

7.4 Answers

A-Level Pure Mathematics, Year 1 Additional Mathematics Coordinate Geometry

7.4.1 Solution to 7.2 Completing the Square (Teaching Video)

$$x^2 + y^2 + 12x - 2y = 27$$

$$\left[x^2 + 12x \right] + \left[y^2 - 2y \right] = 27$$

$$\left[(x + 6)^2 - 36 \right] + \left[(y - 1)^2 - 1 \right] = 27$$

$$(x + 6)^2 + (y - 1)^2 = 27 + 36 + 1$$

$$(x + 6)^2 + (y - 1)^2 = 64$$

This is the equation of a circle, centre $(-6, 1)$ and radius 8

7.4.2 Solution to 7.3 Exercise

Answer 1

(i) $y = x^2 + 8x + 17$

$$= \left[x^2 + 8x \right] + 17$$

$$= \left[(x + 4)^2 - 16 \right] + 17$$

$$= (x + 4)^2 + 1$$

(ii) $y = x^2 + 10x + 7$

$$= \left[x^2 + 10x \right] + 7$$

$$= \left[(x + 5)^2 - 25 \right] + 7$$

$$= (x + 5)^2 - 18$$

(iii) $y = x^2 - 12x + 3$

$$= \left[x^2 - 12x \right] + 3$$

$$= \left[(x - 6)^2 - 36 \right] + 3$$

$$= (x - 6)^2 - 33$$

(iv) $y = x^2 - 6x - 7$

$$= \left[x^2 - 6x \right] - 7$$

$$= \left[(x - 3)^2 - 9 \right] - 7$$

$$= (x - 3)^2 - 16$$

[8 marks]

Answer 2

(i) $x^2 - 4x + y^2 - 8y = 44$

$$\left[x^2 - 4x \right] + \left[y^2 - 8y \right] = 44$$

$$\left[(x - 2)^2 - 4 \right] + \left[(y - 4)^2 - 16 \right] = 44$$

$$(x - 2)^2 + (y - 4)^2 = 64$$

[4 marks]

(ii) Centre is $(2, 4)$

[1 mark]

(iii) Radius is 8

[1 mark]

Answer 3

$$\begin{aligned}
 \text{(i)} \quad y &= (x + 3)^2 + 4 \\
 &= x^2 + 6x + 9 + 4 \\
 &= x^2 + 6x + 13
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad y &= (x + 7)^2 - 10 \\
 &= x^2 + 14x + 49 - 10 \\
 &= x^2 + 14x + 39
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad y &= (x + 1)^2 + 7 \\
 &= x^2 + 2x + 1 + 7 \\
 &= x^2 + 2x + 8
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad y &= \left(x + \frac{1}{2}\right)^2 + 10 \\
 &= x^2 + x + \frac{1}{4} + 10 \\
 &= x^2 + x + 10\frac{1}{4}
 \end{aligned}$$

[8 marks]**Answer 4**

$$\begin{aligned}
 \text{(i)} \quad & x^2 + y^2 + 8x - 14y + 29 = 0 \\
 & [x^2 + 8x] + [y^2 - 14y] + 29 = 0 \\
 & [(x + 4)^2 - 16] + [(y - 7)^2 - 49] + 29 = 0 \\
 & (x + 4)^2 + (y - 7)^2 = 36
 \end{aligned}$$

[4 marks]

$$\text{(ii)} \quad \text{Centre is } (-4, 7)$$

[1 mark]

$$\text{(iii)} \quad \text{Radius is } 6$$

[1 mark]**Answer 5**

$$\begin{aligned}
 & x^2 + y^2 - 4x - 6y + 3 = 0 \\
 & [x^2 - 4x] + [y^2 - 6y] + 3 = 0 \\
 & [(x - 2)^2 - 4] + [(y - 3)^2 - 9] + 3 = 0 \\
 & (x - 2)^2 + (y - 3)^2 = 10
 \end{aligned}$$

This is a circle, centre (2, 3) and radius $\sqrt{10}$ (about 3.16)

[4 marks]

Answer 6**(i)**

$$\begin{aligned}
 x^2 + y^2 - 8x + 2y - 19 &= 0 \\
 [x^2 - 8x] + [y^2 + 2y] - 19 &= 0 \\
 [(x - 4)^2 - 16] + [(y + 1)^2 - 1] - 19 &= 0 \\
 (x - 4)^2 + (y + 1)^2 &= 36
 \end{aligned}$$

[4 marks]**(ii)**

$$\begin{aligned}
 \text{LHS} &= (x - 4)^2 + (y + 1)^2 \\
 &= (8 - 4)^2 + (3 + 1)^2 \quad \text{if the circle passes through } (8, 3) \\
 &= 4^2 + 4^2 \\
 &= 16 + 16 \\
 &= 32 \\
 &< \text{RHS of } 36 \quad \therefore (8, 3) \text{ is inside the circle}
 \end{aligned}$$

□

[2 marks]**Answer 7****(i)** The midpoint of PQ will be the centre, C , of the circle

$$\text{Midpoint } PQ = \left(\frac{15 + 1}{2}, \frac{1 + 3}{2} \right) = (8, 2)$$

[2 marks]**(ii)** The distance PQ will be the diameter of the circle

$$\begin{aligned}
 |PQ| &= \sqrt{(15 - 1)^2 + (1 - 3)^2} \\
 &= \sqrt{14^2 + 2^2} \\
 &= \sqrt{196 + 4} \\
 &= 10\sqrt{2}
 \end{aligned}$$

$$\therefore \text{Radius is } 5\sqrt{2} \quad \square$$

[2 marks]**(iii)** The centre at $(8, 2)$ and radius of $5\sqrt{2}$ give,

$$\begin{aligned}
 (x - 8)^2 + (y - 2)^2 &= 25 \times 2 \\
 x^2 - 16x + 64 + y^2 - 4y + 4 &= 50 \\
 x^2 + y^2 - 16x - 4y + 18 &= 0
 \end{aligned}$$

$$\text{That is, } a = -16, \quad b = -4 \text{ and } c = 18$$

[2 marks]

Answer 8

$$(i) \quad x^2 + y^2 + 41 = 10(x + y)$$

$$[x^2 - 10x] + [y^2 - 10y] + 41 = 0$$

$$[(x - 5)^2 - 25] + [(y - 5)^2 - 25] + 41 = 0$$

$$(x - 5)^2 + (y - 5)^2 = 9$$

[4 marks]**(ii)** Centre is (5, 5)**[1 mark]****(iii)** Radius is 3**[1 mark]****Answer 9**

$$(i) \quad x^2 + y^2 - 2x - 4y - 20 = 0$$

$$[x^2 - 2x] + [y^2 - 4y] - 20 = 0$$

$$[(x - 1)^2 - 1] + [(y - 2)^2 - 4] - 20 = 0$$

$$(x - 1)^2 + (y - 2)^2 = 25$$

This represents a circle with centre (1, 2) and radius 5

[3 marks]**(ii)**

$$(x - 1)^2 + (y - 2)^2 = 25 \text{ and } y = x + 2$$

$$(x - 1)^2 + ((x + 2) - 2)^2 = 25$$

$$(x - 1)^2 + x^2 = 25$$

$$x^2 - 2x + 1 + x^2 = 25$$

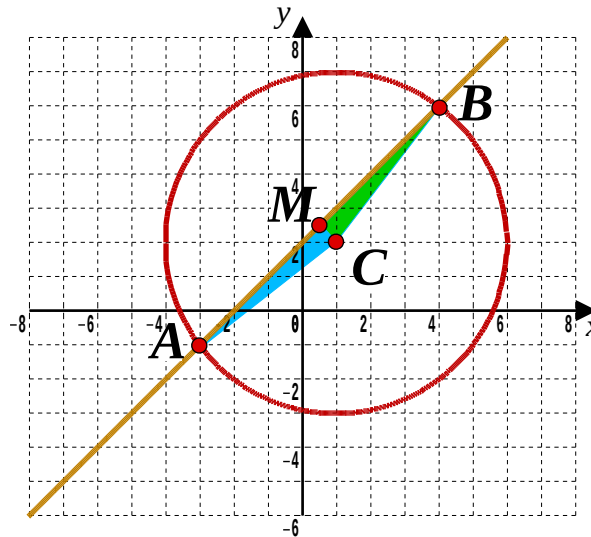
$$2x^2 - 2x - 24 = 0$$

divide through by 2

$$x^2 - x - 12 = 0$$

$$(x - 4)(x + 3) = 0$$

Either $x = 4$ or $x = -3$ $\therefore$ Using, $y = x + 2$, the points are (4, 6), (-3, -1)**[5 marks]****(iii)** A diagram is needed for this part of the question.Using the part (i) answer, draw the circle with centre (1, 2) and radius 5
(using the NEWS method)Add the straight line $y = x + 2$ and the part (ii) intersections A at (-3, -1)
and B at (4, 6)



$$|AB| = \sqrt{(-3 - 4)^2 + (-1 - 6)^2} = \sqrt{7^2 + 7^2} = \sqrt{98} = 7\sqrt{2}$$

Let M be the midpoint at AB in which case $\angle CMA$ is a right angle.

$$\angle ACM = \arcsin\left(\frac{3.5\sqrt{2}}{5}\right) = 81.87 \Rightarrow \angle ACB = 163.7$$

[4 marks]

Answer 10

$$x^2 + y^2 + 4x - 2y - 11 = 0$$

$$[x^2 + 4x] + [y^2 - 2y] - 11 = 0$$

$$[(x + 2)^2 - 4] + [(y - 1)^2 - 1] - 11 = 0$$

$$(x + 2)^2 + (y - 1)^2 = 16$$

(a) Centre is $(-2, 1)$

[2 marks]

(b) Radius is 4

[2 marks]

(c) The circle will cross the y -axis when $x = 0$

$$(0 + 2)^2 + (y - 1)^2 = 16$$

$$4 + (y - 1)^2 = 16$$

$$y - 1 = \pm\sqrt{12}$$

$$y = 1 \pm 2\sqrt{3}$$

The points are thus $(0, 1 - 2\sqrt{3})$, $(0, 1 + 2\sqrt{3})$

[4 marks]

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