

6.5 Answers

GCSE Mathematics Differentiation I

6.5.1 Solution to 6.3 Example (Teaching Video)

$$\text{STEP 1} \quad y = x^2 - 6x + 5$$

$$\frac{dy}{dx} = 2x - 6$$

$$\text{STEP 2} \quad 2x - 6 = 0$$

$$x = 3$$

$$\text{STEP 3} \quad y = 3^2 - 6 \times 3 + 5$$

$$y = -4$$

$\therefore$ Minimum at $(3, -4)$ as the coefficient of x^2 in parabola's equation is positive

[3 marks]

6.5.2 Solution to 6.4 Exercise

Answer 1

$$\text{STEP 1} \quad y = x^2 - 8x + 9$$

$$\frac{dy}{dx} = 2x - 8$$

$$\text{STEP 2} \quad 2x - 8 = 0$$

$$x = 4$$

$$\text{STEP 3} \quad y = 4^2 - 8 \times 4 + 9$$

$$y = -7$$

$\therefore$ Minimum at $(4, -7)$ as the coefficient of x^2 in parabola's equation is positive

[3 marks]

Answer 2

$$\text{STEP 1} \quad y = 6x + 14 - x^2$$

$$\frac{dy}{dx} = 6 - 2x$$

$$\text{STEP 2} \quad 6 - 2x = 0$$

$$x = 3$$

$$\text{STEP 3} \quad y = 6 \times 3 + 14 - 3^2$$

$$y = 23$$

$\therefore$ Maximum at $(3, 23)$ as the coefficient of x^2 in parabola's equation is negative

[3 marks]

Answer 3

$$\text{STEP 1} \quad y = 2x^2 - 20x + 52$$

$$\frac{dy}{dx} = 4x - 20$$

$$\text{STEP 2} \quad 4x - 20 = 0$$

$$x = 5$$

$$\text{STEP 3} \quad y = 2 \times 5^2 - 20 \times 5 + 52$$

$$y = 2$$

$\therefore$ Minimum at (5, 2) as the coefficient of x^2 in parabola's equation is positive

[3 marks]

Answer 4

$$\text{STEP 1} \quad y = 12x - 7 - 3x^2$$

$$\frac{dy}{dx} = 12 - 6x$$

$$\text{STEP 2} \quad 12 - 6x = 0$$

$$x = 2$$

$$\text{STEP 3} \quad y = 12 \times 2 - 7 - 3 \times 2^2$$

$$y = 5$$

$\therefore$ Maximum at (2, 5) as the coefficient of x^2 in parabola's equation is negative

[3 marks]

Answer 5

(a) (i) (-2, 16) (ii) (2, -16)

[1, 1 mark]

(b)

$$\text{STEP 1} \quad y = x^3 - 12x$$

$$\frac{dy}{dx} = 3x^2 - 12$$

$$\text{STEP 2} \quad 3x^2 - 12 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

$$\text{STEP 3} \quad y = (-2)^3 - 12 \times (-2) \quad y = 2^3 - 12 \times 2$$

$$y = 16 \quad y = -16$$

$\therefore$ Maximum at (-2, 16) and a minimum (2, -16)

as coefficient of x^3 in cubic's equation is positive

[4 marks]

Answer 6

(i) STEP 1 $y = x^3 - 27x$

$$\frac{dy}{dx} = 3x^2 - 27$$

STEP 2 $3x^2 - 27 = 0$

$$x^2 = 9$$

$$x = \pm 3$$

STEP 3 $y = (-3)^3 - 27 \times (-3) \quad y = 3^3 - 27 \times 3$

$$y = 54 \quad y = -54$$

$\therefore$ Maximum at $(-3, 54)$ and a minimum $(3, -54)$
 as coefficient of x^3 in cubic's equation is positive

[4 marks]

(ii) STEP 1 $y = (x + 7)(x + 1)$

$$= x^2 + 8x + 7$$

$$\frac{dy}{dx} = 2x + 8$$

STEP 2 $2x + 8 = 0$

$$x = -4$$

STEP 3 $y = (-4 + 7)(-4 + 1)$

$$y = -9$$

$\therefore$ Minimum at $(-4, -9)$ as the coefficient of x^2 in parabola's equation is positive

[4 marks]

(iii) STEP 1 $y = x^4 - 256x$

$$\frac{dy}{dx} = 4x^3 - 256$$

STEP 2 $4x^3 - 256 = 0$

$$x^3 = 64$$

$$x = 4$$

STEP 3 $y = 4^4 - 256 \times 4$

$$y = -768$$

$\therefore$ Stationary point at $(4, -768)$

To show it's a minimum, plot graph or use second derivative

[4 marks]

Answer 7

$$\text{STEP 1} \quad y = x^3 - 6x^2 - 36x + 100$$

$$\frac{dy}{dx} = 3x^2 - 12x - 36$$

$$\text{STEP 2} \quad 3x^2 - 12x - 36 = 0$$

Divide through by 3

$$x^2 - 4x - 12 = 0$$

$$(x + 2)(x - 6) = 0$$

$$\text{Either } x = -2 \text{ or } x = 6$$

$$\text{STEP 3 For } x = -2, \quad y = (-2)^3 - 6 \times (-2)^2 - 36 \times (-2) + 100$$

$$y = 140$$

$$\text{For } x = 6, \quad y = 6^3 - 6 \times 6^2 - 36 \times 6 + 100$$

$$y = -116$$

$\therefore$ Maximum at $(-2, 140)$ and a minimum $(6, -116)$
as coefficient of x^3 in cubic's equation is positive

[6 marks]

$$\text{Answer 8} \quad f(x) = \frac{12}{x} = 12x^{-1}$$

$$\text{(i) } \quad f'(x) = -12x^{-2} \quad \text{which can be written } f'(x) = -\frac{12}{x^2}$$

[2 marks]

$$\text{(ii) } \quad f''(x) = 24x^{-3} \quad \text{which can be written } f''(x) = \frac{24}{x^3}$$

[2 marks]

$$\text{(iii) } \quad (2, 6) \quad \text{(iv) } \quad -3 \quad \text{(v) } \quad \text{Anticlockwise}$$

[2, 2, 2 marks]**Answer 9**

$$\text{STEP 1} \quad y = x^3 + 12x$$

$$\frac{dy}{dx} = 3x^2 + 12$$

$$\text{STEP 2} \quad 3x^2 + 12 = 0$$

$$x^2 = -4 \quad \text{which has no real solutions}$$

$\therefore$ No turning points $\square$

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