5.2 Answers

A-Level Pure Mathematics, Year 1
Additional Mathematics
GCSE
Coordinate Geometry

Answer 1

$$m_{AB} = \frac{6-2}{15-7} = \frac{4}{8} = \frac{1}{2}$$

[2 marks]

Answer 2

$$Midpoint\ AB = \left(\frac{6+4}{2}, \frac{7+(-9)}{2} \right) = (5, -1)$$

[2 marks]

Answer 3

$$D = \sqrt{(15-3)^2 + (4-(-1))^2} = \sqrt{12^2 + 5^2} = 13$$

[2 marks]

Answer 4

(i) $m_{parallel} = -\frac{4}{5}$

(ii) $m_{perpendicular} = \frac{5}{4}$

[1, 1 mark]

Answer 5

$$y = mx + c$$

$$21 = 5 \times 11 + c \Rightarrow c = -34$$

$\therefore$ The line will have equation $y = 5x - 34$

[3 marks]

Answer 6

$$5x - 4y - 12 = 0$$

$$4y = 5x - 12$$

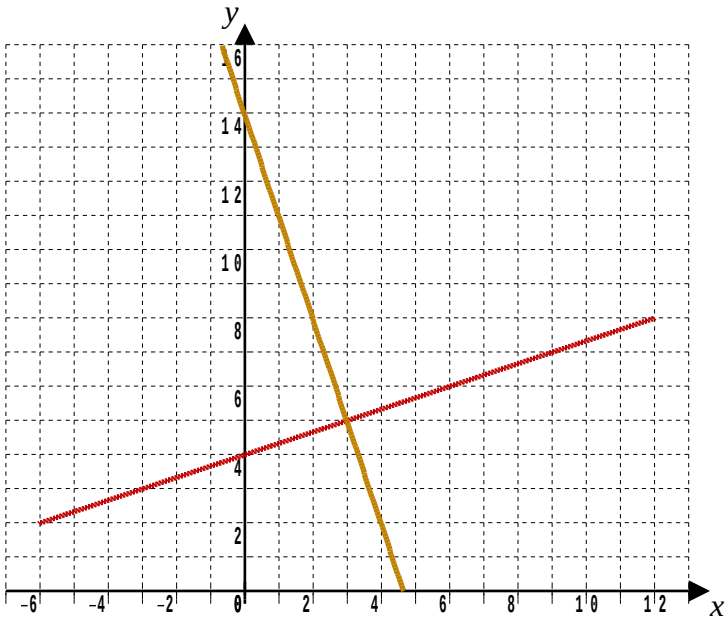
$$y = \frac{5}{4}x - 3$$

The gradient is $\frac{5}{4}$

[2 marks]

Answer 7

(i), (iii)



[3, 2 marks]

(ii)

$$f(-6) = \frac{1}{3} \times (-6) + 4 = 2 \Rightarrow (-6,\ 2) \text{ is an endpoint}$$

$$f(12) = \frac{1}{3} \times 12 + 4 = 8 \Rightarrow (12,\ 8) \text{ is an endpoint}$$

$$\textit{Midpoint} = \left(\frac{12 + (-6)}{2},\ \frac{8 + 2}{2} \right) = (3,\ 5)$$

$$\textit{Gradient} = \frac{1}{3} \text{ from the equation given in the question}$$

Require a line with the sign changed reciprocal gradient through $(3,\ 5)$

$$y = mx + c$$

$$5 = -3 \times 3 + c \Rightarrow c = 14$$

$$\therefore\ y = -3x + 14$$

[5 marks]

Answer 8

$$\textit{Midpoint } BD = \left(\frac{2 + 10}{2}, \frac{7 + 19}{2} \right) = (6,\ 13)$$

$$\textit{Gradient } BD = \frac{7 - 19}{2 - 10} = \frac{-12}{-8} = \frac{3}{2}$$

Require a line with the sign changed reciprocal gradient through $(6,\ 13)$

$$y = mx + c$$

$$13 = -\frac{2}{3} \times 6 + c \Rightarrow c = 17$$

$$\therefore\ y = -\frac{2}{3}x + 17$$

multiply through by 3

$$3y = -2x + 51$$

$$3y + 2x = 51$$

[5 marks]

Answer 9

(i) The sign changed reciprocal gradient is -2

$$y = mx + c$$

$$7 = -2 \times 3 + c \Rightarrow c = 13$$

$\therefore$ The line l has equation $y = -2x + 13$

[3 marks]

(ii)

$$-2x + 13 = \frac{1}{2}x - 2$$

multiply through by 2

$$-4x + 26 = x - 4$$

$$5x = 30$$

$$x = 6 \Rightarrow y = 1$$

The two lines intersect at the point $(6,\ 1)$

[3 marks]

Answer 10

$$m_{AB} = \frac{18 - 3}{2 - (-3)} = \frac{15}{5} = 3$$

$$y = mx + c$$

$$18 = 3 \times 2 + c \Rightarrow c = 12$$

$\therefore$ The line through A and B is $y = 3x + 12$

This line cuts the x-axis when $y = 0 \Rightarrow (-4,\ 0)$

and cuts the y-axis at $(0, 12)$

Square has sides of length $\sqrt{12^2 + 4^4} = \sqrt{160}\ \left(= 4\sqrt{10}\right)$

$\therefore$ Area of the square is 160 units^2

[5 marks]

Answer 11

$$3x + 8 = -2x + 13$$

$$5x = 5$$

$$x = 1 \Rightarrow y = 11$$

So a line through $(1,\ 11)$ with gradient $\frac{1}{2}$ is required

$$y = mx + c$$

$$11 = \frac{1}{2} \times 1 + c \Rightarrow c = \frac{21}{2}$$

$$y = \frac{1}{2}x + \frac{21}{2}$$

$$\therefore\ x - 2y + 21 = 0$$

[6 marks]

Answer 12

$$x^2 + 4x - 5 = 3x + 1$$

$$x^2 + x - 6 = 0$$

$$(x + 3)(x - 2) = 0$$

Either $x = -3$ or $x = 2$

Points of intersection are thus $(-3, -8)$, $(2, 7)$

$$D = \sqrt{(7 - (-8))^2 + (2 - (-3))^2}$$

$$= \sqrt{15^2 + 5^2}$$

$$= \sqrt{250}$$

$$= 5\sqrt{10} \qquad \text{(about 15.8)}$$

[6 marks]

Answer 13

(a) $2x - 3y + 12 = 0 \Rightarrow y = \frac{2}{3}x + 4 \quad \therefore m = \frac{2}{3}$

[1 mark]

(b) $A(-6, 0)$ $B(0, 4)$

l_2 has a sign changed reciprocal gradient of $-\frac{3}{2}$ through $B(0, 4)$

$$\therefore y = -\frac{3}{2}x + 4$$

[3 marks]

(c) $C\left(\frac{8}{3}, 0\right)$

$$\text{Area } \Delta ABC = \frac{1}{2} \times \left(6 + \frac{8}{3}\right) \times 4$$

$$= \frac{1}{2} \times \frac{26}{3} \times 4$$

$$= \frac{52}{3} = 17\frac{1}{3}$$

[4 marks]

Answer 14

(a) $m = \dfrac{8-2}{11-(-1)} = \dfrac{6}{12} = \dfrac{1}{2}$

$$y = mx + c$$

$$8 = \frac{1}{2} \times 11 + c \Rightarrow c = \frac{5}{2}$$

$$\therefore y = \frac{1}{2}x + \frac{5}{2}$$

[3 marks]

(b) l_2 has a sign changed reciprocal gradient of -2 through $R(10,\ 0)$

$$y = mx + c$$

$$0 = -2 \times 10 + c \Rightarrow c = 20$$

$$\therefore y = -2x + 20 \text{ is the equation of } l_2$$

Intersection is when, $\dfrac{1}{2}x + \dfrac{5}{2} = -2x + 20$

multiply through by 2

$$x + 5 = -4x + 40$$

$$5x = 35$$

$$x = 7 \Rightarrow y = 6$$

$$\therefore S(7,\ 6)$$

[4 marks]

(c) $R(10,\ 0),\ S(7,\ 6)$

$$|RS| = \sqrt{(7-10)^2 + (6-0)^2}$$

$$= \sqrt{3^2 + 6^2}$$

$$= \sqrt{45}$$

$$= 3\sqrt{5} \qquad \square$$

[2 marks]

(d) $P(-1,\ 2),\ Q(11,\ 8)$

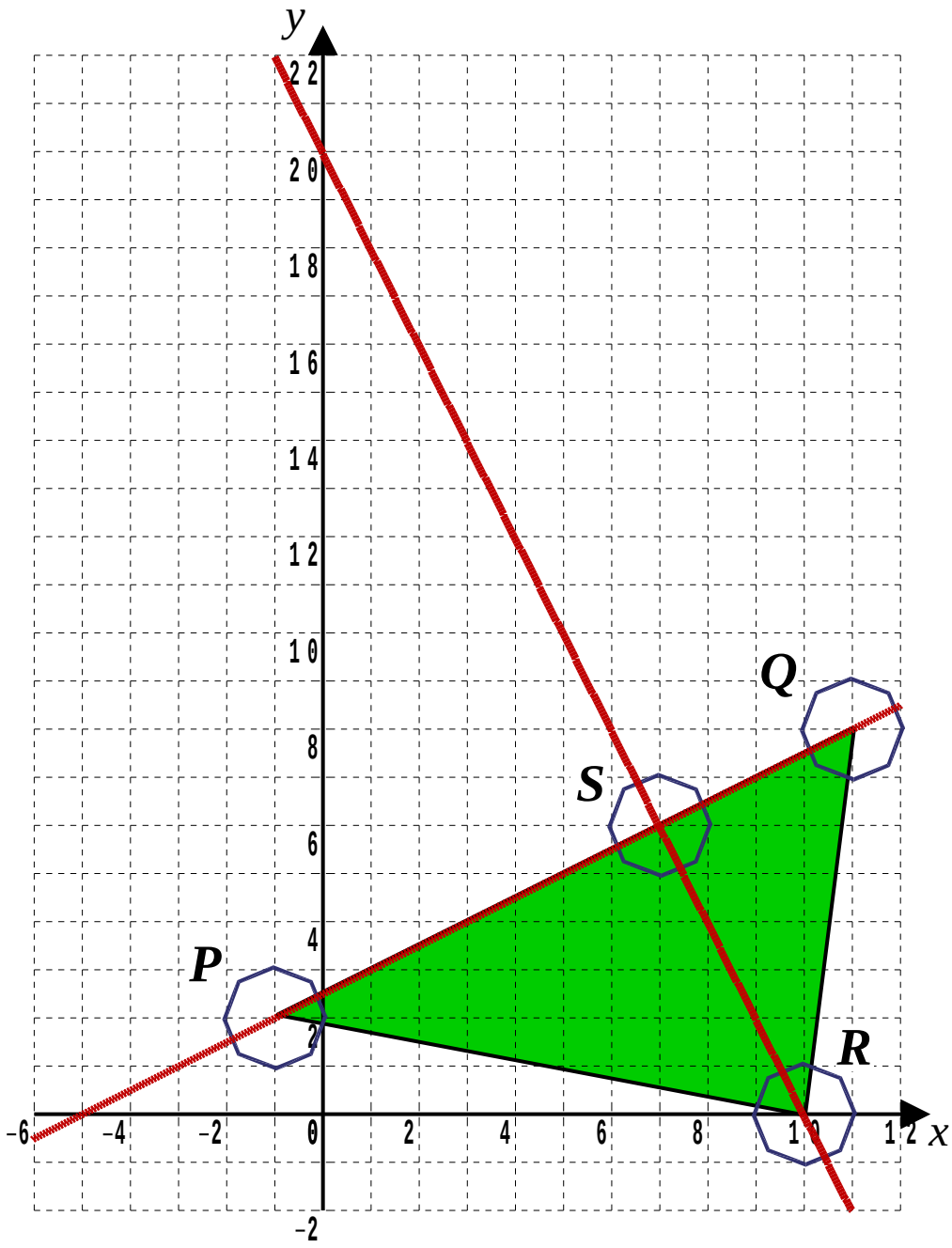
$$|PQ| = \sqrt{(8-2)^2 + (11-(-1))^2}$$

$$= \sqrt{6^2 + 12^2}$$

$$= \sqrt{180}$$

$$= 6\sqrt{5} \qquad \square$$

A diagram is needed to properly see what is going on...



(d) $P(-1,\ 2),\ Q(11,\ 8),\ R(10,\ 0),\ S(7,\ 6)$

$$\text{Area } \triangle PQR = \frac{1}{2} \times |PQ| \times |RS|$$

$$= \frac{1}{2} \times 6\sqrt{5} \times 3\sqrt{5}$$

$$= 45$$

[4 marks]