

6.4 Answers

A-Level Pure Mathematics, Year 1 Additional Mathematics Coordinate Geometry

6.4.1 Solution to 6.2 Example (Teaching Video)

Answer 1

(i) The centre is $(4, -3)$ and the radius is 5

[2 marks]

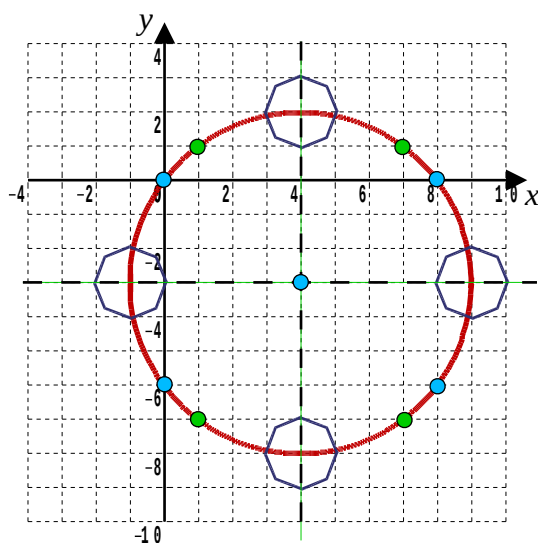
(ii)

$$\begin{aligned}\text{LHS} &= (x - 4)^2 + (y + 3)^2 \\ &= (0 - 4)^2 + (0 + 3)^2 \quad \text{if the circle passes through } (0, 0) \\ &= (-4)^2 + 3^2 \\ &= 16 + 9 \\ &= 25 \\ &= \text{RHS}\end{aligned}$$

$\therefore$ The circle passes through $(0, 0)$ $\square$

[2 marks]

(iii)



[2 marks]

6.4.2 Solutions to 6.3 Exercise

Answer 1

$$(x - 16)^2 + (y - 9)^2 = 100$$

The centre is (16, 9) and the radius is 10

[2 marks]

Answer 2

$$(x + 25)^2 + (y + 121)^2 = 17^2$$

The centre is (-25, -121) and the radius is 17

[2 marks]

Answer 3

(i) $(x - 12)^2 + (y + 5)^2 = 400$

The centre is (12, -5) and the radius is 20

[2 marks]

(ii)
$$\begin{aligned} |OC| &= \sqrt{(12 - 0)^2 + (-5 - 0)^2} \\ &= \sqrt{12^2 + 5^2} \\ &= 13 \end{aligned}$$

Notice that the origin is inside the circle

[2 marks]

Answer 4

Centre (2, 8), Radius 7

$$(x - 2)^2 + (y - 8)^2 = 7^2$$

[2 marks]

Answer 5

Centre (-2.1, 4.8), Radius 3.6

$$(x + 2.1)^2 + (y - 4.8)^2 = 3.6^2$$

[2 marks]

Answer 6

Centre (-1, -5), Radius $\sqrt{13}$

$$(x + 1)^2 + (y + 5)^2 = 13$$

[2 marks]

Answer 7

$$\begin{aligned}
 \text{(i)} \quad \text{Radius} &= \sqrt{(9 - 3)^2 + (7 - 2)^2} \\
 &= \sqrt{6^2 + 5^2} \\
 &= \sqrt{61} \quad (\text{about } 7.81)
 \end{aligned}$$

[2 marks]

$$\begin{aligned}
 \text{(ii)} \quad &\text{Centre } (2, 3), \text{ Radius } \sqrt{61} \\
 &(x - 2)^2 + (y - 3)^2 = 61
 \end{aligned}$$

[2 marks]**Answer 8**

$$\begin{aligned}
 \text{(i)} \quad |AB| &= \sqrt{(5 - 1)^2 + (3 - 1)^2} \\
 &= \sqrt{4^2 + 2^2} \\
 &= \sqrt{20} \\
 &= 2\sqrt{5}
 \end{aligned}$$

[2 marks]

$$\begin{aligned}
 \text{(ii)} \quad \text{Midpoint } AB &= \left(\frac{5 + 1}{2}, \frac{3 + 1}{2} \right) \\
 &= (3, 2)
 \end{aligned}$$

[1 mark]

$$\begin{aligned}
 \text{(iii)} \quad &\text{Centre } (3, 2) \text{ Radius } \sqrt{5} \\
 &(x - 3)^2 + (y - 2)^2 = 5
 \end{aligned}$$

[3 marks]**Answer 9**

$$\begin{aligned}
 \text{For the point } (5, 2), \quad x^2 + y^2 &= 5^2 + 2^2 \\
 &= 25 + 4 \\
 &= 29
 \end{aligned}$$

As $29 < 30$ the point $(5, 2)$ is inside the circle**[3 marks]**

Answer 10

(i) $(10, 8)$ (ii) 10

[1, 1 marks]

(iii) $\text{LHS} = (x - 10)^2 + (y - 8)^2$
 $= (18 - 10)^2 + (14 - 8)^2$ if the circle passes through $(18, 14)$
 $= 8^2 + 6^2$
 $= 100$
 $= \text{RHS} \quad \therefore \text{The circle passes through } (18, 14)$ $\square$

[2 marks]

(iv) Circle crosses the x -axis when $y = 0$ in which case,

$$(x - 10)^2 + (0 - 8)^2 = 100$$

$$x^2 - 20x + 100 + 64 = 100$$

$$x^2 - 20x + 64 = 0$$

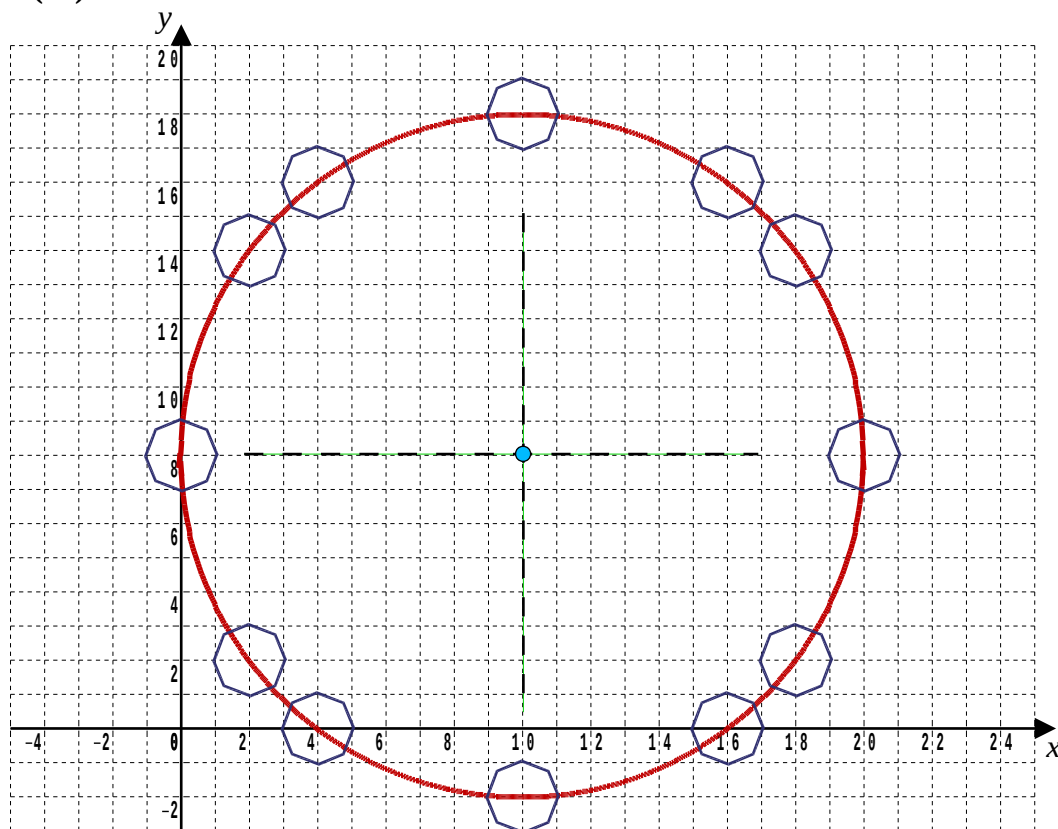
$$(x - 4)(x - 16) = 0$$

$$\text{Either } x - 4 = 0 \text{ or } x - 16 = 0$$

$$x = 4 \text{ or } x = 16, \text{ that is, } (4, 0) \text{ and } (16, 0)$$

[2 marks]

(v)



[3 marks]

Answer 11

$A(1, 10)$, $B(8, 9)$ and $C(7, 2)$

(i) Midpoint AC , $M = \left(\frac{7+1}{2}, \frac{2+10}{2} \right) = (4, 6)$

[1 mark]

(ii) $|AC| = \sqrt{(7-1)^2 + (2-10)^2}$
 $= \sqrt{6^2 + 8^2}$
 $= 10$

$\therefore \text{Radius} = 5$

And the equation of the circle will be $(x-4)^2 + (y-6)^2 = 5^2$

[4 marks]

(iii) LHS $= (x-4)^2 + (y-6)^2$
 $= (8-4)^2 + (9-6)^2$ if the circle passes through $(8, 9)$
 $= 4^2 + 3^2$
 $= 16 + 9$
 $= 25$
 $= \text{RHS} \quad \therefore \text{The circle passes through } B(8, 9)$

□

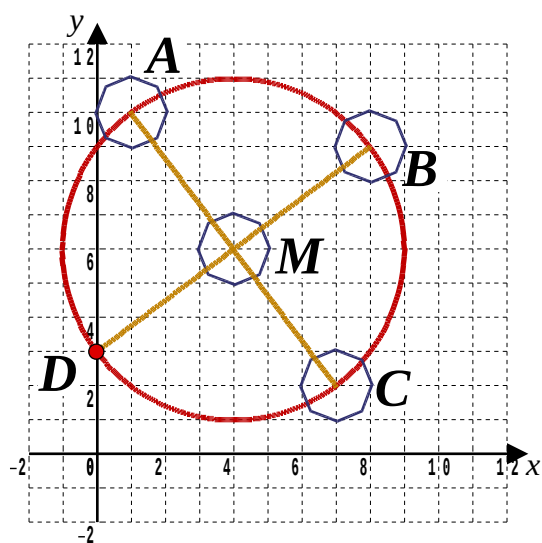
[1 mark]

(iv) $m_{AM} = \frac{\Delta Y}{\Delta X} = \frac{6-10}{4-1} = -\frac{4}{3}$
 $m_{BM} = \frac{\Delta Y}{\Delta X} = \frac{6-9}{4-8} = \frac{3}{4}$

As $m_{AM} \times m_{BM} = -1$ AM is perpendicular to BM □

[3 marks]

(v) A diagram is an excellent idea in answering this question



Here, I've used a vector method,

$$\begin{aligned}\overrightarrow{BM} &= m - b \\ &= \begin{pmatrix} 4 \\ 6 \end{pmatrix} - \begin{pmatrix} 8 \\ 9 \end{pmatrix} = \begin{pmatrix} -4 \\ -3 \end{pmatrix} \\ \overrightarrow{OD} &= \overrightarrow{OM} + \overrightarrow{MD} \quad \text{but } \overrightarrow{BM} = \overrightarrow{MD} \\ &= \begin{pmatrix} 4 \\ 6 \end{pmatrix} + \begin{pmatrix} -4 \\ -3 \end{pmatrix} \\ &= \begin{pmatrix} 0 \\ 3 \end{pmatrix} \\ \therefore D(0, 3)\end{aligned}$$

[3 marks]

Answer 12

(a) Radius 10, Centre (2, 0)

[2 marks]

(b) (i) $(x - 2)^2 + y^2 = 100$ and $y = 2x + 6$

$$(x - 2)^2 + (2x + 6)^2 = 100$$

$$x^2 - 4x + 4 + 4x^2 + 24x + 36 = 100$$

$$5x^2 + 20x - 60 = 0$$

$$x^2 + 4x - 12 = 0$$

$$(x + 6)(x - 2) = 0$$

Either $x = -6$ or $x = 2$

Points of intersection are, $(-6, -6)$, $(2, 10)$

[5 marks]

$$(ii) \quad \text{Midpoint, } M = \left(\frac{2 + (-6)}{2}, \frac{10 + (-6)}{2} \right) = (-2, 2)$$

[1 mark]

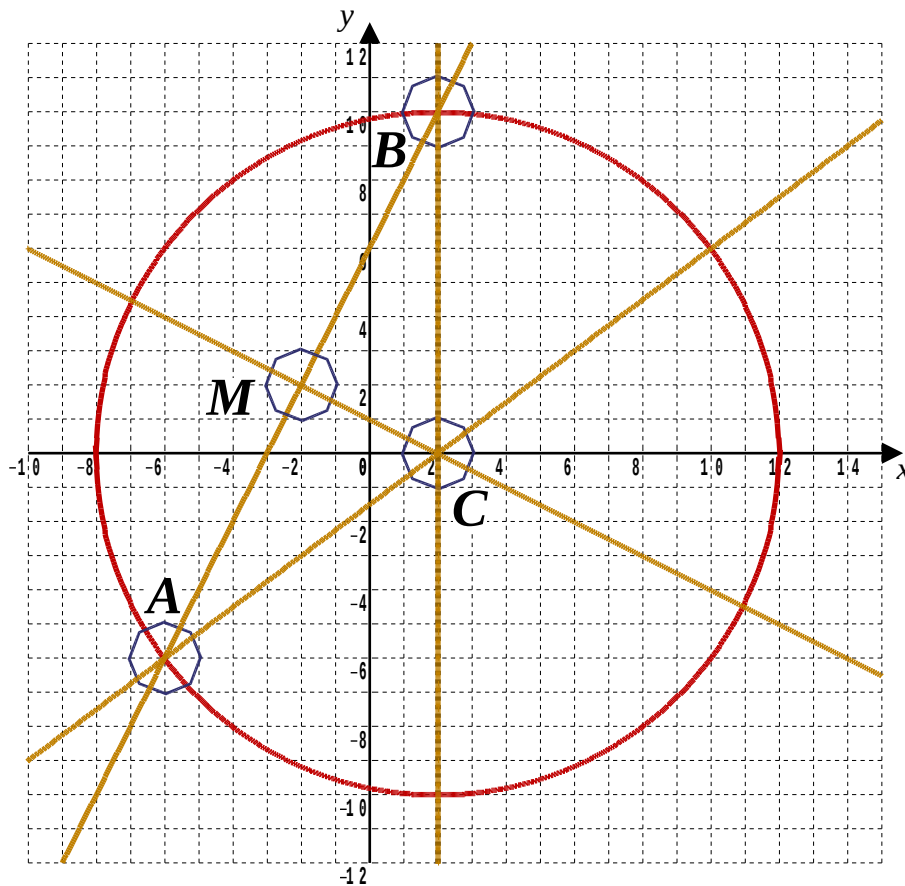
$$(iii) \quad |AB| = \sqrt{(2 - (-6))^2 + (10 - (-6))^2}$$

$$= \sqrt{8^2 + 16^2}$$

$$= 8\sqrt{5}$$

[2 marks]

(c) A diagram is helpful...



$$|MC| = \sqrt{10^2 - (4\sqrt{5})^2}$$

$$= \sqrt{100 - 16 \times 5}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5}$$

[2 marks]

This document is a part of a **Mathematics Community Outreach Project** initiated by Shrewsbury School

It may be freely duplicated and distributed, unaltered, for non-profit educational use

In October 2020, Shrewsbury School was voted "**Independent School of the Year 2020**"

© 2025 Number Wonder

Teachers may obtain detailed worked solutions to the exercises by email from MHShrewsbury@Gmail.com