4.4 Answers to 4.2 Exercise

4.4.1 Solution to 4.2 Example (Teaching Video)

$$k(x) = 2x^9 - 3x^7 + x^5 - 3x^3 + 2x$$

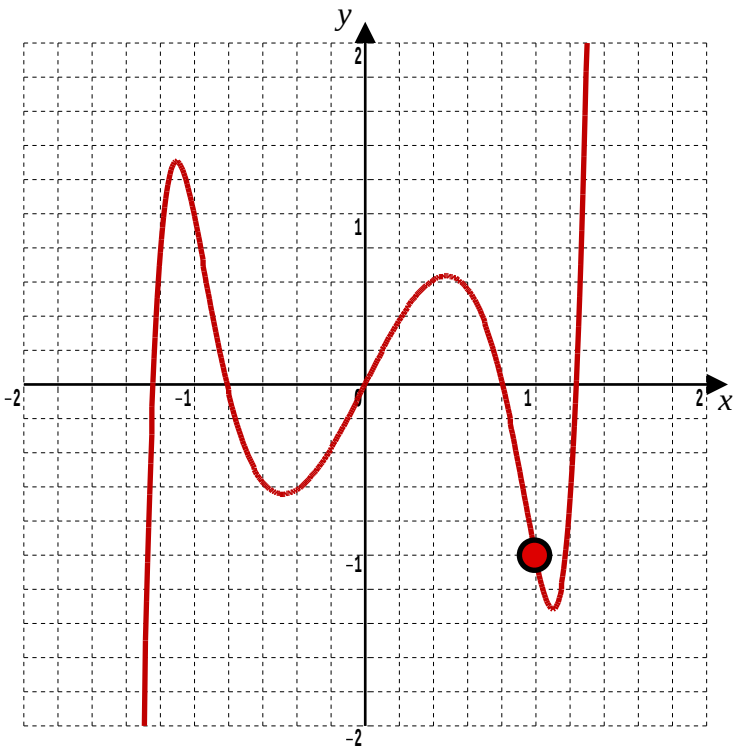
(i) $k'(x) = 18x^8 - 21x^6 + 5x^4 - 9x^2 + 2$

(ii) $k''(x) = 144x^7 - 126x^5 + 20x^3 - 18x$

(iii) $k(1) = 2 - 3 + 1 - 3 + 2$

$= -1$

Thus $(1,\ k(1)) = (1, -1)$



(iv) $k'(1) = 18 - 21 + 5 - 9 + 2$

$= -5$ $\therefore$ Sloping downwards at $(1, -1)$

(v) $k''(1) = 144 - 126 + 20 - 18$

$= 20$ As this is positive curve is bending anticlockwise at $(1, -1)$

[10 marks]

4.4.2 Solutions to 4.3 Exercise

Answer 1

$p(x) = 0.02x^3 + 0.5$

(i) $p'(x) = 0.06x^2$

[2 marks]

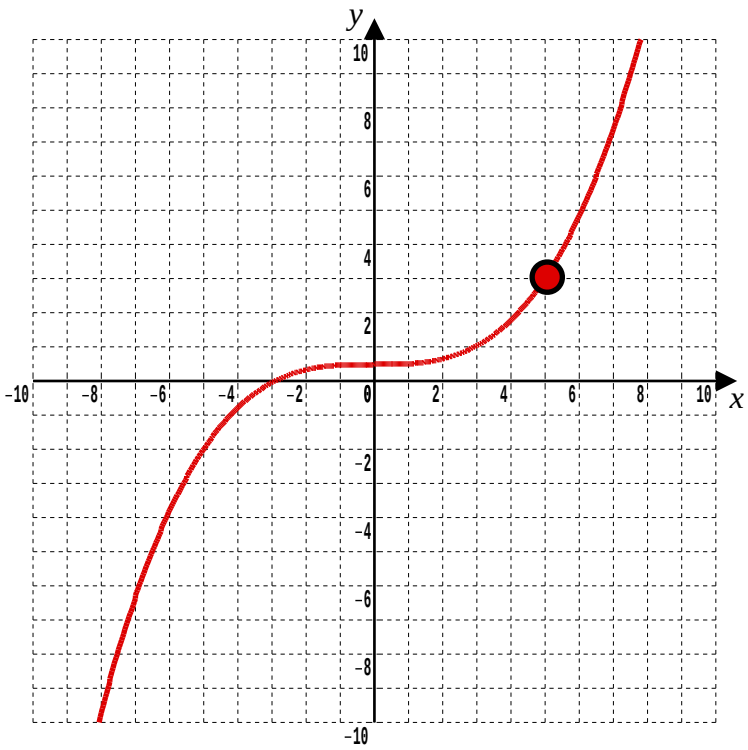
(ii) $p''(x) = 0.12x$

[2 marks]

(iii) $p(5) = 3$

Thus $(5,\ p(5)) = (5,\ 3)$

[2 marks]



(iv) $p'(5) = 1.5$ $\therefore$ Sloping upwards at $(5,\ 3)$

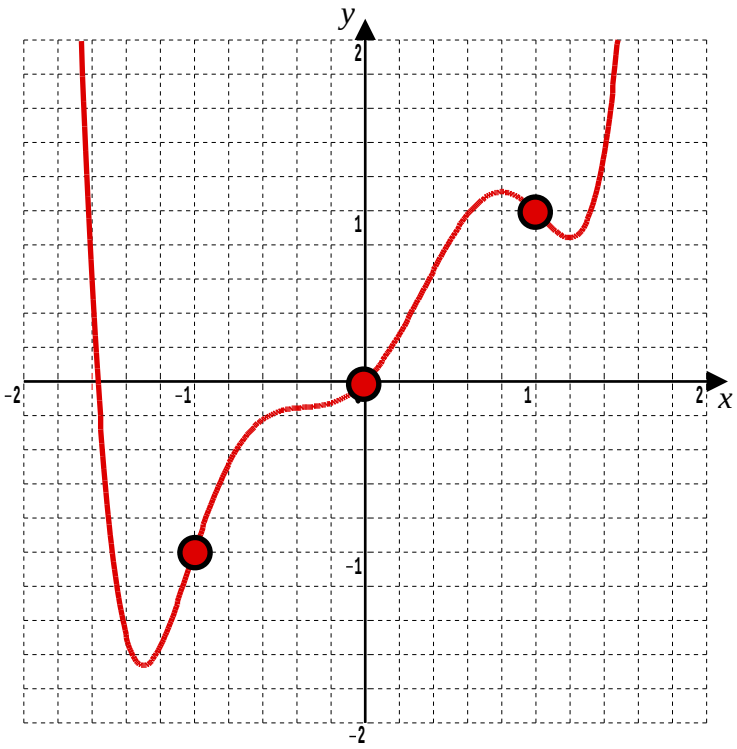
[2 marks]

(v) $p''(5) = 0.6$ As this is positive curve is bending anticlockwise at $(5,\ 3)$

[2 marks]

Answer 2

(a) **(i)** (0, 0) **(ii)** (1, 1) **(iii)** $(-1, -1)$



[6 marks]

(b) $f'(x) = 6x^5 - 12x^3 + 4x + 1$

[2 marks]

(c) **(i)** $f'(0) = 1$ Yes, graph slopes upwards at (0, 0)

(ii) $f'(1) = -1$ Yes, graph slopes downward at (1, 1)

(iii) $f'(-1) = 3$ Yes, graph slopes upward at $(-1, -1)$

[6 marks]

(d) $f''(x) = 30x^4 - 36x^2 + 4$

[2 marks]

(e) **(i)** $f''(0) = 4$ Yes, graph bending anticlockwise at (0, 0)

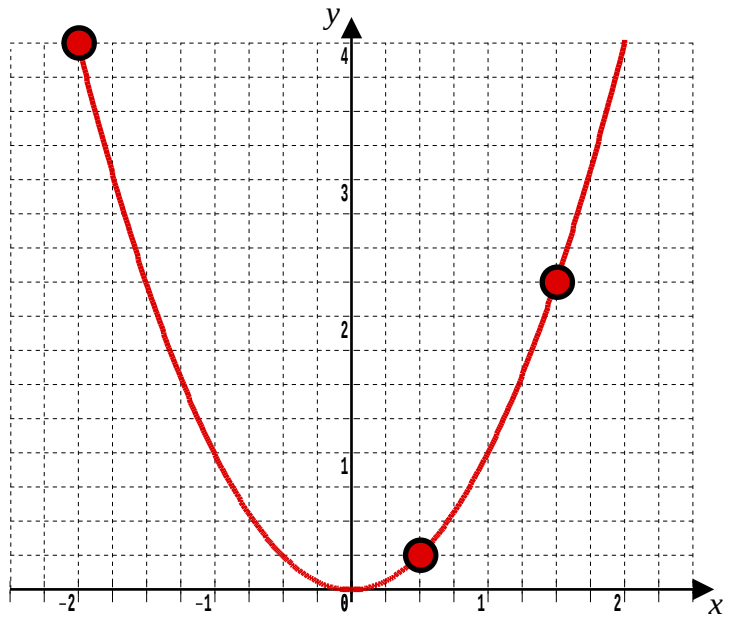
(ii) $f''(1) = -2$ $\therefore$ Graph bending clockwise at (1, 1)

(iii) $f''(-1) = -2$ $\therefore$ Graph bending clockwise at $(-1, -1)$

[6 marks]

Answer 3

(a) **(i)** $(-2,\ 4)$ **(ii)** $\left(\frac{1}{2},\ \frac{1}{4}\right)$ **(iii)** $\left(\frac{3}{2},\ \frac{9}{4}\right)$



[6 marks]

(b) $g'(x) = 2x$

[2 marks]

(c) **(i)** $g'(-2) = 4$

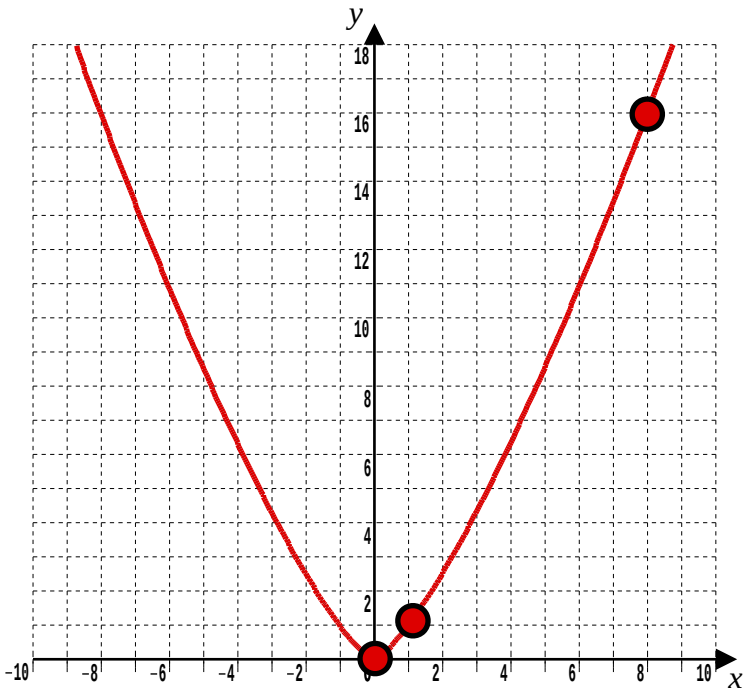
(ii) $g'\left(\frac{1}{2}\right) = 1$

(iii) $g'\left(\frac{3}{2}\right) = 3$

[6 marks]

Answer 4

(a) **(i)** (0, 0) **(ii)** (1, 1) **(iii)** (8, 16)



[6 marks]

(b) $h'(x) = \frac{4}{3}x^{\frac{1}{3}}$

[2 marks]

(c) **(i)** $h'(0) = 0$ Yes, graph flat at (0, 0)

(ii) $h'(1) = \frac{4}{3}$ Yes, graph sloping upwards at (1, 1)

(iii) $h'(8) = \frac{8}{3}$ Yes, graph sloping upwards at (8, 16)

[6 marks]

(d) $h''(x) = \frac{4}{9}x^{-\frac{2}{3}}$

[2 marks]

(e) **(i)** $h''(0) = \frac{4}{9} \times 0^{-\frac{2}{3}} = 0$ □

(ii) $h''(1) = \frac{4}{9}$ Yes, graph bending anticlockwise at (1, 1)

(iii) $h''(8) = \frac{8}{3}$ Yes, graph bending anticlockwise at (8, 16)

[6 marks]

(f) Require $\frac{4}{3}x^{\frac{1}{3}} = 0 \Rightarrow x^{\frac{1}{3}} = \frac{3}{4} \Rightarrow x = \frac{27}{64} \Rightarrow x = 0.421875$

[3 marks]

Answer 5

(i) $\frac{dy}{dx} = 21x^2 + 8x$

(ii) $\frac{dy}{dx} = 13$

(iii) $\frac{dy}{dx} = 25x^9$

(iv) $\frac{dy}{dx} = 0$

(v) $\frac{dy}{dx} = 10x^3$

(vi) $\frac{dy}{dx} = 21x^8$

(vii) $\frac{dy}{dx} = 4.5x^2$

(viii) $\frac{dy}{dx} = 3.2$

(ix) $y = 5x^4 + 2x^3 + 7$

$\frac{dy}{dx} = 20x^3 + 6x^2$

(x) $y = 4x^5 + 7x + 1.2$

$\frac{dy}{dx} = 20x^4 + 7$

(xi) $y = 16x^2 + 24x + 9$

$\frac{dy}{dx} = 32x + 24$

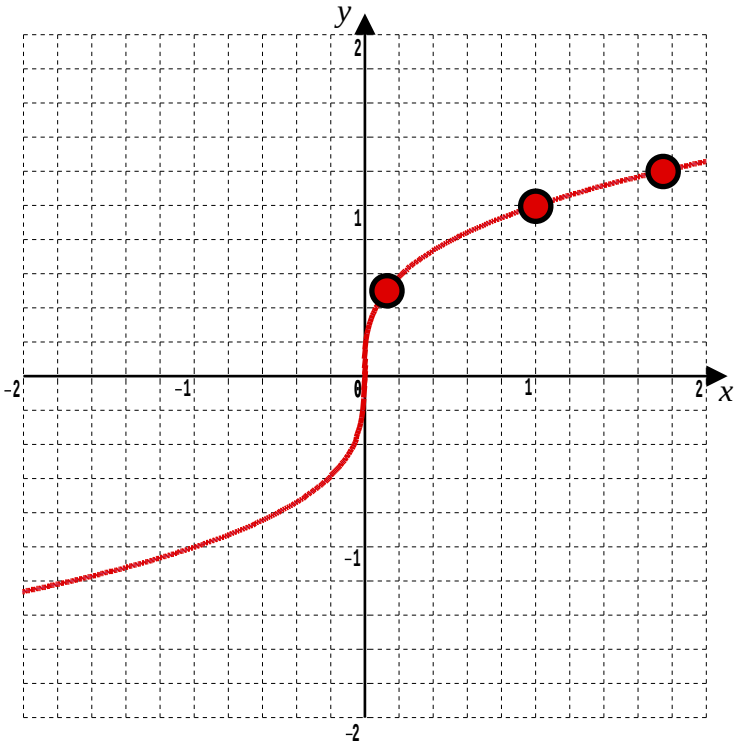
(xii) $y = 9x^2 + 30x + 25$

$\frac{dy}{dx} = 18x + 30$

[12 marks]

Answer 6

(a) **(i)** $\left(\frac{1}{8},\ \frac{1}{2}\right)$ **(ii)** $(1,\ 1)$ **(iii)** $\left(\frac{216}{125},\ \frac{6}{5}\right)$



[6 marks]

(b) $m'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3x^{\frac{2}{3}}}$

[2 marks]

(c) **(i)** $m'\left(\frac{1}{8}\right) = \frac{1}{3} \times 8^{\frac{2}{3}} = \frac{4}{3}$ Yes, sloping up at $\left(\frac{1}{8},\ \frac{1}{2}\right)$

(ii) $m'(1) = \frac{1}{3}$ Yes, sloping up at (1, 1)

(iii) $m'\left(\frac{216}{125}\right) = \frac{1}{3} \times \left(\frac{125}{216}\right)^{\frac{2}{3}} = \frac{25}{108}$ which is about 0.231

Yes, sloping up at $\left(\frac{216}{125},\ \frac{6}{5}\right)$

[6 marks]

(d) $\frac{1}{3}x^{-\frac{2}{3}} = 1 \Leftrightarrow \frac{1}{x^{-\frac{2}{3}}} = 3 \Leftrightarrow x^{\frac{2}{3}} = \frac{1}{3} \Leftrightarrow x = \left(\frac{1}{3}\right)^{\frac{3}{2}}$

Which is about 0.19245...

[3 marks]