

8.5 Answers

A-Level Pure Mathematics, Year 1 Additional Mathematics Coordinate Geometry

8.5.1 Solution to 8.2 Example

The midpoint of the two given points will be the centre of the circle.

$$\left(\frac{5 + (-3)}{2}, \frac{19 + 4}{2} \right) = (1, 11.5)$$

Half the distance between the two given points will be the radius.

$$\begin{aligned} r &= \frac{1}{2} \sqrt{\Delta y^2 + \Delta x^2} \\ &= \frac{1}{2} \sqrt{(19 - 4)^2 + (5 - (-3))^2} \\ &= \frac{1}{2} \sqrt{15^2 + 8^2} \\ &= \frac{17}{2} \\ &= 8.5 \end{aligned}$$

Thus, the equation of the circle is;

$$(x - 1)^2 + (y - 11.5)^2 = 8.5^2$$

8.5.2 Solutions to 8.4 Exercise

Answer 1

$$(a) \quad \text{Midpoint } AB = \left(\frac{13 + 5}{2}, \frac{11 + (-1)}{2} \right) = (9, 5)$$

[2 marks]

$$(b) \quad |AB| = \sqrt{(13 - 5)^2 + (11 - (-1))^2} = \sqrt{8^2 + 12^2} = 4\sqrt{13}$$

$$\text{Centre } (9, 5) \quad \text{Radius } 2\sqrt{13}$$

$$\text{The Equation of the circle is, } (x - 9)^2 + (y - 5)^2 = 52$$

[4 marks]

Answer 2

$$\text{Midpoint} = \left(\frac{3 + (-1)}{2}, \frac{6 + 4}{2} \right) = (1, 5)$$

$$\begin{aligned} \text{Length of diameter} &= \sqrt{(3 - (-1))^2 + (6 - 4)^2} \\ &= \sqrt{4^2 + 2^2} = 2\sqrt{5} \end{aligned}$$

$$\text{The Equation of the circle is, } (x - 1)^2 + (y - 5)^2 = 5$$

[6 marks]

Answer 3

$$\begin{aligned} \text{Radius} &= \sqrt{(0 - (-1))^2 + (0 - 7)^2} \\ &= \sqrt{1^2 + 7^2} = \sqrt{50} = 5\sqrt{2} \end{aligned}$$

The Equation of the circle is, $(x + 1)^2 + (y - 7)^2 = 50$

[4 marks]

Answer 4

(a) $|AB| = \sqrt{(3 - 4)^2 + (5 - 0)^2} = \sqrt{1^2 + 5^2} = \sqrt{26}$

[2 marks]

(b) $\text{Midpoint } AB = \left(\frac{3 + 4}{2}, \frac{5 + 0}{2} \right) = (3.5, 2.5)$

[2 marks]

(c) The Equation of the circle is,

$$\begin{aligned} (x - 3.5)^2 + (y - 2.5)^2 &= \left(\frac{\sqrt{26}}{2} \right)^2 \\ (x - 3.5)^2 + (y - 2.5)^2 &= \frac{26}{4} \\ (x - 3.5)^2 + (y - 2.5)^2 &= 6.5 \end{aligned}$$

[3 marks]

Answer 5

(a) $m_{AB} = \frac{1 - (-2)}{3 - 1} = \frac{3}{2} \therefore l \text{ will have gradient } -\frac{2}{3}$

$$y = mx + c$$

$$1 = -\frac{2}{3} \times 3 + c \Rightarrow c = 3$$

$$l \text{ has equation } y = -\frac{2}{3}x + 3$$

[4 marks]

(b) As P lies on l , $y = -\frac{2}{3} \times 6 + 3$

$$= -4 + 3$$

$$= -1 \text{ as expected } \square$$

[1 mark]

(c) Centre $P(6, -1)$, Radius is distance between P and A

$$\begin{aligned} \text{Radius} &= \sqrt{(6 - 1)^2 + (-1 - (-2))^2} \\ &= \sqrt{5^2 + 1^2} = \sqrt{26} \end{aligned}$$

The circle has equation $(x - 6)^2 + (y + 1)^2 = 26$

[4 marks]

Answer 6

$$(a) \quad r = \sqrt{9^2 + (-13)^2} = \sqrt{250} = 5\sqrt{10}$$

[2 marks]

$$(b) \quad x^2 + y^2 = 250$$

[1 mark]

$$(c) \quad k = 2 \times (-13) + 3 \times 9 = 1$$

[1 mark]

$$(d) \quad \text{From } 2y + 3x = 1 \Rightarrow y = -\frac{3}{2}x + \frac{1}{2}$$

$$x^2 + y^2 = 250$$

$$x^2 + \left(-\frac{3}{2}x + \frac{1}{2}\right)^2 = 250$$

multiply through by 4

$$4x^2 + (-3x + 1)^2 = 1000$$

$$4x^2 + 9x^2 - 6x + 1 = 1000$$

$$13x^2 - 6x - 999 = 0$$

It's known that $x = 9$ must be a solution

$$(13x + 111)(x - 9) = 0$$

$$\therefore B \text{ has } x \text{ coordinate } -\frac{111}{13}$$

$$y_B = -\frac{3}{2} \times -\frac{111}{13} + \frac{1}{2} = -\frac{173}{13}$$

[6 marks]

Answer 7

$$(a) \quad x^2 + y^2 - 6x + 4y = 12$$

$$[x^2 - 6x] + [y^2 + 4y] = 12$$

$$[(x - 3)^2 - 9] + [(y + 2)^2 - 4] = 12$$

$$(x - 3)^2 + (y + 2)^2 = 25$$

The circle has centre $(3, -2)$ and radius 5

[5 marks]

$$(b) \quad |PQ| = \sqrt{(7 - (-1))^2 + (-5 - 1)^2}$$

$$= \sqrt{8^2 + 6^2} = 10$$

As this is twice the radius of 5, PQ is a diameter of C

[2 marks]

(c) R must lie on the circle, by the circle theorem that any angle which stands on a diameter is 90° (argued backwards)

As R is on the y -axis, $x = 0$

$$(0 - 3)^2 + (y + 2)^2 = 25$$

$$(y + 2)^2 = 25 - 9$$

$$y + 2 = \pm 4$$

$$y = -6 \text{ or } 2$$

Question states that R is on the positive y -axis, so $R(0, 2)$

[4 marks]

Answer 8

(a) By the circle theorem that any angle which stands on a diameter is 90°

$$\begin{aligned}
 m_{PQ} \times m_{RQ} &= -1 \\
 \frac{10 - 2}{9 - (-3)} \times \frac{4 - 10}{a - 9} &= -1 \\
 \frac{8}{12} \times \frac{-6}{a - 9} &= -1 \\
 \frac{-4}{a - 9} &= -1 \\
 a - 9 &= 4 \\
 a &= 13 \quad \square
 \end{aligned}$$

[3 marks]

(b) $|PR| = \sqrt{(13 - (-3))^2 + (4 - 2)^2}$

$$= \sqrt{16^2 + 2^2} = \sqrt{260} = 2\sqrt{65} \quad \therefore \text{Radius} = \sqrt{65}$$

$$\text{Midpoint } AB = \left(\frac{13 + (-3)}{2}, \frac{4 + 2}{2} \right) = (5, 3)$$

The Equation of the circle is, $(x - 5)^2 + (y - 3)^2 = 65$

[5 marks]