7.2 Answers

Answers 1

$$\text{Total surface area} = 2x^2 + 4x(12 - 3x)$$
$$= 2x^2 + 48x - 12x^2$$
$$= 48x - 10x^2$$
$$\frac{dA}{dx} = 48 - 20x$$
$$48 - 20x = 0$$
$$x = 2.4 \text{ cm}$$
$$A_{MAX} = 48 \times 2.4 - 10 \times 2.4^2$$
$$= 115.2 - 57.6$$
$$= 57.6 \text{ cm}^2$$

$\therefore$ Maximum at (2.4, 57.6) as the coefficient of x^2 in the parabola's equation is negative.

[5 marks]

Answer 2

$$y = x^2 + 3x$$

(a) $\dfrac{dy}{dx} = 2x + 3$

[2 marks]

(b) When $x = -4$, $\dfrac{dy}{dx} = -5$

[2 marks]

(c) $2x + 3 = 0 \Rightarrow x = -\dfrac{3}{2}$

$$y = \left(-\frac{3}{2}\right)^2 + 3\left(-\frac{3}{2}\right)$$
$$= -\frac{9}{4}$$

$\therefore$ Minimum point is $\left(-\dfrac{3}{2}, -\dfrac{9}{4}\right)$

[3 marks]

Answer 3

$$y = x^3 + 3x^2 - 24x$$

(a) $\dfrac{dy}{dx} = 3x^2 + 6x - 24$

[3 marks]

(b)

$$3x^2 + 6x - 24 = 0$$

Divide through by 3

$$x^2 + 2x - 8 = 0$$

$$(x + 4)(x - 2) = 0$$

Either $x = -4$ or $x = 2$

For $x = -4$, $y = (-4)^3 + 3 \times (-4)^2 - 24 \times (-4)$

$= -64 + 48 + 96$

$= 80$

For $x = 2$, $y = 2^3 + 3 \times 2^2 - 24 \times 2$

$= 8 + 12 - 48$

$= -28$

$\therefore$ Turning points are $(-4,\ 80)$, $(2, -28)$

[5 marks]

Answer 4

$$y = x^3 - x^2 - 8x + 12$$

(a) $\dfrac{dy}{dx} = 3x^2 - 2x - 8$

[2 marks]

(b)

$$3x^2 - 2x - 8 = 0$$

$$(3x + 4)(x - 2) = 0$$

Either $x = -\dfrac{4}{3}$ or $x = 2$

[3 marks]

(c) For $x = -\dfrac{4}{3}$, $y = \left(-\dfrac{4}{3}\right)^3 - \left(-\dfrac{4}{3}\right)^2 - 8 \times \left(-\dfrac{4}{3}\right) + 12$

$= \left(\dfrac{500}{27}\right)$ $\therefore$ Tangent is not at $\left(-\dfrac{4}{3},\ \dfrac{500}{27}\right)$

For $x = 2$, $y = 2^3 - 2^2 - 8 \times 2 + 12$

$= 0$ $\therefore$ Tangent is at $(2,\ 0)$

That is, the x-axis is a tangent to the curve, C

[2 marks]

Answer 5

(a) **(i)** $\dfrac{dy}{dx} = 16x$

(ii) $y = 2x^{-1} \Rightarrow \dfrac{dy}{dx} = -2x^{-2}$

(b) $y = 8x^2 + \dfrac{2}{x} \Rightarrow \dfrac{dy}{dx} = 16x - \dfrac{2}{x^2}$

$$16x - \frac{2}{x^2} = 0$$

Multiply through by x^2

$$16x^3 - 2 = 0$$

$$x^3 = \frac{1}{8}$$

$$x = \frac{1}{2}$$

$$y = 8 \times \left(\frac{1}{2}\right)^2 + 2 \div \left(\frac{1}{2}\right)$$

$$= 8 \times \frac{1}{4} + 2 \times 2$$

$$= 6$$

$\therefore$ Turning point is at $\left(\dfrac{1}{2},\ 6\right)$

[4 marks]

Answer 6

$$y = x^2 + \frac{16}{x} \Rightarrow \frac{dy}{dx} = 2x - \frac{16}{x^2}$$

$$2x - \frac{16}{x^2} = 0$$

Multiply through by x^2

$$2x^3 - 16 = 0$$

$$x^3 = 8$$

$$x = 2$$

$$y = 2^2 + \frac{16}{2}$$

$$= 12$$

$\therefore$ Turning point is at $(2,\ 12)$

[4 marks]

Answer 7

(a) $V = (3 - x)(2x + 5)(x + 1)$

$= (3 - x)\left(2x^2 + 7x + 5\right)$

Multiplication grid :

	$2x^2$	$7x$	5
3	$6x^2$	$21x$	15
$-x$	$-2x^3$	$-7x^2$	$-5x$

$= 15 + 16x - x^2 - 2x^3$ □

[3 marks]

(b) $\dfrac{dV}{dx} = 16 - 2x - 6x^2$

$16 - 2x - 6x^2 = 0$

Divide through by -2

$3x^2 + x - 8 = 0$

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 3 \times (-8)}}{2 \times 3}$$

$$= \frac{-1 \pm \sqrt{97}}{6}$$

Either $x = -1.808$ or $x = 1.475$

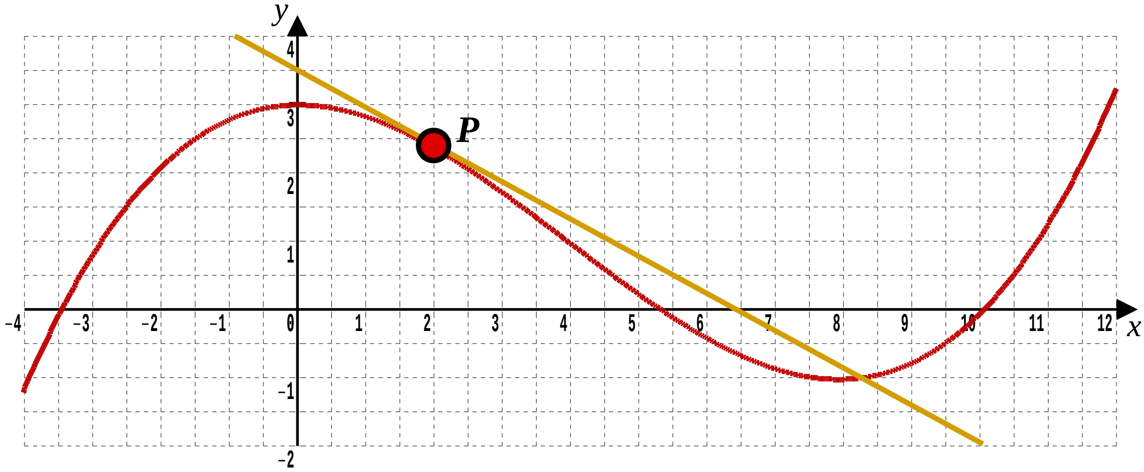
The negative value of x does not make sense in terms of the physical problem as the length $(x + 1)$ cm would then be negative.

$\therefore$ The maximum must correspond to $x = 1.475$

[5 marks]

Answer 8

(a) **(i)**



For the tangent drawn between (0, 3.6) and (10, – 1.8)

$$m = \frac{\Delta Y}{\Delta X}$$

$$= \frac{3.6 - (-1.8)}{0 - 10}$$

$$= -0.54 \qquad \text{Accept } (-0.4) \leqslant m \leqslant (-0.7)$$

[3 marks]

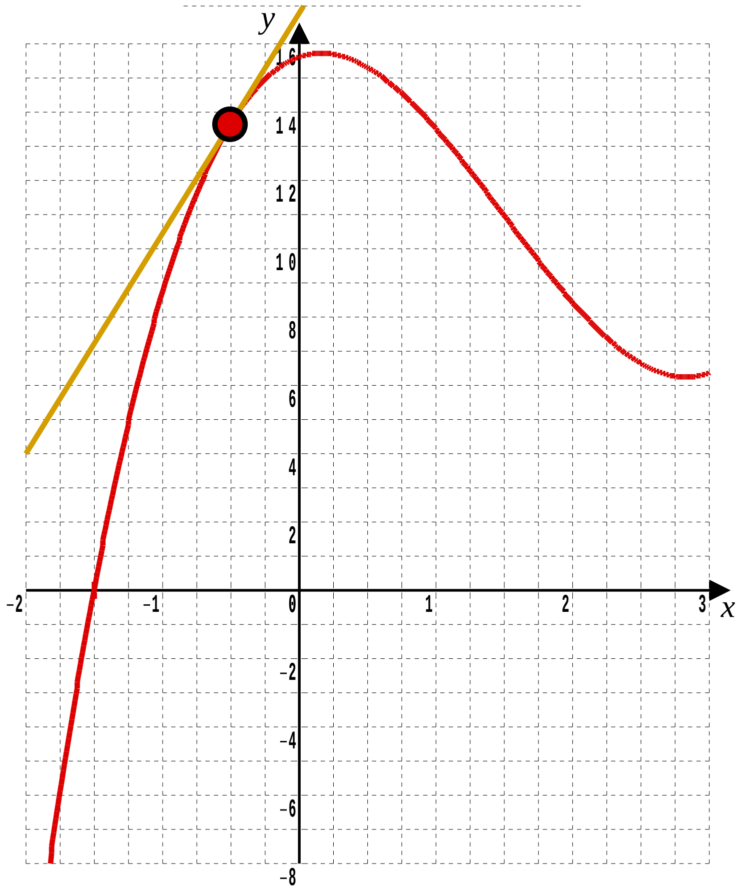
(ii) $y = -0.54x + 3.6$

[2 marks]

(b) $k = -1,\ 3,$

[2 marks]

Answer 9



For the tangent drawn between $(0,\ 17)$ and $(-2,\ 4)$

$$m = \frac{\Delta Y}{\Delta X}$$

$$= \frac{17 - 4}{0 - (-2)}$$

$$= 6.5 \qquad \text{Accept } \ 5 \leqslant m \leqslant 8$$

[3 marks]