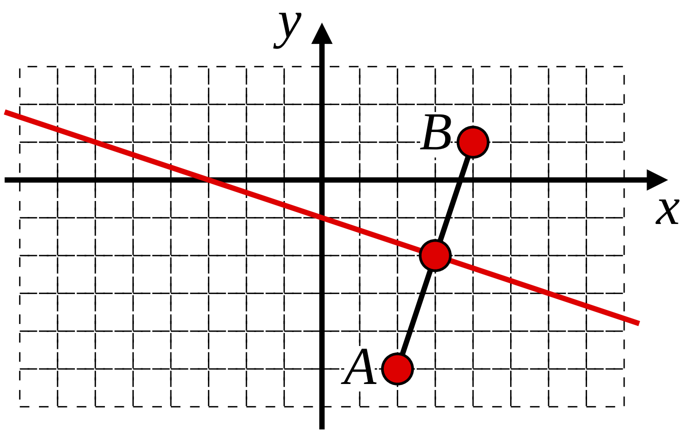
4.4 Answers

A-Level Pure Mathematics, Year 1
Additional Mathematics
GCSE
Coordinate Geometry

4.4.1 Solutions to 4.2 Example (Teaching Video)



$$\textit{Midpoint of } AB = \left(\frac{x_B + x_A}{2}, \frac{y_B + y_A}{2} \right)$$

$$= \left(\frac{4 + 2}{2}, \frac{1 + (-5)}{2} \right)$$

$$= (3, -2)$$

$$\textit{Gradient of } AB = \frac{\Delta Y}{\Delta X} = \frac{1 - (-5)}{4 - 2} = \frac{6}{2} = 3$$

The line perpendicular to AB will have gradient $-\frac{1}{3}$

("Sign changed reciprocal")

Equation of perpendicular bisector:

$$y = mx + c$$

$$-2 = -\frac{1}{3} \times 3 + c$$

$$\therefore c = -1$$

Thus, $\quad y = -\frac{1}{3}x - 1$

[4 marks]

4.4.2 Solutions to 4.3 Exercise

Answer 1

(i) $\frac{2}{3}$ **(ii)** $-\frac{3}{2}$

[1, 1 mark]

Answer 2

(i) $y = -\frac{2}{5}x + \frac{4}{5}$ **(ii)** $-\frac{2}{5}$ **(iii)** $\frac{5}{2}$

[1, 1, 1 mark]

Answer 3

$$y = \frac{5}{3}x - \frac{2}{3}$$

(i) $\frac{5}{3}$ **(ii)** $-\frac{3}{5}$

[2, 1 mark]

Answer 4

$2x + 3y = 5 \Rightarrow y = -\frac{2}{3}x + \frac{5}{3}$ which has gradient $-\frac{2}{3}$

Require a line with the sign changed reciprocal gradient through $(3, 4)$

$$y = mx + c$$

$$4 = \frac{3}{2} \times 3 + c \Rightarrow c = -\frac{1}{2}$$

$$\therefore y = \frac{3}{2}x - \frac{1}{2}$$

[3 marks]

Answer 5

$$Midpoint\ PQ = \left(\frac{9 + (-1)}{2}, \frac{0 + 6}{2} \right) = (4, 3)$$

$$Gradient\ PQ = \frac{0 - 6}{9 - (-1)} = -\frac{6}{10} = -\frac{3}{5}$$

Require a line with the sign changed reciprocal gradient through $(4, 3)$

$$y = mx + c$$

$$3 = \frac{5}{3} \times 4 + c \Rightarrow c = -\frac{11}{3}$$

$$\therefore y = \frac{5}{3}x - \frac{11}{3} \Rightarrow 5x - 3y - 11 = 0$$

[5 marks]

Answer 6

(a) $3x + 5y - 2 = 0 \Rightarrow y = -\frac{3}{5}x + \frac{2}{5}$

$\therefore$ The line l_1 has a gradient of $-\frac{3}{5}$

[2 marks]

(b) Require l_2 to have sign changed reciprocal gradient through $(3, \ 1)$

$$y = mx + c$$

$$1 = \frac{5}{3} \times 3 + c \Rightarrow c = -4$$

$$\therefore y = \frac{5}{3}x - 4$$

[3 marks]

Answer 7

(a) Put $x = 3$ into $y = 5 - 2x \Rightarrow y = -1$

$\therefore$ As the answer was -1, the point $(3, -1)$ is on L

[1 mark]

(b) The sign changed reciprocal gradient through $(3, -1)$ leads to,

$$y = mx + c$$

$$-1 = \frac{1}{2} \times 3 + c \Rightarrow c = -\frac{5}{2}$$

$$\therefore y = \frac{1}{2}x - \frac{5}{2} \Rightarrow x - 2y - 5 = 0$$

[4 marks]

Answer 8

(i) $m_{AB} = \dfrac{4 - (-1)}{0 - (-5)} = \dfrac{5}{5} = 1$ $\quad$ $m_{CD} = \dfrac{(-2) - 3}{2 - 7} = \dfrac{-5}{-5} = 1$

$\therefore$ As $m_{AB} = m_{CD}$, AB is parallel to CD

$m_{BC} = \dfrac{3 - 4}{7 - 0} = \dfrac{-1}{7}$ $\quad$ $m_{DA} = \dfrac{-2 - (-1)}{2 - (-5)} = \dfrac{-1}{7}$

$\therefore$ As $m_{BC} = m_{DA}$, BC is parallel to DA

Taken together, these two results show that $ABCD$ is a parallelogram ☐

[2 marks]

(ii) $|AB| = \sqrt{(0 - (-5))^2 + (4 - (-1))^2} = \sqrt{50}$

$|BC| = \sqrt{(7 - 0)^2 + (3 - 4)^2} = \sqrt{50}$

$|CD| = \sqrt{(2 - 7)^2 + (-2 - 3)^2} = \sqrt{50}$

$|DA| = \sqrt{(2 - (-5))^2 + ((-1) - (-2))^2} = \sqrt{50}$

$\therefore$ As $|AB| = |BC| = |CD| = |DA|$, $ABCD$ is a rhombus ☐

(All sides have the same length, $\sqrt{50}$)

[2 marks]

(iii) As $m_{AB} \times m_{BC} \neq -1$ the lines AB and BC are not perpendicular which they would be, if $ABCD$ where a square.

$\therefore$ $ABCD$ is not a square ☐

[2 marks]

Answer 9

(a) $2y - 3x - k = 0$ with $A(1,\ 4)$

$$\Rightarrow 2 \times 4 - 3 \times 1 - k = 0$$

$$k = 5$$

[1 mark]

(b) $2y - 3x - 5 = 0 \Rightarrow y = \dfrac{3}{2}x + \dfrac{5}{2}$

$\therefore$ The gradient of L_1 is $\dfrac{3}{2}$

[2 marks]

(c) The sign changed reciprocal gradient through $A(1,\ 4)$ leads to,

$$y = mx + c$$

$$4 = -\frac{2}{3} \times 1 + c \Rightarrow c = \frac{14}{3}$$

$$\therefore y = -\frac{2}{3}x + \frac{14}{3} \Rightarrow 2x + 3y - 14 = 0$$

[4 marks]

(d) Where a line crosses the x-axis, $y = 0$

$$\therefore 2x + 3 \times 0 - 14 = 0$$

$$2x = 14$$

$$x = 7 \qquad \Rightarrow B(7,\ 0)$$

[2 marks]

(e) $A(1,\ 4),\ B(7,\ 0)$

$$|AB| = \sqrt{(7-1)^2 + (0-4)^2}$$

$$= \sqrt{6^2 + 4^2}$$

$$= \sqrt{52}$$

$$= 2\sqrt{13}$$

[2 marks]

Answer 10

(a) $3x + 2y - 8 = 0 \Rightarrow y = -\frac{3}{2}x + 4$

$\therefore$ the line l_2 has a gradient of $-\frac{3}{2}$

[2 marks]

(b)

$$3x + 2 = -\frac{3}{2}x + 4$$

multiple through by 2

$$6x + 4 = -3x + 8$$

$$9x = 4$$

$$x = \frac{4}{9}$$

$$y = 3 \times \frac{4}{9} + 2$$

$$= \frac{10}{3}$$

$\therefore$ P is the point $\left(\frac{4}{9},\ \frac{10}{3}\right)$

[3 marks]

(c) To find the point A, $1 = 3x + 2 \Rightarrow x = -\frac{1}{3} \Rightarrow A\left(-\frac{1}{3},\ 1\right)$

To find the point B, $1 = -\frac{3}{2}x + 4$

multiply through by 2

$$2 = -3x + 8$$

$$x = 2 \Rightarrow B(2,\ 1)$$

$$Area\ \Delta = \frac{1}{2}bh$$

$$= \frac{1}{2} \times \left(2 - \left(-\frac{1}{3}\right)\right)\left(\frac{10}{3} - 1\right)$$

$$= \frac{1}{2} \times \frac{7}{3} \times \frac{7}{3}$$

$$= \frac{49}{18} \qquad \text{(about 2.72)}$$

[4 marks]