3.3 Answers

GCSE Mathematics
Differentiation I

3.3.1 Solutions (3.1 Teaching Video)

$$y = x^3 - 3x + 1$$

$$\therefore \quad \frac{dy}{dx} = 3x^2 - 3$$

At a turning point, the gradient has a value of zero.

$$3x^2 - 3 = 0$$

Divide through by 3

$$x^2 - 1 = 0$$

$$(x + 1)(x - 1) = 0 \qquad \text{Difference of two squares}$$

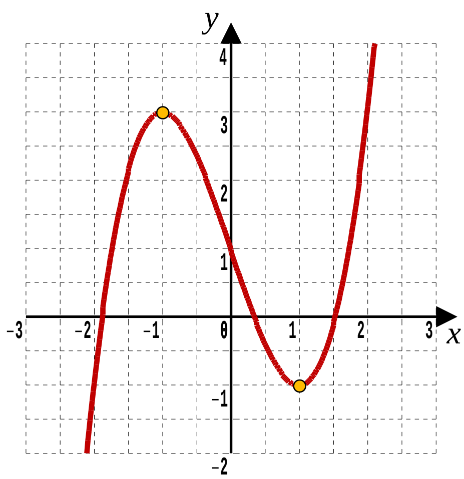
Either $x + 1 = 0$ or $x - 1 = 0$

$x = -1$ $\qquad$ $x = 1$

To get ***points*** substitute these values of *x* back into the point equation of the curve. (NOT the gradient equation of the curve !)

$$\begin{aligned} y &= x^3 - 3x + 1 \\ &= (-1)^3 - 3(-1) + 1 \\ &= -1 + 3 + 1 \\ &= 3 \end{aligned} \qquad \begin{aligned} y &= x^3 - 3x + 1 \\ &= 1^3 - 3 \times 1 + 1 \\ &= 1 - 3 + 1 \\ &= -1 \end{aligned}$$

The turning points are thus, $(-1,\ 3)$, $(1, -1)$



[6 marks]

3.3.2 Solutions (3.2 Exercise)

Answer 1

(i) $y = 7x^5$ $\qquad \dfrac{dy}{dx} = 35x^4$

(ii) $y = 8x + 1.5$ $\qquad \dfrac{dy}{dx} = 8$

(iii) $y = 5x^8 + 17x - 11$ $\qquad \dfrac{dy}{dx} = 40x^7 + 17$

[6 marks]

Answer 2

(a)

$$y = 5x^2 - 30x$$

$$\therefore \quad \frac{dy}{dx} = 10x - 30$$

[2 marks]

(b) At a turning point, the gradient has a value of zero.

$$10x - 30 = 0$$

$$x = 3$$

To get a ***point*** substitute this value of x back into the point equation of the curve. (NOT the gradient equation of the curve !)

$$\begin{aligned} y &= 5x^2 - 30x \\ &= 5 \times 3^2 - 30 \times 3 \\ &= -45 \end{aligned}$$

The turning point is thus $(3, -45)$

[5 marks]

Answer 3

(i) $y = 24x^2 - 12x^4$ $\qquad \dfrac{dy}{dx} = 48x - 48x^3$

(ii) $y = 13$ $\qquad \dfrac{dy}{dx} = 0$

(iii) $y = \dfrac{5}{x^3} \Rightarrow y = 5x^{-3}$

$$\frac{dy}{dx} = -15x^{-4} \Rightarrow \frac{dy}{dx} = -\frac{15}{x^4}$$

[6 marks]

Answer 4

(a) $y = 4x^2 + 16x + 21$

$$\therefore \quad \frac{dy}{dx} = 8x + 16$$

[2 marks]

(b) At a turning point, the gradient has a value of zero.

$$8x + 16 = 0$$

$$x = -2$$

To get a ***point*** substitute this value of x back into the equation of the curve.

$$y = 4x^2 + 16x + 21$$

$$= 4 \times (-2)^2 + 16 \times (-2) + 21$$

$$= 16 - 32 + 21$$

$$= 5$$

The turning point is thus, $(-2,\ 5)$

[5 marks]

Answer 5

$$y = x^3\left(4x^8 - 7x\right) \Rightarrow y = 4x^{11} - 7x^4 \Rightarrow \frac{dy}{dx} = 44x^{10} - 28x^3$$

[4 marks]

Answer 6

(a) $y = x^3 + 9x^2 + 15x$

$$\therefore \quad \frac{dy}{dx} = 3x^2 + 18x + 15$$

(b) At a turning point, the gradient has a value of zero.

$$3x^2 + 18x + 15 = 0$$

Divide through by 3

$$x^2 + 6x + 5 = 0$$

$$(x + 5)(x + 1) = 0$$

Either $x = 5$ or $x = -1$

To get ***points*** substitute these values of x back into the equation of the curve.

$$y = x^3 + 9x^2 + 15x$$

$$= (-5)^3 + 9(-5)^2 + 15(-5)$$

$$= -125 + 225 - 75$$

$$= 25$$

$$y = x^3 + 9x^2 + 15x$$

$$= (-1)^3 + 9(-1)^2 + 15(-1)$$

$$= -1 + 9 - 15$$

$$= -7$$

The turning points are $(-5,\ 25)$, $(-1, -7)$

[6 marks]

Answer 7

$$y = 10x^2 + 9x + 5$$

$$\therefore \quad \frac{dy}{dx} = 20x + 9$$

At a minimum point, the gradient has a value of zero.

$$20x + 9 = 0$$

$$x = -\frac{9}{20} \text{ or } -0.45$$

To get a ***point*** substitute this value of x back into the equation of the curve.

$$y = 10x^2 + 9x + 5$$

$$= 10 \times \left(-\frac{9}{20}\right)^2 + 9 \times \left(-\frac{9}{20}\right) + 5$$

$$= \frac{119}{40} \text{ } or \text{ } 2.975$$

The turning point is thus, $(-0.45, \ 2.975)$

[4 marks]

Answer 8

(a)

$$y = x^3 + 6x^2 + 5$$

$$\therefore \quad \frac{dy}{dx} = 3x^2 + 12x$$

[2 marks]

(b) At a turning point, the gradient has a value of zero.

$$3x^2 + 12x = 0$$

Divide through by 3

$$x^2 + 4x = 0$$

$$x(x + 4) = 0$$

Either $x = 0$ or $x = -4$

To get ***points*** substitute these values of x back into the equation of the curve

$$y = x^3 + 6x^2 + 5 \qquad\qquad y = x^3 + 6x^2 + 5$$

$$= (0)^3 + 6(0)^2 + 5 \qquad\qquad = (-4)^3 + 6(-4)^2 + 5$$

$$= 5 \qquad\qquad = -64 + 96 + 5$$

$$= 37$$

The turning points are thus, $(0, \ 5)$, $(-4, \ 37)$

[6 marks]