

5.2 Answers

GCSE Mathematics Differentiation I

Answer 1

- | | | | | | |
|---------|----|----------|-------|---------|-----------------|
| (i) | 64 | (ii) | − 216 | (iii) | $\frac{1}{32}$ |
| (iv) | 10 | (v) | 2 | (vi) | − 1 |
| (vii) | 1 | (viii) | 0 | (ix) | $\frac{25}{81}$ |

[9 marks]

Answer 2

- | | | | | | |
|--------|-----------|--------|-------------|---------|-----------------|
| (i) | (0, 0) | (ii) | (1, 0) | (iii) | (2, 6) |
| (iv) | (4, 60) | (v) | (10, 990) | (vi) | (− 10, − 990) |

[6 marks]

Answer 3

- | | | | |
|---------|------------------------------|--------|---------------|
| (i) | $\frac{1}{9}$ | (ii) | $\frac{4}{7}$ |
| (iii) | $\frac{4}{10} = \frac{2}{5}$ | (iv) | − 1 |

[4 marks]

Answer 4

$$w(x) = x^8 + 3x^5 - 4x^3 - 3x^2 + 1$$

$$(i) \quad w'(x) = 8x^7 + 15x^4 - 12x^2 - 6x$$

[2 marks]

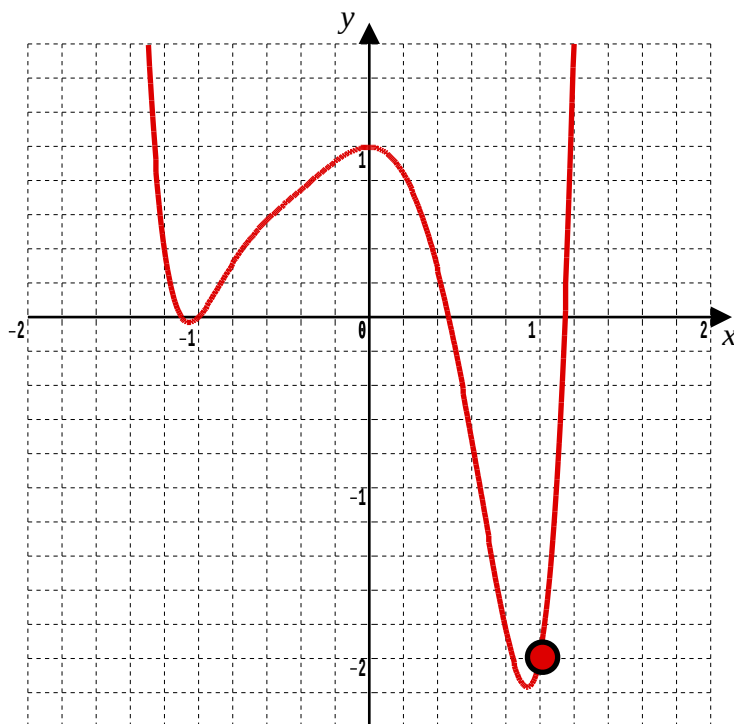
$$(ii) \quad w''(x) = 56x^6 + 60x^3 - 24x - 6$$

[2 marks]

$$(iii) \quad w(1) = 1 + 3 - 4 - 3 + 1 \\ = -2$$

$$\text{Thus } (1, w(1)) = (1, -2)$$

[2 marks]



$$(iv) \quad w'(1) = 8 + 15 - 12 - 6 \\ = 5 \therefore \text{Sloping upwards at } (1, -2)$$

[2 marks]

$$(v) \quad w''(1) = 56 + 60 - 24 - 6 \\ = 86$$

As this is positive the curve is bending anticlockwise at $(1, -2)$

[2 marks]

Answer 5

$$(i) \quad y = 24x^5 \qquad \therefore \quad \frac{dy}{dx} = 120x^4$$

[1 mark]

$$(ii) \quad y = 4x^{-3} \qquad \therefore \quad \frac{dy}{dx} = -12x^{-4}$$

$$\text{which means } \frac{dy}{dx} = -\frac{12}{x^4}$$

[1 mark]

$$(iii) \quad y = 8x + 3 \qquad \therefore \quad \frac{dy}{dx} = 8$$

[1 mark]

$$(iv) \quad y = (2x + 3)^2$$

$$y = 4x^2 + 12x + 9 \qquad \therefore \quad \frac{dy}{dx} = 8x + 12$$

[2 marks]

$$(v) \quad y = \sqrt{x}$$

$$y = x^{\frac{1}{2}} \qquad \therefore \quad \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$$

$$\text{which means } \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

[2 marks]

$$(vi) \quad y = \frac{1}{x^4}$$

$$y = x^{-4} \qquad \therefore \quad \frac{dy}{dx} = -4x^{-5}$$

$$\text{which means } \frac{dy}{dx} = -\frac{4}{x^5}$$

[2 marks]**Answer 6**

$$(i) \quad \frac{25}{121} \qquad (ii) \quad \frac{64}{1000000} \text{ or } 0.000064$$

[2, 2 marks]

$$(iii) \quad \frac{5}{4} \qquad (iv) \quad -\frac{64}{27}$$

[2, 2 marks]

Answer 7

$$s(x) = \frac{8}{x^3} + \frac{x^2}{4} = 8x^{-3} + \frac{x^2}{4}$$

$$(i) \quad s'(x) = -24x^{-4} + \frac{x}{2}$$

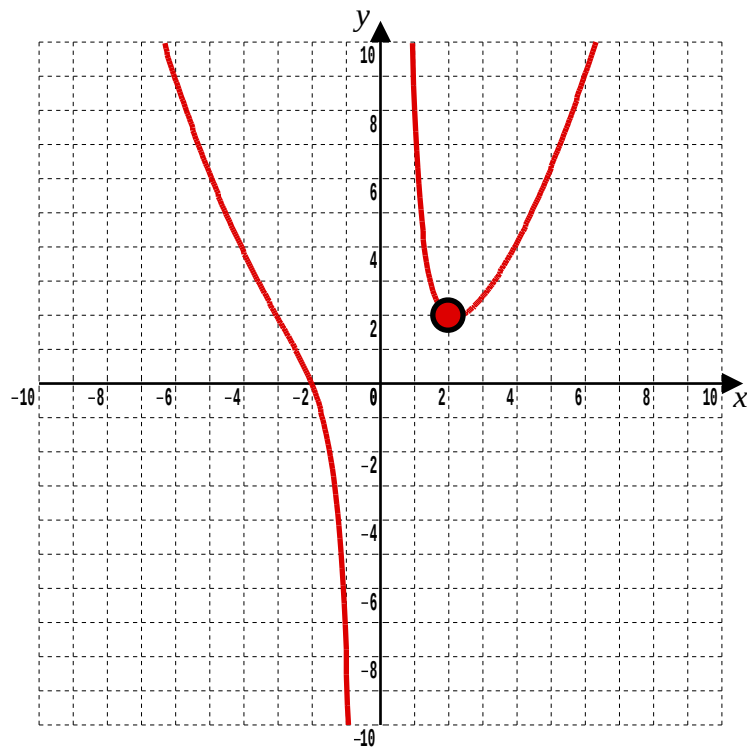
[3 marks]

$$(ii) \quad s''(x) = 96x^{-5} + \frac{1}{2}$$

[3 marks]

$$(iii) \quad s(2) = \frac{8}{2^3} + \frac{2^2}{4} = 2 \quad \text{Thus } (2, w(2)) = (2, 2)$$

[2 marks]



$$(iv) \quad s'(2) = -\frac{24}{2^4} + \frac{2}{2} = -\frac{1}{2} \therefore \text{Sloping downwards at } (2, 2)$$

[2 marks]

$$(v) \quad s''(1) = \frac{96}{2^5} + \frac{1}{2} = 3.5$$

As this is positive the curve is bending anticlockwise at (2, 2)

[2 marks]

Answer 8

$$(i) \quad y = 9x^4 - 8x^{-2} \qquad \therefore \quad \frac{dy}{dx} = 36x^3 + 16x^{-3}$$

$$\text{which means } \frac{dy}{dx} = 36x^3 + \frac{16}{x^3}$$

[3 marks]

$$(ii) \quad y = 22x^4 + \frac{12}{x^4}$$

$$y = 22x^4 + 12x^{-4} \qquad \therefore \quad \frac{dy}{dx} = 88x^3 - 48x^{-5}$$

$$\text{which means } \frac{dy}{dx} = 88x^3 - \frac{48}{x^5}$$

[3 marks]

$$(iii) \quad y = x^8(4x^3 + 7x^2)$$

$$y = 4x^{11} + 7x^{10} \qquad \therefore \quad \frac{dy}{dx} = 44x^{10} + 70x^9$$

[3 marks]

$$(iv) \quad y = \frac{1}{5x^2}$$

$$y = \frac{1}{5}x^{-2} \qquad \therefore \quad \frac{dy}{dx} = -\frac{2}{5}x^{-3}$$

$$\text{which means } \frac{dy}{dx} = -\frac{2}{5x^3}$$

[3 marks]

$$(v) \quad y = \frac{7x^3}{11} \qquad \therefore \quad \frac{dy}{dx} = \frac{21x^2}{11}$$

[3 marks]

$$(vi) \quad y = \frac{x^9 + 6x^5}{2x^3}$$

$$y = \frac{x^9}{2x^3} + \frac{6x^5}{2x^3}$$

$$y = \frac{x^6}{2} + 3x^2 \qquad \therefore \quad \frac{dy}{dx} = 3x^5 + 6x$$

[3 marks]