

2.4 Answers

A-Level Pure Mathematics, Year 1
Additional Mathematics
GCSE
Coordinate Geometry

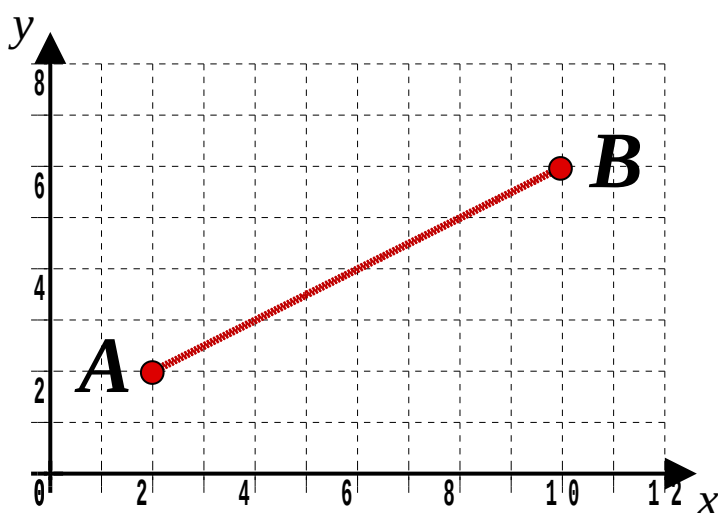
2.4.1 Solution to 2.2 Example (Teaching Video)

$$f(x) = \frac{1}{2}x + 1, \quad x \in \mathbb{R}, \quad 2 \leq x \leq 10$$

Endpoints are,

$$f(2) = \frac{1}{2} \times 2 + 1 = 2 \Rightarrow A(2, 2)$$

$$f(10) = \frac{1}{2} \times 10 + 1 = 6 \Rightarrow B(10, 6)$$



[3 marks]

$$\begin{aligned} |AB| &= \sqrt{(\Delta X)^2 + (\Delta Y)^2} \\ &= \sqrt{(10 - 2)^2 + (6 - 2)^2} \\ &= \sqrt{8^2 + 4^2} \\ &= \sqrt{80} \\ &= 4\sqrt{5} \quad \text{is the length of the line segment} \end{aligned}$$

[2 marks]

$$\begin{aligned} \text{Midpoint } AB &= \left(\frac{x_B + x_A}{2}, \frac{y_B + y_A}{2} \right) \\ &= \left(\frac{10 + 2}{2}, \frac{6 + 2}{2} \right) \\ &= (6, 4) \end{aligned}$$

[2 marks]

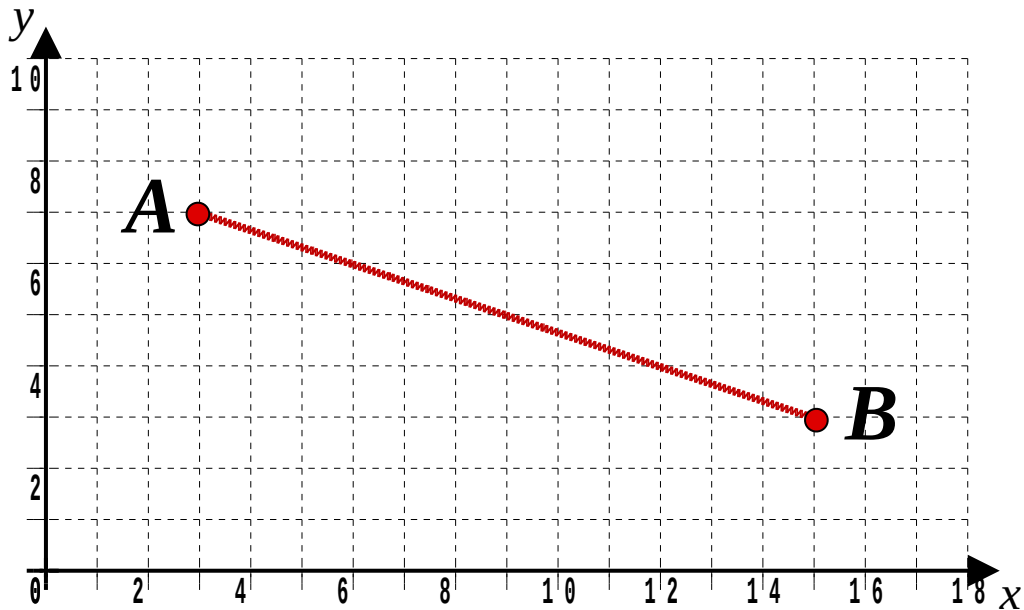
2.4.2 Solutions to 2.3 Exercise

Answer 1

$$(i) \quad g(x) = -\frac{1}{3}x + 8, \quad x \in \mathbb{R}, \quad 3 \leq x \leq 15$$

$$\text{Endpoint } A : g(3) = -\frac{1}{3} \times 3 + 8 = 7 \Rightarrow A(3, 7)$$

$$\text{Endpoint } B : g(15) = -\frac{1}{3} \times 15 + 8 = 3 \Rightarrow B(15, 3)$$



[3 marks]

$$\begin{aligned} (ii) \quad |AB| &= \sqrt{(\Delta X)^2 + (\Delta Y)^2} \\ &= \sqrt{(15 - 3)^2 + (3 - 7)^2} \\ &= \sqrt{12^2 + 4^2} = \sqrt{160} = 4\sqrt{10} \end{aligned}$$

[2 marks]

$$\begin{aligned} (iii) \quad \text{Midpoint } AB &= \left(\frac{x_B + x_A}{2}, \frac{y_B + y_A}{2} \right) \\ &= \left(\frac{15 + 3}{2}, \frac{3 + 7}{2} \right) \\ &= (9, 5) \end{aligned}$$

[2 marks]

Answer 2

$$\begin{aligned} |AB| &= \sqrt{(\Delta X)^2 + (\Delta Y)^2} \\ &= \sqrt{(9 - (-15))^2 + (5 - (-2))^2} \\ &= \sqrt{24^2 + 7^2} = 25 \end{aligned}$$

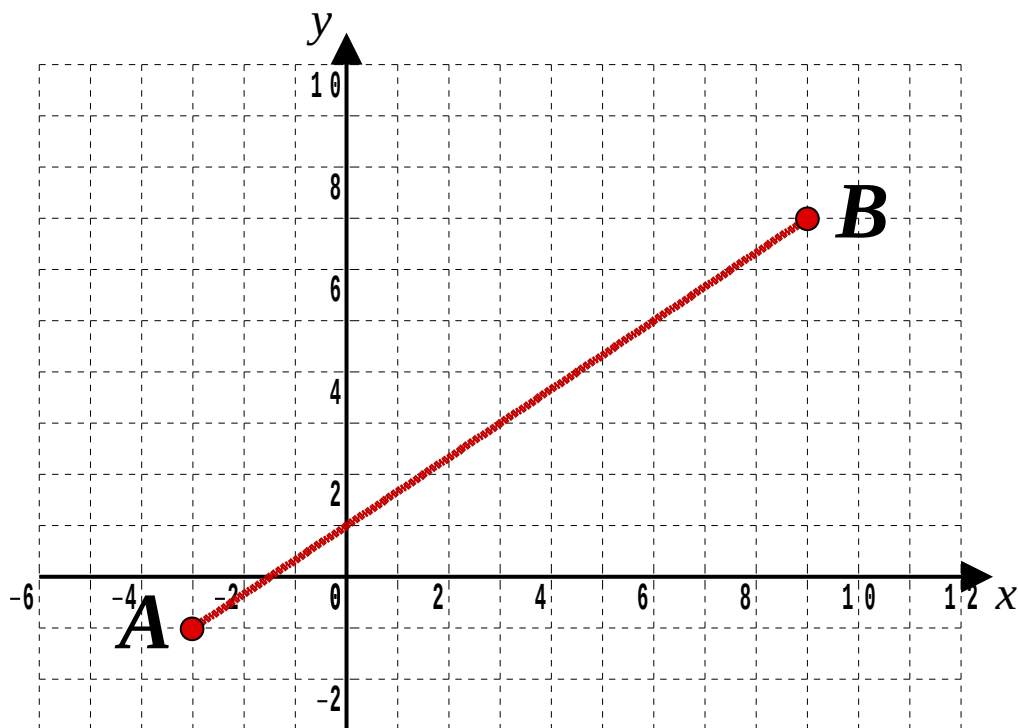
[3 marks]

Answer 3

(i)
$$h(x) = \frac{2}{3}x + 1, \quad x \in \mathbb{R}, \quad -3 \leq x \leq 9$$

Endpoint A : $h(-3) = \frac{2}{3} \times (-3) + 1 = -1 \Rightarrow A(-3, -1)$

Endpoint B : $h(9) = \frac{2}{3} \times 9 + 1 = 7 \Rightarrow B(9, 7)$



[3 marks]

(ii)
$$\begin{aligned} |AB| &= \sqrt{(\Delta X)^2 + (\Delta Y)^2} \\ &= \sqrt{(9 - (-3))^2 + (7 - (-1))^2} \\ &= \sqrt{12^2 + 8^2} = \sqrt{208} = 4\sqrt{13} \end{aligned}$$

[2 marks]

(iii)
$$\begin{aligned} \text{Midpoint } AB &= \left(\frac{x_B + x_A}{2}, \frac{y_B + y_A}{2} \right) \\ &= \left(\frac{9 + (-3)}{2}, \frac{7 + (-1)}{2} \right) \\ &= (3, 3) \end{aligned}$$

[2 marks]

Answer 4

$$4y - 12x + 3 = 0 \Rightarrow y = 3x - \frac{3}{4}$$

[2 marks]

Answer 5

$$\begin{aligned}
 \text{(a)} \quad \text{Midpoint } AB &= \left(\frac{x_B + x_A}{2}, \frac{y_B + y_A}{2} \right) \\
 &= \left(\frac{13 + 5}{2}, \frac{1 + (-4)}{2} \right) \\
 &= \left(9, -\frac{3}{2} \right)
 \end{aligned}$$

[2 marks]**(b)** Gradient is -3 **[1 mark]****(c)** Put $x = 100$ into $y = 2 - 3x \Rightarrow y = -298$ $\therefore$ As the answer was not -302 , the point $(100, -302)$ is not on L **[1 mark]****Answer 6**

$$2y + 8x = 5 \Rightarrow y = -4x + \frac{5}{2} \text{ which has gradient of } -4$$

$$y = mx + c$$

$$3 = -4 \times 2 + c$$

$$c = 11$$

 $\therefore L$ has equation $y = -4x + 11$ **[3 marks]****Answer 7**

$$\begin{aligned}
 \text{(i)} \quad |AB| &= \sqrt{(\Delta X)^2 + (\Delta Y)^2} \\
 &= \sqrt{((-3) - 1)^2 + (7 - 5)^2} \\
 &= \sqrt{4^2 + 2^2} \\
 &= \sqrt{20} \\
 &= 2\sqrt{5} \quad \text{is the length of the line segment}
 \end{aligned}$$

[2 marks]

$$\begin{aligned}
 \text{(ii)} \quad \text{Midpoint } AB &= \left(\frac{x_B + x_A}{2}, \frac{y_B + y_A}{2} \right) \\
 &= \left(\frac{(-3) + 1}{2}, \frac{7 + 5}{2} \right) \\
 &= (-1, 6)
 \end{aligned}$$

[1 mark]

Answer 8

(i) $2x + 5y = 7 \Rightarrow y = -\frac{2}{5}x + \frac{7}{5}$ which has gradient of $-\frac{2}{5}$

$$y = mx + c$$

$$5 = -\frac{2}{5} \times 1 + c$$

$$c = \frac{27}{5}$$

$\therefore$ The line through A has equation $y = -\frac{2}{5}x + \frac{27}{5}$

[2 marks]

(ii)

$$y = -\frac{2}{5}x + \frac{27}{5}$$

$$p = -\frac{2}{5} \times 3 + \frac{27}{5}$$

$$= \frac{21}{5}$$

[2 marks]

Answer 9

(a) Putting all of the lines into the $y = mx + c$ format;

Line A $y = 2x + 3$

Line B $2y = 6 - 3x \Rightarrow y = -\frac{3}{2}x + 3$

Line C $4x - 2y = 3 \Rightarrow y = 2x - \frac{3}{2}$

Line D $y = 3 - 2x$

$\therefore$ Line A and Line C are parallel, both with a gradient of 2

[2 marks]

(b)

$$y = mx + c$$

$$3 = -\frac{5}{2} \times 1 + c$$

$$c = \frac{11}{2}$$

The line is thus, $y = -\frac{5}{2}x + \frac{11}{2}$

$$2y = -5x + 11$$

$$5x + 2y = 11 \text{ is the requested format}$$

[3 marks]

Answer 10**(a)**

$$m = \frac{\Delta Y}{\Delta X}$$

$$= \frac{-3 - 4}{8 - (-6)}$$

$$= -\frac{7}{14}$$

$$= -\frac{1}{2}$$

$$y = mx + c$$

$$4 = -\frac{1}{2} \times (-6) + c$$

$$c = 1$$

$$\text{The line is thus, } y = -\frac{1}{2}x + 1$$

$$\text{That is } x + 2y - 2 = 0$$

[4 marks]**(b)**

$$|AB| = \sqrt{(\Delta X)^2 + (\Delta Y)^2}$$

$$= \sqrt{(8 - (-6))^2 + (-3 - 4)^2}$$

$$= \sqrt{14^2 + 7^2}$$

$$= \sqrt{245}$$

$$= 7\sqrt{5} \quad \text{is the length of the line segment}$$

[3 marks]

Answer 11**(a)**

$$\begin{aligned}
 m &= \frac{\Delta Y}{\Delta X} & y &= mx + c \\
 &= \frac{0 - 4}{2 - 7} & 0 &= \frac{4}{5} \times 2 + c \\
 &= \frac{4}{5} & c &= -\frac{8}{5}
 \end{aligned}$$

The line is thus, $y = \frac{4}{5}x - \frac{8}{5}$

That is, $4x - 5y - 8 = 0$

[3 marks]**(b)**

$$\begin{aligned}
 |AB| &= \sqrt{(\Delta X)^2 + (\Delta Y)^2} \\
 &= \sqrt{(2 - 7)^2 + (0 - 4)^2} \\
 &= \sqrt{5^2 + 4^2} \\
 &= \sqrt{41}
 \end{aligned}$$

[3 marks]**(c)** $t = 8$ **[1 mark]****(d)**

$$\begin{aligned}
 \text{Area } \Delta &= \frac{1}{2}bh \\
 &= \frac{1}{2} \times 8 \times (7 - 2) \\
 &= 20
 \end{aligned}$$

[2 marks]