -2$$

[1 mark]

(iii)

Perpendicular bisector will have gradient $\frac{1}{2}$

$$y = mx + c$$

$$0 = \frac{1}{2} \times 4 + c \Rightarrow c = -2$$

Perpendicular bisector has equation $y = \frac{1}{2}x - 2$

[2 marks]

(iv)

$$Midpoint = \left(\frac{2+9}{2}, \frac{4+5}{2} \right) = \left(\frac{11}{2}, \frac{9}{2} \right)$$

$$m = \frac{\Delta Y}{\Delta X} = \frac{4-5}{2-9} = \frac{1}{7}$$

Perpendicular bisector will have gradient -7

$$y = mx + c$$

$$\frac{9}{2} = -7 \times \frac{11}{2} + c \Rightarrow c = 43$$

Perpendicular bisector has equation $y = -7x + 43$

[4 marks]

(v)

$$\frac{1}{2}x - 2 = -7x + 43$$

multiply through by 2

$$x - 4 = -14x + 86$$

$$15x = 90$$

$$x = 6$$

Centre of the rotation is $(6, 1)$

[4 marks]

Answer 11

(i) $P(5,\ 1)$

[1 mark]

(ii) $V(2,\ 1),\ P(5,\ 1)$

$$Midpoint\ VP = \left(\frac{5+2}{2},\ \frac{1+1}{2}\right) = M\left(\frac{7}{2},\ 1\right)$$

[2 marks]

(iii) $T(5,\ 10)$

$$y = x^2 - 4x + 5$$

$$\frac{dy}{dx} = 2x - 4$$

$$\text{When } x = 5,\ \frac{dy}{dx} = 2 \times 5 - 4$$

$$= 6$$

$$y = mx + c$$

$$10 = 6 \times 5 + c \Rightarrow c = -20$$

Equation of the tangent at T is $y = 6x - 20$

[4 marks]

(iv)

$$y = 6x - 20 \qquad M\left(\frac{7}{2},\ 1\right)$$

$$\text{Does } y = 1 \text{ when } x = \frac{7}{2}\ ?$$

$$\text{RHS} = 6x - 20$$

$$= 6 \times \frac{7}{2} - 20$$

$$= 1$$

$$= \text{LHS}$$

(v)

- Draw a line, L, parallel to the x-axis through V
- Drop a perpendicular line from T to the line, L, calling the intersection P
- Calculate the midpoint, M, of VP
- Draw a line through T and M which will be the tangent at T

[3 marks]

Out Of Interest

I have a proof that this will always work.