13.2 Answers

A-Level Pure Mathematics, Year 1
Additional Mathematics
Coordinate Geometry

Answer 1

Centre $(-11,\ 13)$ Radius 7

[2 marks]

Answer 2

Working out the distance between the centre and the origin, which will be the radius...

$$R = \sqrt{(15-0)^2 + (8-0)^2}$$
$$= \sqrt{15^2 + 8^2}$$
$$= 17$$
$$\therefore\ (x-15)^2 + (y+8)^2 = 17^2$$

[2 marks]

Answer 3

$$x^2 + y^2 - 12x + 10y - 20 = 0$$
$$[x^2 - 12x] + [y^2 + 10y] - 20 = 0$$
$$[(x-6)^2 - 36] + [(y+5)^2 - 25] - 20 = 0$$
$$(x-6)^2 + (y+5)^2 = 81$$
$$\therefore\ \text{Centre } (6, -5) \text{ Radius } 9$$

[5 marks]

Answer 4

(i) $y = 9x^5 - 7x^3 + 4 \quad\Rightarrow\quad \dfrac{dy}{dx} = 45x^4 - 21x^2$

(ii) $y = 100x^{0.35} \quad\Rightarrow\quad \dfrac{dy}{dx} = 35x^{-0.65}$

(iii) $y = \dfrac{2}{3}x^{-9} \quad\Rightarrow\quad \dfrac{dy}{dx} = -6x^{-10}$

[2, 2, 2 marks]

Answer 5

(i)

$$y = \frac{x^3}{16} - 3$$
$$\frac{dy}{dx} = \frac{3x^2}{16}$$

[2 marks]

(ii) When $x = 4$, $\dfrac{dy}{dx} = 3$

[1 mark]

(iii)

$$y = mx + c$$

$$1 = 3 \times 4 + c \Rightarrow c = -11$$

Equation of the tangent is $y = 3x - 11$

[2 marks]

(iv) Normal's "sign changed reciprocal" gradient will be $-\dfrac{1}{3}$

[1 mark]

(v)

$$y = mx + c$$

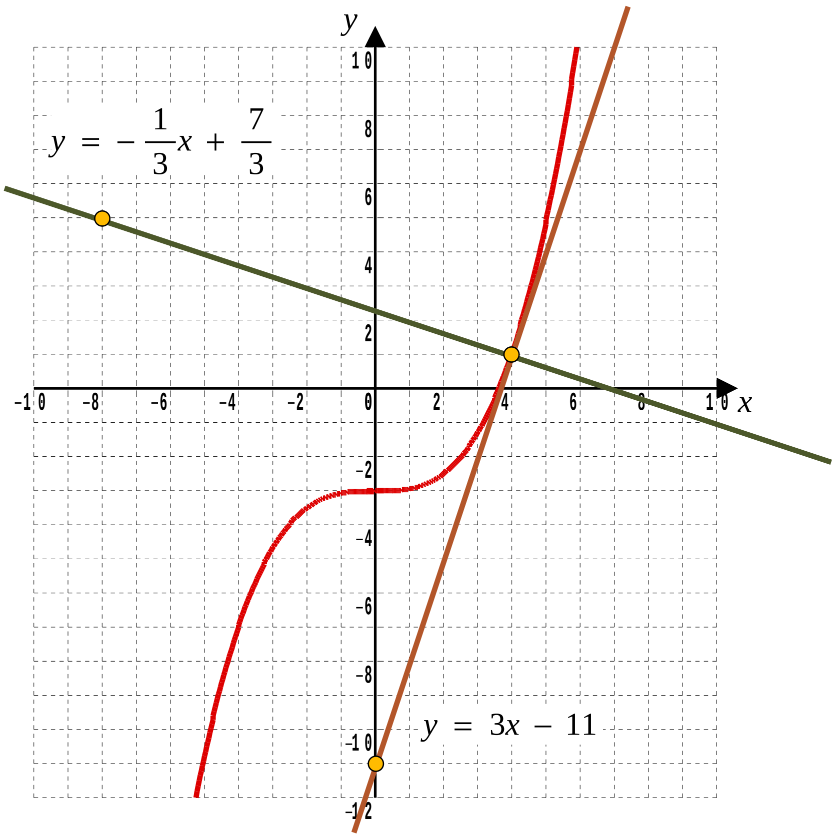
$$1 = -\frac{1}{3} \times 4 + c \Rightarrow c = \frac{7}{3}$$

Equation of the normal is $y = -\dfrac{1}{3}x + \dfrac{7}{3}$

Alternatively $3y + x - 7 = 0$

[2 marks]

(vi)



[2 marks]

Answer 6

$$\begin{aligned} \text{LHS} &= x^2 + (y - 2)^2 \\ &= 7^2 + (6 - 2)^2 \qquad \text{if the circle passes through } (7,\ 6) \\ &= 7^2 + 4^2 \\ &= 49 + 16 \\ &= 65 \\ &> \text{RHS} \end{aligned}$$

$\therefore\ (7,\ 6)$ is outside the circle ☐

[3 marks]

Answer 7

$$(x + 5)^2 + (y - 3)^2 = 32 \quad \text{and} \quad y = 4x + 3$$

$$(x + 5)^2 + (4x + 3 - 3)^2 = 32$$

$$x^2 + 10x + 25 + 16x^2 = 32$$

$$17x^2 + 10x - 7 = 0$$

$$(x + 1)(17x - 7) = 0$$

Either $x = -1$ or $x = \dfrac{7}{17}$

Points of intersection are $(-1, -1)$, $\left(\dfrac{7}{17},\ \dfrac{79}{17}\right)$

[6 marks]

Answer 8

(i) $$|AB| = \sqrt{(6-(-2))^2 + (8-4)^2}$$
$$= \sqrt{8^2 + 4^2}$$
$$= \sqrt{80}$$
$$= 4\sqrt{5}$$

(This is the diameter)

[2 marks]

(ii) $$Midpoint\ AB = \left(\frac{6+(-2)}{2}, \ \frac{8+4}{2}\right) = (2, \ 6)$$

[2 marks]

(iii) $$(x-2)^2 + (y-6)^2 = 20$$

[3 marks]

(iv) $$m_{AB} = \frac{\Delta Y}{\Delta X} = \frac{8-4}{6-(-2)} = \frac{4}{8} = \frac{1}{2}$$

[2 marks]

(v) For the tangent, the gradient is -2 through $B(6, \ 8)$

$$y = mx + c$$
$$8 = -2 \times 6 + c \Rightarrow c = 20$$

The tangent's equation is $y = -2x + 20$

[3 marks]

(vi) The circle will cut the y-axis when $x = 0$

$$(x-2)^2 + (y-6)^2 = 20$$
$$(0-2)^2 + (y-6)^2 = 20$$
$$(y-6)^2 = 16$$
$$y - 6 = \pm 4$$

Either $y = 2$ or $y = 10$

Points of intersection are $(0, \ 2)$, $(0, \ 10)$

[3 marks]

Answer 9

The distance, D, between the circle's centre and the origin is,

$$D = \sqrt{(20 - 0)^2 + (15 - 0)^2}$$
$$= \sqrt{20^2 + 15^2}$$
$$= \sqrt{625}$$
$$= 25$$

D is the hypotenuse of a right angled triangle with the circle's radius being one of the shorter sides, and the length sought, X, being the other.

$$X = \sqrt{25^2 - 7^2}$$
$$= \sqrt{625 - 49}$$
$$= \sqrt{576}$$
$$= 24$$

[6 marks]

Answer 10

(a) Centre $A(2, 1)$, Radius is distance between A and B

$$\text{Radius} = \sqrt{(10 - 2)^2 + (7 - 1)^2}$$
$$= \sqrt{8^2 + 6^2}$$
$$= 10$$

The circle has equation $(x - 2)^2 + (y - 1)^2 = 100$

[4 marks]

(b) $m_{AB} = \dfrac{\Delta Y}{\Delta X} = \dfrac{7 - 1}{10 - 2} = \dfrac{6}{8} = \dfrac{3}{4}$

The line l_1 will have gradient $-\dfrac{4}{3}$ through $B(10, 7)$

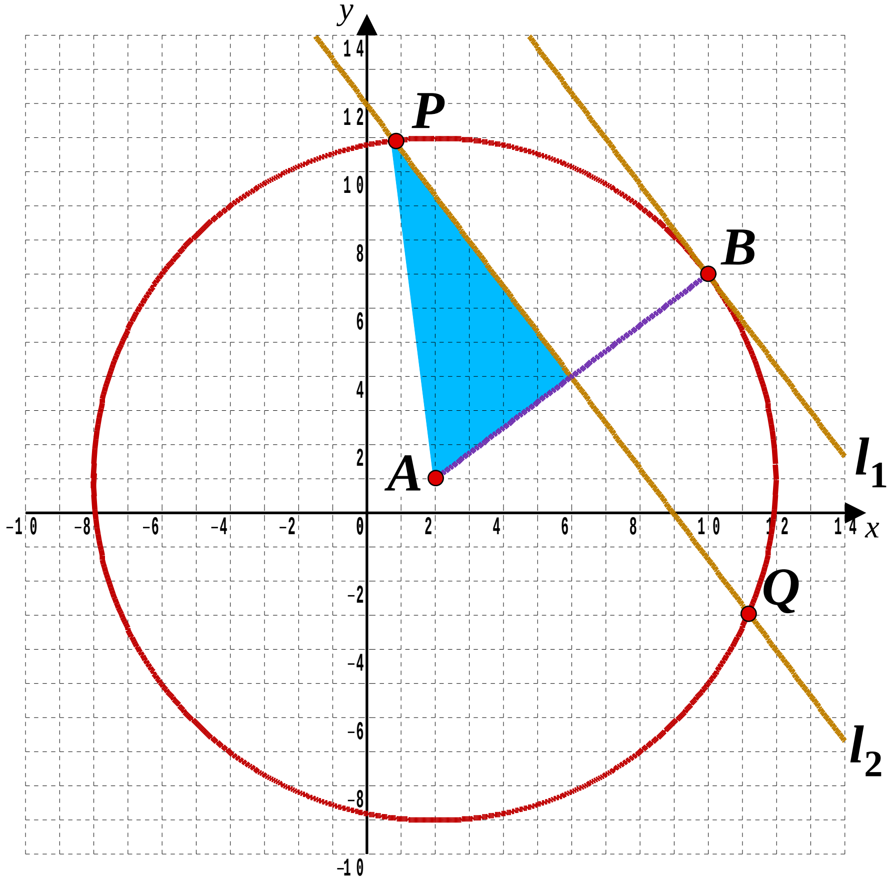
$$y = mx + c$$
$$7 = -\frac{4}{3} \times 10 + c \Rightarrow c = \frac{61}{3}$$

l_1 has equation $y = -\dfrac{4}{3}x + \dfrac{61}{3}$

Alternatively, $4x + 3y - 61 = 0$

[4 marks]

(c) A rough sketch is helpful in seeing how to answer this part of the question !



The blue triangle has a hypotenuse that is the radius of the circle, 10 and its other side is half the radius running from *A* to *B*

$$\frac{1}{2}\,|PQ| = \sqrt{10^2 - 5^2}$$

$$= \sqrt{100 - 25}$$

$$= \sqrt{75}$$

$$= 5\sqrt{3}$$

$$|PQ| = 10\sqrt{3}$$

[3 marks]

Answer 11

(a)

$$f(x) = \frac{(4 + 3\sqrt{x})^2}{x}$$

$$= \frac{16 + 24\sqrt{x} + 9x}{x}$$

$$= \frac{16}{x} + \frac{24\sqrt{x}}{x} + \frac{9x}{x}$$

$$= 16x^{-1} + \frac{24\sqrt{x}}{\sqrt{x} \times \sqrt{x}} + 9$$

$$= 16x^{-1} + \frac{24}{\sqrt{x}} + 9$$

$$= 16x^{-1} + \frac{24}{x^{\frac{1}{2}}} + 9$$

$$= 16x^{-1} + 24x^{-\frac{1}{2}} + 9$$

That is, $A = 16$, $B = 24$, $C = 9$ and $k = -\frac{1}{2}$

[4 marks]

(b)

$$f'(x) = -16x^{-2} - 12x^{-\frac{3}{2}}$$

$$= -\frac{16}{x^2} - \frac{12}{x^{\frac{3}{2}}}$$

[2 marks]

(c) When $x = 4$,

$$f(4) = \frac{(4 + 3\sqrt{4})^2}{4} = \frac{100}{4} = 25$$

$$f'(4) = -\frac{16}{4^2} - \frac{12}{4^{\frac{3}{2}}} = -1 - \frac{12}{8} = -\frac{5}{2}$$

$$y = mx + c$$

$$25 = -\frac{5}{2} \times 4 + c \Rightarrow c = 35$$

$$y = -\frac{5}{2} + 35$$

Alternatively, $5x + 2y - 70 = 0$

[4 marks]

Answer 12

$$y = \sqrt{2}\,x^2 - 6\sqrt{x} + 4\sqrt{2}$$

$$= \sqrt{2}\,x^2 - 6x^{\frac{1}{2}} + 4\sqrt{2}$$

$$\frac{dy}{dx} = \sqrt{2} \times 2x - 6 \times \frac{1}{2}x^{-\frac{1}{2}}$$

$$\text{When } x = 2,\ \frac{dy}{dx} = \sqrt{2} \times 2 \times 2 - 3 \times \frac{1}{\sqrt{2}}$$

$$= 4\sqrt{2} - \frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= 4\sqrt{2} - \frac{3\sqrt{2}}{2}$$

$$= \frac{8\sqrt{2} - 3\sqrt{2}}{2}$$

$$= \frac{5\sqrt{2}}{2}$$

$$= \frac{5}{2}\sqrt{2}$$

$$\text{That is, } a = \frac{5}{2}$$

[4 marks]