## 6.5 Answers



### 6.5.1 Solution to 6.3 Example (Teaching Video)

STEP 1
$$y = x^2 - 6x + 5$$
$$\frac{dy}{dx} = 2x - 6$$

STEP 2
$$2x - 6 = 0$$
$$x = 3$$

STEP 3
$$y = 3^2 - 6 \times 3 + 5$$
$$y = -4$$

$\therefore$ Minimum at ( 3, – 4 ) as the coefficient of $x^2$ in parabola's equation is positive

**[ 3 marks ]**

### 6.5.2 Solution to 6.4 Exercise

**Answer 1**

STEP 1
$$y = x^2 - 8x + 9$$
$$\frac{dy}{dx} = 2x - 8$$

STEP 2
$$2x - 8 = 0$$
$$x = 4$$

STEP 3
$$y = 4^2 - 8 \times 4 + 9$$
$$y = -7$$

$\therefore$ Minimum at ( 4, – 7 ) as the coefficient of $x^2$ in parabola's equation is positive

**[ 3 marks ]**

**Answer 2**

STEP 1
$$y = 6x + 14 - x^2$$
$$\frac{dy}{dx} = 6 - 2x$$

STEP 2
$$6 - 2x = 0$$
$$x = 3$$

STEP 3
$$y = 6 \times 3 + 14 - 3^2$$
$$y = 23$$

$\therefore$ Maximum at ( 3, 23 ) as the coefficient of $x^2$ in parabola's equation is negative

**[ 3 marks ]**

**Answer 3**

STEP 1 $$y = 2x^2 - 20x + 52$$

$$\frac{dy}{dx} = 4x - 20$$

STEP 2 $$4x - 20 = 0$$

$$x = 5$$

STEP 3 $$y = 2 \times 5^2 - 20 \times 5 + 52$$

$$y = 2$$

$\therefore$ Minimum at $(5, 2)$ as the coefficient of $x^2$ in parabola's equation is positive

**[ 3 marks ]**

**Answer 4**

STEP 1 $$y = 12x - 7 - 3x^2$$

$$\frac{dy}{dx} = 12 - 6x$$

STEP 2 $$12 - 6x = 0$$

$$x = 2$$

STEP 3 $$y = 12 \times 2 - 7 - 3 \times 2^2$$

$$y = 5$$

$\therefore$ Maximum at $(2, 5)$ as the coefficient of $x^2$ in parabola's equation is negative

**[ 3 marks ]**

**Answer 5**

**( a )** **( i )** $(-2, 16)$ **( ii )** $(2, -16)$

**[ 1, 1 mark ]**

**( b )**

STEP 1 $$y = x^3 - 12x$$

$$\frac{dy}{dx} = 3x^2 - 12$$

STEP 2 $$3x^2 - 12 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

STEP 3 $$y = (-2)^3 - 12 \times (-2) \qquad y = 2^3 - 12 \times 2$$

$$y = 16 \qquad y = -16$$

$\therefore$ Maximum at $(-2, 16)$ and a minimum $(2, -16)$

as coefficient of $x^3$ in cubic's equation is positive

**[ 4 marks ]**

**Answer 6**

**( i )** STEP 1

$$y = x^3 - 27x$$

$$\frac{dy}{dx} = 3x^2 - 27$$

STEP 2

$$3x^2 - 27 = 0$$

$$x^2 = 9$$

$$x = \pm 3$$

STEP 3

$$y = (-3)^3 - 27 \times (-3) \qquad y = 3^3 - 27 \times 3$$

$$y = 54 \qquad y = -54$$

$\therefore$ Maximum at $(-3, 54)$ and a minimum $(3, -54)$

as coefficient of $x^3$ in cubic's equation is positive

**[ 4 marks ]**

**( ii )** STEP 1

$$y = (x + 7)(x + 1)$$

$$= x^2 + 8x + 7$$

$$\frac{dy}{dx} = 2x + 8$$

STEP 2

$$2x + 8 = 0$$

$$x = -4$$

STEP 3

$$y = (-4 + 7)(-4 + 1)$$

$$y = -9$$

$\therefore$ Minimum at $(-4, -9)$ as the coefficient of $x^2$ in parabola's equation is positive

**[ 4 marks ]**

**( iii )** STEP 1

$$y = x^4 - 256x$$

$$\frac{dy}{dx} = 4x^3 - 256$$

STEP 2

$$4x^3 - 256 = 0$$

$$x^3 = 64$$

$$x = 4$$

STEP 3

$$y = 4^4 - 256 \times 4$$

$$y = -768$$

$\therefore$ Stationary point at $(4, -768)$

To show it's a minimum, plot graph or use second derivative

**[ 4 marks ]**

**Answer 7**

STEP 1 $$y = x^3 - 6x^2 - 36x + 100$$

$$\frac{dy}{dx} = 3x^2 - 12x - 36$$

STEP 2 $$3x^2 - 12x - 36 = 0$$

Divide through by 3

$$x^2 - 4x - 12 = 0$$

$$(x + 2)(x - 6) = 0$$

Either $x = -2$ or $x = 6$

STEP 3 For $x = -2$, $$y = (-2)^3 - 6 \times (-2)^2 - 36 \times (-2) + 100$$

$$y = 140$$

For $x = 6$, $$y = 6^3 - 6 \times 6^2 - 36 \times 6 + 100$$

$$y = -116$$

$\therefore$ Maximum at $(-2,\ 140)$ and a minimum $(6, -116)$

as coefficient of $x^3$ in cubic's equation is positive

**[ 6 marks ]**

**Answer 8** $$f(x) = \frac{12}{x} = 12x^{-1}$$

**( i )** $f'(x) = -12x^{-2}$ which can be written $f'(x) = -\dfrac{12}{x^2}$

**[ 2 marks ]**

**( ii )** $f''(x) = 24x^{-3}$ which can be written $f''(x) = \dfrac{24}{x^3}$

**[ 2 marks ]**

**( iii )** $(2,\ 6)$ **( iv )** $-3$ **( v )** Anticlockwise

**[ 2, 2, 2 marks ]**

**Answer 9**

STEP 1 $$y = x^3 + 12x$$

$$\frac{dy}{dx} = 3x^2 + 12$$

STEP 2 $$3x^2 + 12 = 0$$

$x^2 = -4$ which has no real solutions

$\therefore$ No turning points ☐