

3.3 Answers

GCSE Mathematics Differentiation I

3.3.1 Solutions (3.1 Teaching Video)

$$y = x^3 - 3x + 1$$

$$\therefore \frac{dy}{dx} = 3x^2 - 3$$

At a turning point, the gradient has a value of zero.

$$3x^2 - 3 = 0$$

Divide through by 3

$$x^2 - 1 = 0$$

$$(x + 1)(x - 1) = 0 \quad \text{Difference of two squares}$$

$$\text{Either } x + 1 = 0 \quad \text{or} \quad x - 1 = 0$$

$$x = -1 \quad \quad \quad x = 1$$

To get **points** substitute these values of x back into the point equation of the curve. (NOT the gradient equation of the curve !)

$$y = x^3 - 3x + 1$$

$$= (-1)^3 - 3(-1) + 1$$

$$= -1 + 3 + 1$$

$$= 3$$

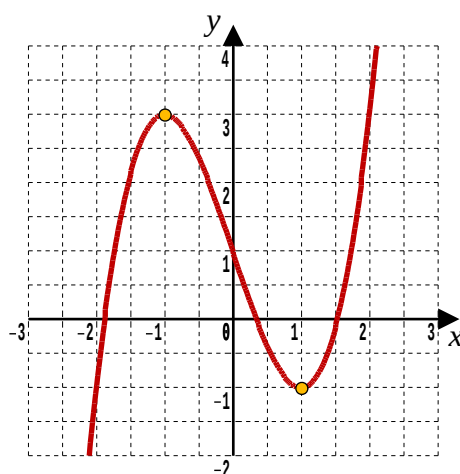
$$y = x^3 - 3x + 1$$

$$= 1^3 - 3 \times 1 + 1$$

$$= 1 - 3 + 1$$

$$= -1$$

The turning points are thus, $(-1, 3)$, $(1, -1)$



[6 marks]

3.3.2 Solutions (3.2 Exercise)

Answer 1

$$\begin{array}{ll} \text{(i)} & y = 7x^5 \qquad \frac{dy}{dx} = 35x^4 \\ \text{(ii)} & y = 8x + 1.5 \qquad \frac{dy}{dx} = 8 \\ \text{(iii)} & y = 5x^8 + 17x - 11 \qquad \frac{dy}{dx} = 40x^7 + 17 \end{array}$$

[6 marks]

Answer 2

(a)

$$\begin{aligned} y &= 5x^2 - 30x \\ \therefore \frac{dy}{dx} &= 10x - 30 \end{aligned}$$

[2 marks]

(b) At a turning point, the gradient has a value of zero.

$$10x - 30 = 0$$

$$x = 3$$

To get a **point** substitute this value of x back into the point equation of the curve. (NOT the gradient equation of the curve !)

$$\begin{aligned} y &= 5x^2 - 30x \\ &= 5 \times 3^2 - 30 \times 3 \\ &= -45 \end{aligned}$$

The turning point is thus (3, -45)

[5 marks]

Answer 3

$$\begin{array}{ll} \text{(i)} & y = 24x^2 - 12x^4 \qquad \frac{dy}{dx} = 48x - 48x^3 \\ \text{(ii)} & y = 13 \qquad \frac{dy}{dx} = 0 \\ \text{(iii)} & y = \frac{5}{x^3} \Rightarrow y = 5x^{-3} \\ & \frac{dy}{dx} = -15x^{-4} \Rightarrow \frac{dy}{dx} = -\frac{15}{x^4} \end{array}$$

[6 marks]

Answer 4

$$(a) \quad y = 4x^2 + 16x + 21$$

$$\therefore \frac{dy}{dx} = 8x + 16$$

[2 marks]**(b)** At a turning point, the gradient has a value of zero.

$$8x + 16 = 0$$

$$x = -2$$

To get a **point** substitute this value of x back into the equation of the curve.

$$\begin{aligned} y &= 4x^2 + 16x + 21 \\ &= 4 \times (-2)^2 + 16 \times (-2) + 21 \\ &= 16 - 32 + 21 \\ &= 5 \end{aligned}$$

The turning point is thus, $(-2, 5)$ **[5 marks]****Answer 5**

$$y = x^3(4x^8 - 7x) \Rightarrow y = 4x^{11} - 7x^4 \Rightarrow \frac{dy}{dx} = 44x^{10} - 28x^3$$

[4 marks]**Answer 6**

$$(a) \quad y = x^3 + 9x^2 + 15x$$

$$\therefore \frac{dy}{dx} = 3x^2 + 18x + 15$$

(b) At a turning point, the gradient has a value of zero.

$$3x^2 + 18x + 15 = 0$$

Divide through by 3

$$x^2 + 6x + 5 = 0$$

$$(x + 5)(x + 1) = 0$$

$$\text{Either } x = 5 \quad \text{or} \quad x = -1$$

To get **points** substitute these values of x back into the equation of the curve.

$$\begin{aligned} y &= x^3 + 9x^2 + 15x & y &= x^3 + 9x^2 + 15x \\ &= (-5)^3 + 9(-5)^2 + 15(-5) & &= (-1)^3 + 9(-1)^2 + 15(-1) \\ &= -125 + 225 - 75 & &= -1 + 9 - 15 \\ &= 25 & &= -7 \end{aligned}$$

The turning points are $(-5, 25)$, $(-1, -7)$ **[6 marks]**

Answer 7

$$y = 10x^2 + 9x + 5$$

$$\therefore \frac{dy}{dx} = 20x + 9$$

At a minimum point, the gradient has a value of zero.

$$20x + 9 = 0$$

$$x = -\frac{9}{20} \text{ or } -0.45$$

To get a **point** substitute this value of x back into the equation of the curve.

$$\begin{aligned} y &= 10x^2 + 9x + 5 \\ &= 10 \times \left(-\frac{9}{20}\right)^2 + 9 \times \left(-\frac{9}{20}\right) + 5 \\ &= \frac{119}{40} \text{ or } 2.975 \end{aligned}$$

The turning point is thus, $(-0.45, 2.975)$

[4 marks]

Answer 8

(a) $y = x^3 + 6x^2 + 5$

$$\therefore \frac{dy}{dx} = 3x^2 + 12x$$

[2 marks]

(b) At a turning point, the gradient has a value of zero.

$$3x^2 + 12x = 0$$

Divide through by 3

$$x^2 + 4x = 0$$

$$x(x + 4) = 0$$

$$\text{Either } x = 0 \text{ or } x = -4$$

To get **points** substitute these values of x back into the equation of the curve

$y = x^3 + 6x^2 + 5$	$y = x^3 + 6x^2 + 5$
$= (0)^3 + 6(0)^2 + 5$	$= (-4)^3 + 6(-4)^2 + 5$
$= 5$	$= -64 + 96 + 5$
	$= 37$

The turning points are thus, $(0, 5)$, $(-4, 37)$

[6 marks]

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