
DIFFERENTIATION
dy
dx

D I F F E R E N T I A T I O N
The Basics and Applications

Lesson 1

GCSE Mathematics
Differentiation I

1.1 Indices

The mathematical topic of differentiation is about the gradient of curves. It is built upon the assumption that the topic of indices has been thoroughly mastered and so, by way of preparation, this lesson will revise the necessary background material.

Without using a calculator, determine the value of each of the following.
Give exact answers by using vulgar fractions where it is helpful to do so.

(i) $\left(\frac{3}{7}\right)^{2}$ **(ii)** 3^{-2} **(iii)** π^{0}

(iv) $\left(\frac{3}{2}\right)^{-3}$ **(v)** $\sqrt{\left(\frac{16}{9}\right)}$ **(vi)** 0.125^{2}

(vii) $\left(\frac{1}{2}\right)^{-5}$ **(viii)** $16^{\frac{1}{2}}$ **(ix)** $8^{\frac{2}{3}}$

[9 marks]

Now watching the teaching video to see if your answers are correct.

Teaching Video : http://www.NumberWonder.co.uk/v9036/1.mp4

1.2 A Useful "Flip"

Power Inversion Rule

$$\left(\frac{p}{q}\right)^{-m} = \left(\frac{q}{p}\right)^{m} \qquad p,\ q \neq 0$$

1.3 Exercise

Marks Available : 76

Question 1

Without using a calculator, determine the value of each of the following.
You may leave answers as vulgar fractions where it is helpful to do so.

(i) 4^2 **(ii)** 2^8 **(iii)** 7^1

(iv) $\left(\frac{1}{2}\right)^5$ **(v)** $\left(\frac{2}{5}\right)^3$ **(vi)** 0.25^2

(vii) $\sqrt{36}$ **(viii)** 14^0 **(ix)** $81^{\frac{1}{2}}$

(x) $25^{\frac{1}{2}}$ **(xi)** $125^{\frac{1}{3}}$ **(xii)** $\left(\frac{9}{4}\right)^{\frac{1}{2}}$

(xiii) $6^2 + 8^2$ **(xiv)** $10^3 \times 13^2$ **(xv)** 7^{-2}

(xvi) $\sqrt{\frac{1}{3^2 + 4^2}}$ **(xvii)** $\left(\frac{\pi}{1 + \sqrt{\frac{1}{\pi}}}\right)^0$ **(xviii)** $4^{\frac{3}{2}}$

(xix) $8^{\frac{2}{3}}$ **(xx)** 2^{-3} **(xxi)** $\left(\frac{1}{3}\right)^2$

(xxii) $\left(\frac{3}{7}\right)^{-1}$ **(xxiii)** 0.1^2 **(xxiv)** $\sqrt{\sqrt{256}}$

(xxv) $\left(\frac{3}{10}\right)^3$ **(xxvi)** $27^{\frac{2}{3}}$

[26 marks]

Question 2

Without using a calculator, determine the value of each of the following.
You may leave answers as vulgar fractions where it is helpful to do so.

(i) $(-3)^2$ **(ii)** 2^7 **(iii)** $\left(\frac{1}{20}\right)^3$

(iv) $\left(-\frac{1}{2}\right)^4$ **(v)** $\left(\frac{3}{5}\right)^{-3}$ **(vi)** $400^{\frac{1}{2}}$

(vii) $(-1)^{42}$ **(viii)** $(-2)^3$ **(ix)** $169^{\frac{1}{2}}$

(x) $81^{0.5}$ **(xi)** $64^{\frac{1}{3}}$ **(xii)** $\left(\frac{1}{25}\right)^{\frac{1}{2}}$

(xiii) $9^2 + 12^2$ **(xiv)** $10^4 \times 14^2$ **(xv)** $\sqrt{\frac{1}{12^2 + 5^2}}$

(xvi) $1^{-0.5}$ **(xvii)** $\left(\frac{sin\,(4\pi)}{cos\,(2\pi) + \sqrt{\frac{1}{\pi}}}\right)^0$

(xviii) $9^{\frac{3}{2}}$ **(xix)** $(-8)^{\frac{2}{3}}$ **(xx)** 2^{-5}

(xxi) $\left(-\frac{5}{6}\right)^{-2}$ **(xxii)** $\left(\frac{4}{9}\right)^{\frac{1}{2}}$ **(xxiii)** $\left(\frac{1}{100}\right)^3$

(xxiv) $\sqrt{\sqrt{81}}$ **(xxv)** $\left(-\frac{10}{3}\right)^2$ **(xxvi)** $125^{\frac{2}{3}}$

[26 marks]

Question 3
Without using a calculator, for the curve with equation

$$y = x^2 + x$$

write down points on the curve with the x values given.

(1, _____) (4, _____) (100, _____) (– 1, _____)

[4 marks]

Question 4
Without using a calculator, for the curve with equation;

$$y = x^3 - 5$$

write down points on the curve with the x values given.

(0, _____) (1, _____) (– 1, _____) (5, _____)

[4 marks]

Question 5
Without using a calculator, for the curve with equation

$$y = x^{\frac{3}{2}} + 3$$

write down points on the curve with the x values given.

(0, _____) (1, _____) (4, _____) (9, _____)

[4 marks]

Question 6
Without using a calculator, for the curve with equation

$$y = 2 + x^{-1}$$

write down points on the curve with the x values given.

(1, _____) (– 1 , _____) (2, _____) (10, _____)

[4 marks]

Question 7
Without using a calculator, for the curve with equation

$$y = x^2 + x^{\frac{1}{2}}$$

write down points on the curve with the x values given.

(1, ____) (4, ____) (100, ____) $\left(\frac{1}{4}, \text{ ____} \right)$

[4 marks]

Question 8

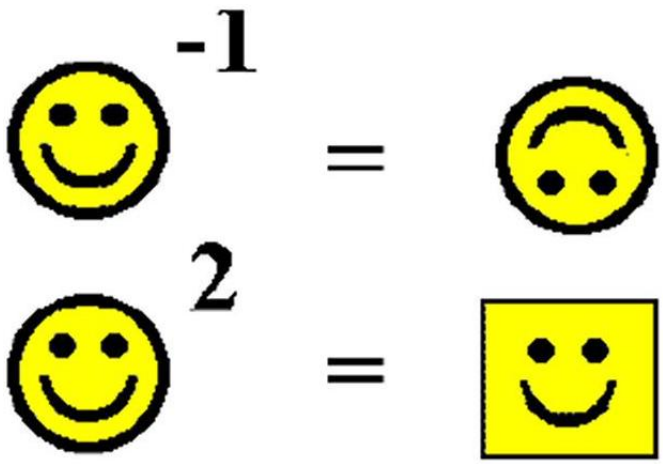
Without using a calculator, for the curve with equation

$$y = x^3 + 4x^2$$

write down points on the curve with the x or y values given.

(1, _____) (3, _____) (0, _____) (– 1, _____)

[4 marks]



1.4 Homework

Marks Available : 46

Question 1

Without using a calculator, determine the value of each of the following.
You may leave answers as fractions where it is helpful to do so.

(i) 5^3 **(ii)** $(-2)^8$ **(iii)** $(-1)^3$

(iv) $\left(\frac{1}{3}\right)^3$ **(v)** $\left(\frac{4}{25}\right)^{\frac{1}{2}}$ **(vi)** 0.5^2

(vii) $\left(\sqrt{\pi^4}\right)^0$ **(viii)** $(-1)^4$ **(ix)** $144^{\frac{1}{2}}$

(x) $625^{\frac{1}{4}}$ **(xi)** $1^{\frac{1}{3}}$ **(xii)** $\left(\frac{1}{9}\right)^{\frac{1}{2}}$

(xiii) 11^2 **(xiv)** $\left(\frac{8}{7}\right)^2$ **(xv)** $\sqrt{\frac{13^2 - 12^2}{3^2 + 4^2}}$

(xvi) 20^3 **(xvii)** $\left(\frac{log(\pi)}{\pi + \sqrt{\frac{1}{\pi}}}\right)^0$ **(xviii)** $16^{\frac{3}{2}}$

(xix) $27^{\frac{2}{3}}$ **(xx)** 21^{-1} **(xxi)** $\left(-\frac{3}{8}\right)^2$

(xxii) $\left(\frac{7}{9}\right)^{-1}$ **(xxiii)** $\sqrt{\sqrt{10000}}$ **(xxiv)** $(-1)^{101}$

(xxv) $\left(\frac{1}{2}\right)^{-3}$ **(xxvi)** $125^{\frac{2}{3}}$

[26 marks]

Question 2
Without using a calculator, for the curve with equation

$$y = x^2 + 3$$

write down points on the curve with the x values given.

(1, _____) (4, _____) (100, _____) (– 1, _____)

[4 marks]

Question 3
Without using a calculator, for the curve with equation;

$$y = x^{0.5} + x$$

write down points on the curve with the x values given.

(0, _____) (1, _____) (9, _____) (100, _____)

[4 marks]

Question 4
Without using a calculator, for the curve with equation

$$y = 6x^{-1}$$

write down points on the curve with the x values given.

(1, _____) (2, _____) (3, _____) (6, _____)

[4 marks]

Question 5
Without using a calculator, for the curve with equation

$$y = 64x^{-2}$$

write down points on the curve with the x values given.

(1, _____) (2 , _____) (4, _____) (8, _____)

[4 marks]

Question 6
Without using a calculator, for the curve with equation

$$y = 12x^{\frac{1}{2}}$$

write down points on the curve with the x values given.

(1, ____) (4, ____) (36, ____) $\left(\frac{1}{4}, \text{____}\right)$

[4 marks]

1.5 Answers

GCSE Mathematics
Differentiation I

1.5.1 Solutions (1.1 Teaching Video)

(i) $\frac{9}{49}$ **(ii)** $\frac{1}{9}$ **(iii)** 1

(iv) $\frac{8}{27}$ **(v)** $\frac{4}{3}$ **(vi)** $0.125 = \left(\frac{1}{8}\right)^2 = \frac{1}{64}$

(vii) 32 **(viii)** 4 **(ix)** 4

[9 marks]

1.5.2 Solutions (1.3 Exercise)

Answer 1

(i) 16 **(ii)** 256 **(iii)** 7

(iv) $\frac{1}{32}$ **(v)** $\frac{8}{125}$ **(vi)** $\frac{1}{16}$

(vii) 6 **(viii)** 1 **(ix)** 9

(x) 5 **(xi)** 5 **(xii)** $\frac{3}{2}$

(xiii) 100 **(xiv)** 169 000 **(xv)** $\frac{1}{49}$

(xvi) $\frac{1}{5}$ **(xvii)** 1 **(xviii)** 8

(xix) 4 **(xx)** $\frac{1}{8}$ **(xxi)** $\frac{1}{9}$

(xxii) $\frac{7}{3}$ **(xxiii)** $\frac{1}{100}$ **(xxiv)** 4

(xxv) $\frac{27}{1000}$ **(xxvi)** 9

[26 marks]

Answer 2

(i) 9 **(ii)** 128 **(iii)** $\frac{1}{8000}$

(iv) $\frac{1}{16}$ **(v)** $\frac{125}{27}$ **(vi)** 20

(vii) 1 **(viii)** -8 **(ix)** 13

(x) 9 **(xi)** 4 **(xii)** $\frac{1}{5}$

(xiii) 225 **(xiv)** 1 960 000 **(xv)** $\frac{1}{13}$

(xvi) 1 **(xvii)** 1

(xviii) 27 **(xix)** 4 **(xx)** $\frac{1}{32}$

(xxi) $\frac{36}{25}$ **(xxii)** $\frac{2}{3}$ **(xxiii)** $\frac{1}{1000\,000}$

(xxiv) 3 **(xxv)** $\frac{100}{9}$ **(xxvi)** 25

[26 marks]

Answer 3

$$y = x^2 + x$$

(1, 2) (4, 20) (100, 10100) (-1 , 0)

[4 marks]

Answer 4

$$y = x^3 - 5$$

(0, -5) (1, -4) (-1, -6) (5, 120)

[4 marks]

Answer 5

$$y = x^{\frac{3}{2}} + 3$$

(0, 3) (1, 4) (4, 11) (9, 30)

[4 marks]

Answer 6

$$y = 2 + x^{-1}$$

(1, 3) (-1 , 1) (2, 2.5) (10, 2.1)

[4 marks]

Answer 7

$$y = x^2 + x^{\frac{1}{2}}$$

(1,2) (4, 18) (100, 10010) $\left(\frac{1}{4}, \frac{9}{16} \right)$

[4 marks]

Answer 8

$$y = x^3 + 4x^2$$

(1, 5) (3, 63) (0, 0) (-1, 3)

[4 marks]

1.5.3 Solutions (1.4 Homework)

GCSE Mathematics
Differentiation I

Answer 1

(i) 125	**(ii)** 256	**(iii)** -1
(iv) $\frac{1}{27}$	**(v)** $\frac{2}{5}$	**(vi)** 0.25
(vii) 1	**(viii)** 1	**(ix)** 12
(x) 5	**(xi)** 1	**(xii)** $\frac{1}{3}$
(xiii) 121	**(xiv)** $\frac{64}{49}$	**(xv)** 1
(xvi) 8000	**(xvii)** 1	**(xviii)** 64
(xix) 9	**(xx)** $\frac{1}{21}$	**(xxi)** $\frac{9}{64}$
(xxii) $\frac{9}{7}$	**(xxiii)** 10	**(xxiv)** -1
(xxv) 8	**(xxvi)** 25	

[26 marks]

Answer 2 $y = x^2 + 3$

(1, 4) (4, 19) (100, 10003) $(-1, 4)$

[4 marks]

Answer 3 $y = x^{0.5} + x$

(0, 0) (1, 2) (9, 12) (100, 110)

[4 marks]

Answer 4

$y = 6x^{-1}$

(1, 6) (2, 3) (3, 2) (6, 1)

[4 marks]

Answer 5 $y = 64x^{-2}$

(1, 64) (2, 16) (4, 4) (8, 1)

[4 marks]

Answer 6 $y = 12x^{\frac{1}{2}}$

(1, 12) (4, 24) (36, 72) $\left(\frac{1}{4}, 6\right)$

[4 marks]

Lesson 2

GCSE Mathematics
Differentiation I

2.1 Nuts & Bolts Differentiation

To begin with, without worrying about why it works, we are going to look at a technique that allows us to take the equation of a simple curve, and straight away, without any working, write down a gradient equation for that curve.

The Power Rule

If $y = x^n$ then $\frac{dy}{dx} = n x^{n-1}$ for any constant, n

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2.2 Examples

Teaching Video : http://www.NumberWonder.co.uk/v9036/2.mp4

The teaching video will walk you through the following examples.

(i) $y = 7x^4 + 3x^2$ $\frac{dy}{dx} =$

(ii) $y = \frac{1}{2}x^8 - \frac{3}{4}x^6$ $\frac{dy}{dx} =$

(iii) $y = 5$ $\frac{dy}{dx} =$

(iv) $y = 7x$ $\frac{dy}{dx} =$

(v) $y = \frac{2}{3}x + \frac{1}{3}$ $\frac{dy}{dx} =$

(vi) $y = 3x^7 - x^5 + 0.2x + 3$ $\frac{dy}{dx} =$

[6 marks]

2.3 Exercise

Marks Available : 56

Question 1

For each of these equations, write down the corresponding gradient equation.

(i) $y = 6x$ $\qquad \frac{dy}{dx} =$

(ii) $y = 4x + 5$ $\qquad \frac{dy}{dx} =$

(iii) $y = -3x + 17$ $\qquad \frac{dy}{dx} =$

(iv) $y = 3 - 17x$ $\qquad \frac{dy}{dx} =$

(v) $y = 4x^2$ $\qquad \frac{dy}{dx} =$

(vi) $y = 3x^2 - 7x$ $\qquad \frac{dy}{dx} =$

(vii) $y = 6x^2 + 2x - 5$ $\qquad \frac{dy}{dx} =$

(viii) $y = 10 - 5x - 3x^2$ $\qquad \frac{dy}{dx} =$

[16 marks]

Question 2

Find $f'(x)$ given that $f(x) = \frac{3}{2}x^2 - \frac{5}{2}x^4$

[2 marks]

Question 3

Find $g'(x)$ given that $g(x) = \frac{4}{3}x^6 + \frac{2}{3}x^2 - \frac{5}{6}$

[2 marks]

Question 4

Find $h'(x)$ given that $h(x) = \frac{3}{4}x^2 + \frac{1}{4}x + 2$

[2 marks]

Question 5

For each of the following equations, determine $\frac{dy}{dx}$

(i) $y = 2x^3$ $\frac{dy}{dx} =$

(ii) $y = 4x^3 + x + 1$ $\frac{dy}{dx} =$

(iii) $y = 7x - 5x^3$ $\frac{dy}{dx} =$

(iv) $y = 4x^2 - 9x^4$ $\frac{dy}{dx} =$

(v) $y = 10x^6 - 12x^5$ $\frac{dy}{dx} =$

(vi) $y = x^2(10 - 7x)$ $\frac{dy}{dx} =$

HINT : First, expand the brackets, then differentiate

(vii) $y = (x + 4)(2x - 7)$ $\frac{dy}{dx} =$

(viii) $y = \left(x^2 - 3\right)\left(2x - 1\right)$ $\frac{dy}{dx} =$

[16 marks]

Question 6

$$p(x) = \frac{4}{x}$$

By first rewriting this as $p(x) = 4x^{-1}$ find $p'(x)$

[2 marks]

Question 7

$$q(x) = \frac{10}{x^2}$$

Find $q'(x)$

[2 marks]

Question 8

$$v(x) = 4x - \frac{5}{x}$$

Find $v'(x)$

[2 marks]

Question 9

$$w(x) = 3x^3 + \frac{2}{x^3}$$

Find $w'(x)$

[3 marks]

Question 10

$$e(x) = \frac{1}{4x}$$

Find $e'(x)$

[3 marks]

Question 11

$$k(x) = \frac{3}{4x^2} + 5$$

Find $k'(x)$

[3 marks]

Question 12

$$m(x) = \frac{5}{2x} - \frac{3}{2x^2}$$

Find $m'(x)$

[3 marks]

2.4 Homework

GCSE Mathematics
Differentiation I

Marks Available : 26

Question 1

For each of the following equations, first expand the brackets, then determine $\frac{dy}{dx}$

(i) $y = 13\left(x^3 + 2\right)$ $\frac{dy}{dx} =$

(ii) $y = x^3\left(x^2 + 1\right)$ $\frac{dy}{dx} =$

(iii) $y = 3x^3\left(2x + 5\right)$ $\frac{dy}{dx} =$

(iv) $y = (x + 5)(x + 3)$ $\frac{dy}{dx} =$

(v) $y = \left(4x^3 + 3\right)\left(x^2 + 7\right)$ $\frac{dy}{dx} =$

[10 marks]

Question 2

For each of these equations, write down the corresponding gradient equation.

(i) $y = -7x + 12$ $\frac{dy}{dx} =$

(ii) $y = x^{-3}$ $\frac{dy}{dx} =$

(iii) $y = 6x^{-5} + 19x^5$ $\frac{dy}{dx} =$

(iv) $y = \frac{1}{x^7}$ $\frac{dy}{dx} =$

(v) $y = \frac{4}{5x^3}$ $\frac{dy}{dx} =$

(vi) $y = x^{2.5} - 8x^{1.5}$ $\frac{dy}{dx} =$

(vii) $y = 6x^{\frac{5}{2}} + x^{\frac{1}{3}}$ $\frac{dy}{dx} =$

(viii) $y = \sqrt{x}$ $\frac{dy}{dx} =$

[16 marks]

2.5 Answers

2.5.1 Solutions (2.2 Teaching Video)

(i) $y = 7x^4 + 3x^2 \quad \therefore \quad \frac{dy}{dx} = 28x^3 + 6x$

(ii) $y = \frac{1}{2}x^8 - \frac{3}{4}x^6 \quad \therefore \quad \frac{dy}{dx} = 4x^7 - \frac{9}{2}x^5$

(iii) $y = 5 \quad \therefore \quad \frac{dy}{dx} = 0$

(iv) $y = 7x \quad \therefore \quad \frac{dy}{dx} = 7$

(v) $y = \frac{2}{3}x + \frac{1}{3} \quad \therefore \quad \frac{dy}{dx} = \frac{2}{3}$

(vi) $y = 3x^7 - x^5 + 0.2x + 3 \quad \therefore \quad \frac{dy}{dx} = 21x^6 - 5x^4 + 0.2$

[6 marks]

2.5.2 Solutions (2.3 Exercise)

Answer 1

(i) $y = 6x \quad \therefore \quad \frac{dy}{dx} = 6$

(ii) $y = 4x + 5 \quad \therefore \quad \frac{dy}{dx} = 4$

(iii) $y = -3x + 17 \quad \therefore \quad \frac{dy}{dx} = -3$

(iv) $y = 3 - 17x \quad \therefore \quad \frac{dy}{dx} = -17$

(v) $y = 4x^2 \quad \therefore \quad \frac{dy}{dx} = 8x$

(vi) $y = 3x^2 - 7x \quad \therefore \quad \frac{dy}{dx} = 6x - 7$

(vii) $y = 6x^2 + 2x - 5 \quad \therefore \quad \frac{dy}{dx} = 12x + 2$

(viii) $y = 10 - 5x - 3x^2 \quad \therefore \quad \frac{dy}{dx} = -5 - 6x$

[16 marks]

Answer 2

$f(x) = \frac{3}{2}x^2 - \frac{5}{2}x^4 \quad \therefore \quad f'(x) = 3x - 10x^3$

[2 marks]

Answer 3

$$g(x) = \frac{4}{3}x^6 + \frac{2}{3}x^2 - \frac{5}{6} \qquad \therefore \quad g'(x) = 8x^5 + \frac{4}{3}x$$

[2 marks]

Answer 4

$$h(x) = \frac{3}{4}x^2 + \frac{1}{4}x + 2 \qquad \therefore \quad h'(x) = \frac{3}{2}x + \frac{1}{4}$$

[2 marks]

Answer 5

(i) $y = 2x^3 \qquad \therefore \quad \frac{dy}{dx} = 6x^2$

(ii) $y = 4x^3 + x + 1 \qquad \therefore \quad \frac{dy}{dx} = 12x^2 + 1$

(iii) $y = 7x - 5x^3 \qquad \therefore \quad \frac{dy}{dx} = 7 - 15x^2$

(iv) $y = 4x^2 - 9x^4 \qquad \therefore \quad \frac{dy}{dx} = 8x - 36x^3$

(v) $y = 10x^6 - 12x^5 \qquad \therefore \quad \frac{dy}{dx} = 60x^5 - 60x^4$

(vi) $y = x^2(10 - 7x)$

$y = 10x^2 - 7x^3 \qquad \therefore \quad \frac{dy}{dx} = 20x - 21x^2$

(vii) $y = (x + 4)(2x - 7)$

$y = 2x^2 + x - 28 \qquad \therefore \quad \frac{dy}{dx} = 4x + 1$

(viii) $y = (x^2 - 3)(2x - 1)$

$y = 2x^3 - x^2 - 6x + 3 \qquad \therefore \quad \frac{dy}{dx} = 6x^2 - 2x - 6$

[16 marks]

Answer 6

$$p(x) = \frac{4}{x} \Rightarrow p(x) = 4x^{-1}$$

$$\therefore \quad p'(x) = -4x^{-2} \Rightarrow p'(x) = -\frac{4}{x^2}$$

[2 marks]

Answer 7

$$q(x) = \frac{10}{x^2} \Rightarrow q(x) = 10x^{-2}$$

$$\therefore\ q'(x) = -20x^{-3} \Rightarrow q'(x) = -\frac{20}{x^3}$$

[2 marks]

Answer 8

$$v(x) = 4x - \frac{5}{x} \Rightarrow v(x) = 4x - 5x^{-1}$$

$$\therefore\ v'(x) = 4 + 5x^{-2} \Rightarrow v'(x) = 4 + \frac{5}{x^2}$$

[2 marks]

Answer 9

$$w(x) = 3x^3 + \frac{2}{x^3} \Rightarrow w(x) = 3x^3 + 2x^{-3}$$

$$\therefore\ w'(x) = 9x^2 - 6x^{-4} \Rightarrow w'(x) = 9x^2 - \frac{6}{x^4}$$

[3 marks]

Answer 10

$$e(x) = \frac{1}{4x} \Rightarrow e(x) = \frac{1}{4}x^{-1}$$

$$\therefore\ e'(x) = -\frac{1}{4}x^{-2} \Rightarrow e'(x) = -\frac{1}{4x^2}$$

[3 marks]

Answer 11

$$k(x) = \frac{3}{4x^2} + 5 \Rightarrow k(x) = \frac{3}{4}x^{-2} + 5$$

$$\therefore\ k'(x) = -\frac{6}{4}x^{-3} \Rightarrow k'(x) = -\frac{3}{2x^3}$$

[3 marks]

Answer 12

$$m(x) = \frac{5}{2x} - \frac{3}{2x^2} \Rightarrow m(x) = \frac{5}{2}x^{-1} - \frac{3}{2}x^{-2}$$

$$\therefore\ m'(x) = -\frac{5}{2}x^{-2} + 3x^{-3} \Rightarrow m'(x) = -\frac{5}{2x^2} + \frac{3}{x^3}$$

[3 marks]

2.5.3 Solutions (2.4 Homework)

Answer 1

(i) $y = 13\left(x^3 + 2\right)$

$y = 13x^3 + 26 \qquad \therefore \quad \dfrac{dy}{dx} = 39x^2$

(ii) $y = x^3\left(x^2 + 1\right)$

$y = x^5 + x^3 \qquad \therefore \quad \dfrac{dy}{dx} = 5x^4 + 3x^2$

(iii) $y = 3x^3\,(2x + 5)$

$y = 6x^4 + 15x^3 \qquad \therefore \quad \dfrac{dy}{dx} = 24x^3 + 45x^2$

(iv) $y = (x + 5)(x + 3)$

$y = x^2 + 8x + 15 \qquad \therefore \quad \dfrac{dy}{dx} = 2x + 8$

(v) $y = \left(4x^3 + 3\right)\left(x^2 + 7\right)$

$y = 4x^5 + 28x^3 + 3x^2 + 21 \qquad \therefore \quad \dfrac{dy}{dx} = 20x^4 + 84x^2 + 6x$

[10 marks]

Answer 2

(i) $y = -7x + 12$

$\therefore \quad \dfrac{dy}{dx} = -7$

(ii) $y = x^{-3}$

$\therefore \quad \dfrac{dy}{dx} = -3x^{-4} \Rightarrow \dfrac{dy}{dx} = -\dfrac{3}{x^4}$

(iii) $y = 6x^{-5} + 19x^5$

$\therefore \quad \dfrac{dy}{dx} = -30x^{-6} + 95x^4 \Rightarrow \dfrac{dy}{dx} = -\dfrac{30}{x^6} + 95x^4$

(iv) $y = \frac{1}{x^7} \Rightarrow y = x^{-7}$

$$\therefore \quad \frac{dy}{dx} = -7x^{-8} \Rightarrow \frac{dy}{dx} = -\frac{7}{x^8}$$

(v) $y = \frac{4}{5x^3} \Rightarrow y = \frac{4}{5}x^{-3}$

$$\therefore \quad \frac{dy}{dx} = -\frac{12}{5}x^{-4} \Rightarrow \frac{dy}{dx} = -\frac{12}{5x^4}$$

(vi) $y = x^{2.5} - 8x^{1.5}$

$$\therefore \quad \frac{dy}{dx} = 2.5x^{1.5} - 12x^{0.5}$$

(vii) $y = 6x^{\frac{5}{2}} + x^{\frac{1}{3}}$

$$\therefore \quad \frac{dy}{dx} = 15x^{\frac{3}{2}} + \frac{1}{3}x^{-\frac{2}{3}} \Rightarrow \frac{dy}{dx} = 15x^{\frac{3}{2}} + \frac{1}{3x^{\frac{2}{3}}}$$

(viii) $y = \sqrt{x} \Rightarrow y = x^{\frac{1}{2}}$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

[16 marks]

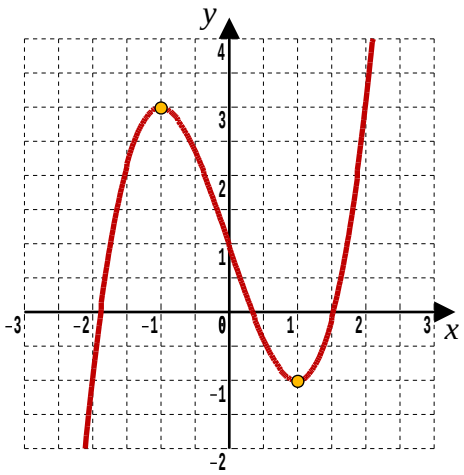
Lesson 3

GCSE Mathematics
Differentiation I

3.1 Turning Points

A turning point is a point of a graph where the gradient "turns through zero".

For example, the graph of $y = x^3 - 3x + 1$ has two turning points.



Remembering that a graph is read from left to right, at $(-1,\ 3)$ the gradient turns from being positive (before $x = -1$) through zero (at $x = -1$) to being negative (after $x = -1$). At $(1, -1)$ the gradient turns from being negative (before $x = 1$) through zero (at $x = 1$) to being positive (after $x = 1$).
The graph thus has two turning points, one at $(-1,\ 3)$ and the other at $(1, -1)$.
How can these be found without drawing a graph ?

Teaching Video : http://www.NumberWonder.co.uk/v9036/3.mp4

After watching the video,
set out the method of
solution here

☞

[6 marks]

3.2 Exercise

Marks Available : 50

Question 1

For each of these equations, determine $\frac{dy}{dx}$

(i) $y = 7x^5$ $\frac{dy}{dx} =$

(ii) $y = 8x + 1.5$ $\frac{dy}{dx} =$

(iii) $y = 5x^8 + 17x - 11$ $\frac{dy}{dx} =$

[6 marks]

Question 2

$y = 5x^2 - 30x$

(a) Find $\frac{dy}{dx}$

[2 marks]

(b) Find the coordinates of the turning point by solving the equation;

$$\frac{dy}{dx} = 0$$

Show your working clearly.

[5 marks]

Question 3

Differentiate each of the following,

(i) $y = 24x^2 - 12x^4$

(ii) $y = 13$

(iii) $y = \frac{5}{x^3}$

[6 marks]

Question 4

$y = 4x^2 + 16x + 21$

(a) Find $\frac{dy}{dx}$

[2 marks]

(b) Find the coordinates of the turning point by solving the equation;

$$\frac{dy}{dx} = 0$$

Show your working clearly.

[5 marks]

Question 5

By first expanding the brackets, find the derivative of,

$$y = x^3\left(4x^8 - 7x\right)$$

[4 marks]

Question 6

$y = x^3 + 9x^2 + 15x$

(a) Find $\frac{dy}{dx}$

[2 marks]

The curve with equation $y = x^3 + 9x^2 + 15x$ has two turning points.

(b) Work out the coordinates of these two turning points.
Show your working clearly.

[6 marks]

Question 7

GCSE Question, 9th January 2017, Paper 3H, Q18

The curve with equation

$$y = 10x^2 + 9x + 5$$

has a minimum at point *A*

Find the coordinates of *A.*
Show your working clearly.

HINT : The minimum is a turning point.

[4 marks]

Question 8

GCSE Question, 4th June 2015, Paper 4H, Q20

$y = x^3 + 6x^2 + 5$

(a) Find $\frac{dy}{dx}$

[2 marks]

The curve with equation $y = x^3 + 6x^2 + 5$ has two turning points.

(b) Work out the coordinates of these two turning points.
Show your working clearly.

[6 marks]

3.3 Answers

3.3.1 Solutions (3.1 Teaching Video)

$$y = x^3 - 3x + 1$$

$$\therefore \quad \frac{dy}{dx} = 3x^2 - 3$$

At a turning point, the gradient has a value of zero.

$$3x^2 - 3 = 0$$

Divide through by 3

$$x^2 - 1 = 0$$

$$(x + 1)(x - 1) = 0 \qquad \text{Difference of two squares}$$

Either $x + 1 = 0$ or $x - 1 = 0$

$x = -1$ $\qquad$ $x = 1$

To get ***points*** substitute these values of x back into the point equation of the curve. (NOT the gradient equation of the curve !)

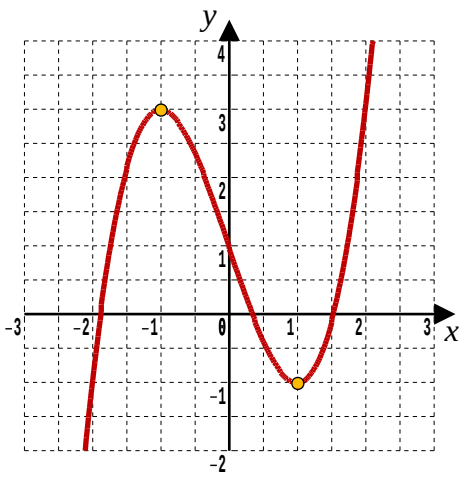
$$y = x^3 - 3x + 1 \qquad\qquad y = x^3 - 3x + 1$$

$$= (-1)^3 - 3(-1) + 1 \qquad\qquad = 1^3 - 3 \times 1 + 1$$

$$= -1 + 3 + 1 \qquad\qquad = 1 - 3 + 1$$

$$= 3 \qquad\qquad = -1$$

The turning points are thus, $(-1,\ 3)$, $(1, -1)$



[6 marks]

3.3.2 Solutions (3.2 Exercise)

Answer 1

(i) $y = 7x^5$ $\qquad \dfrac{dy}{dx} = 35x^4$

(ii) $y = 8x + 1.5$ $\qquad \dfrac{dy}{dx} = 8$

(iii) $y = 5x^8 + 17x - 11$ $\qquad \dfrac{dy}{dx} = 40x^7 + 17$

[6 marks]

Answer 2

(a)

$$y = 5x^2 - 30x$$

$$\therefore \quad \frac{dy}{dx} = 10x - 30$$

[2 marks]

(b) At a turning point, the gradient has a value of zero.

$$10x - 30 = 0$$

$$x = 3$$

To get a ***point*** substitute this value of x back into the point equation of the curve. (NOT the gradient equation of the curve !)

$$y = 5x^2 - 30x$$

$$= 5 \times 3^2 - 30 \times 3$$

$$= -45$$

The turning point is thus $(3, -45)$

[5 marks]

Answer 3

(i) $y = 24x^2 - 12x^4$ $\qquad \dfrac{dy}{dx} = 48x - 48x^3$

(ii) $y = 13$ $\qquad \dfrac{dy}{dx} = 0$

(iii) $y = \dfrac{5}{x^3} \Rightarrow y = 5x^{-3}$

$$\frac{dy}{dx} = -15x^{-4} \Rightarrow \frac{dy}{dx} = -\frac{15}{x^4}$$

[6 marks]

Answer 4

(a) $y = 4x^2 + 16x + 21$

$$\therefore \quad \frac{dy}{dx} = 8x + 16$$

[2 marks]

(b) At a turning point, the gradient has a value of zero.

$8x + 16 = 0$

$x = -2$

To get a ***point*** substitute this value of x back into the equation of the curve.

$$\begin{aligned} y &= 4x^2 + 16x + 21 \\ &= 4 \times (-2)^2 + 16 \times (-2) + 21 \\ &= 16 - 32 + 21 \\ &= 5 \end{aligned}$$

The turning point is thus, $(-2,\ 5)$

[5 marks]

Answer 5

$$y = x^3\left(4x^8 - 7x\right) \Rightarrow y = 4x^{11} - 7x^4 \Rightarrow \frac{dy}{dx} = 44x^{10} - 28x^3$$

[4 marks]

Answer 6

(a) $y = x^3 + 9x^2 + 15x$

$$\therefore \quad \frac{dy}{dx} = 3x^2 + 18x + 15$$

(b) At a turning point, the gradient has a value of zero.

$3x^2 + 18x + 15 = 0$

Divide through by 3

$x^2 + 6x + 5 = 0$

$(x + 5)(x + 1) = 0$

Either $x = 5$ or $x = -1$

To get ***points*** substitute these values of x back into the equation of the curve.

$$\begin{aligned} y &= x^3 + 9x^2 + 15x \\ &= (-5)^3 + 9(-5)^2 + 15(-5) \\ &= -125 + 225 - 75 \\ &= 25 \end{aligned}$$

$$\begin{aligned} y &= x^3 + 9x^2 + 15x \\ &= (-1)^3 + 9(-1)^2 + 15(-1) \\ &= -1 + 9 - 15 \\ &= -7 \end{aligned}$$

The turning points are $(-5,\ 25)$, $(-1, -7)$

[6 marks]

Answer 7

$$y = 10x^2 + 9x + 5$$

$$\therefore \quad \frac{dy}{dx} = 20x + 9$$

At a minimum point, the gradient has a value of zero.

$$20x + 9 = 0$$

$$x = -\frac{9}{20} \text{ or } -0.45$$

To get a ***point*** substitute this value of x back into the equation of the curve.

$$y = 10x^2 + 9x + 5$$

$$= 10 \times \left(-\frac{9}{20}\right)^2 + 9 \times \left(-\frac{9}{20}\right) + 5$$

$$= \frac{119}{40} \text{ or } 2.975$$

The turning point is thus, $(-0.45,\ 2.975)$

[4 marks]

Answer 8

(a) $$y = x^3 + 6x^2 + 5$$

$$\therefore \quad \frac{dy}{dx} = 3x^2 + 12x$$

[2 marks]

(b) At a turning point, the gradient has a value of zero.

$$3x^2 + 12x = 0$$

Divide through by 3

$$x^2 + 4x = 0$$

$$x(x + 4) = 0$$

Either $x = 0$ or $x = -4$

To get ***points*** substitute these values of x back into the equation of the curve

$$y = x^3 + 6x^2 + 5 \qquad y = x^3 + 6x^2 + 5$$

$$= (0)^3 + 6(0)^2 + 5 \qquad = (-4)^3 + 6(-4)^2 + 5$$

$$= 5 \qquad = -64 + 96 + 5$$

$$= 37$$

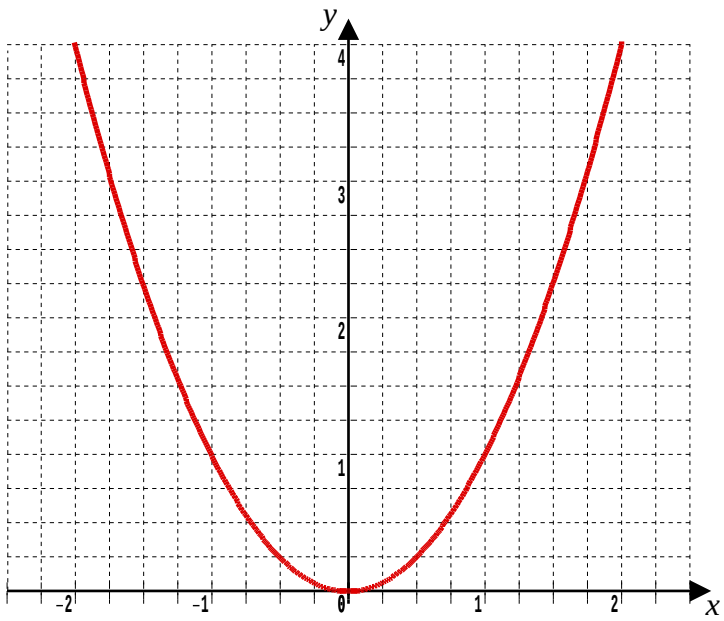
The turning points are thus, $(0,\ 5)$, $(-4,\ 37)$

[6 marks]

4.1 Point, Gradient and Bend

The graph shown below is of the function $g(x) = x^2$
Previously, interest would have focussed on the ***points*** on the curve.
In fact, the curve itself may have been plotted by working out ***points*** that are on the curve and joining them up, dot-to-dot fashion.

Clearly, $g(x) = x^2$, is best though of as being the ***points*** function.



The gradient of the left side of the curve is negative; it slopes downward as the eye starts to moves from left to right. There is then a turning point at (0, 0) where the gradient turns through zero before sloping upwards with positive gradient as the eye continues moving left to right.
Although the gradient is changing, it is not changing randomly. It changes according to the ***gradient*** function which is obtained by differentiation.

$$g(x) = x^2 \Rightarrow g'(x) = 2x$$

When $x = 2.5$, for example, the gradient is $2 \times 2.5 = 5$

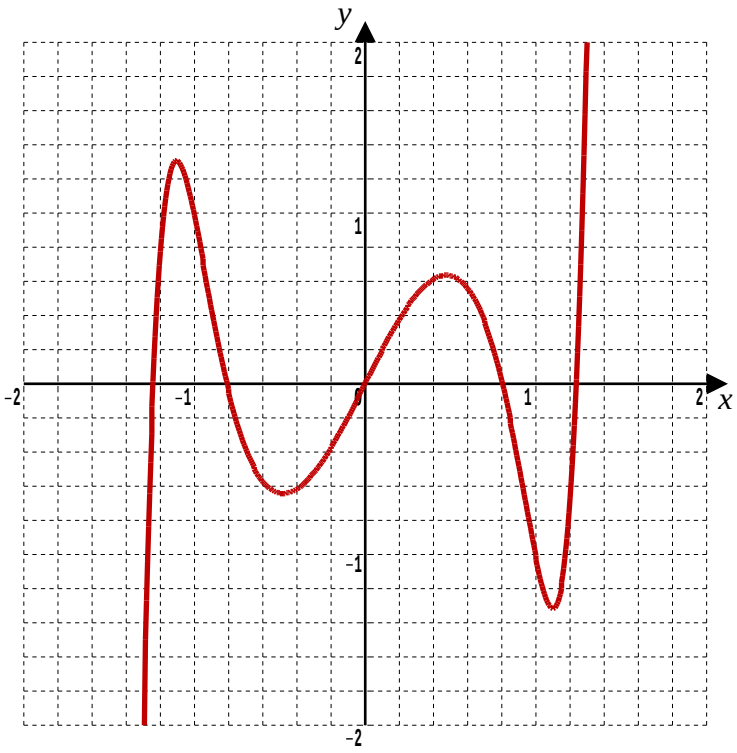
In other words, I can get the value of the gradient at a point $(x, g(x))$ on the curve by putting the x part of the coordinate into the ***gradient*** function, $g'(x)$.
Amazingly, by differentiating a second time, a ***bend*** function is obtained.
Putting a value of x into this determines if the curve is bending anticlockwise (positive answer) or clockwise (negative answer).

$$g(x) = x^2 \Rightarrow g'(x) = 2x \Rightarrow g''(x) = 2 \text{ (Positive)}$$

In this case the ***bend*** function is always 2 (Positive) which means the graph always has an anticlockwise curve to it, (moving left to right) as can be seen from the graph.

4.2 Example

The graph is of the ***points*** function, $k(x) = 2x^9 - 3x^7 + x^5 - 3x^3 + 2x$



Teaching Video : http://www.NumberWonder.co.uk/v9036/4a.mp4 (Part 1)
http://www.NumberWonder.co.uk/v9036/4b.mp4 (Part 2)

<= Part 1 Part 2 =>

The video will guide you through the following questions,

(i) Write down the ***gradient*** function, $k'(x)$

(ii) Write down the ***bend*** function, $k''(x)$

(iii) Use the appropriate function to find the ***point*** on the graph where $x = 1$

(iv) Use the appropriate function to find the ***gradient*** of the curve when $x = 1$

(v) Determine if the curve has anticlockwise or clockwise ***bend*** when $x = 1$

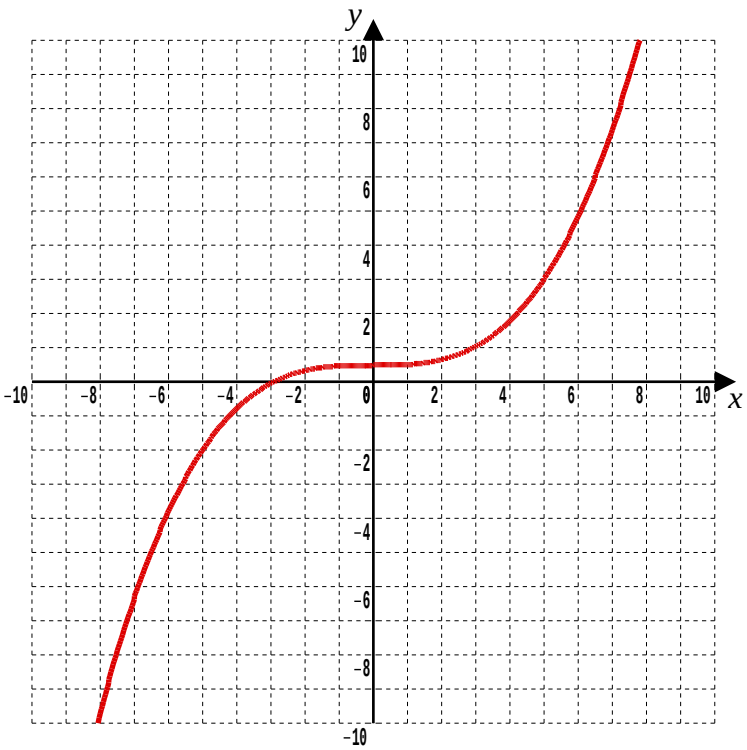
[10 marks]

4.3 Exercise

Marks Available : 100

Question 1

The graph is of the ***points*** function $p(x) = 0.02x^3 + 0.5$



(i) Write down the ***gradient*** function, $p'(x)$

[2 marks]

(ii) Write down the ***bend*** function, $p''(x)$

[2 marks]

(iii) Use the appropriate function to find the ***point*** on the graph where $x = 5$

[2 marks]

(iv) Use the appropriate function to find the ***gradient*** of the curve when $x = 5$

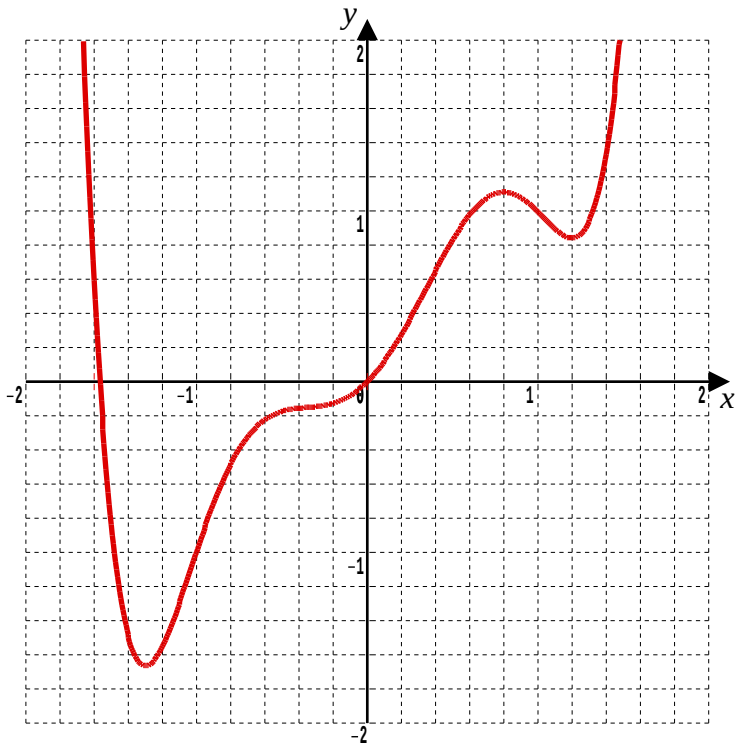
[2 marks]

(v) Determine if the curve has anticlockwise or clockwise ***bend*** when $x = 5$

[2 marks]

Question 2

The graph below is of the function $f(x) = x^6 - 3x^4 + 2x^2 + x$



This curve is to be analysed mathematically, with the graph provided so that the mathematical answers can be seen to be plausible.

Think of $f(x) = x^6 - 3x^4 + 2x^2 + x$ as the ***points*** function

(a) Work out

(i) $f(0)$ and indicate where the ***point*** $(0,\ f(0))$ is on the graph.

[2 marks]

(ii) $f(1)$ and indicate where the ***point*** $(1,\ f(1))$ is on the graph.

[2 marks]

(iii) $f(-1)$ and indicate where the ***point*** $(-1,\ f(-1))$ is on the graph.

[2 marks]

(b) Differentiate, $f(x) = x^6 - 3x^4 + 2x^2 + x$ to obtain the ***gradient*** function of the curve, $f'(x)$

[2 marks]

(c) Work out,

(i) $f'(0)$ and check that the ***gradient*** has the up, flat or down slope at the point $(0, f(0))$ on the graph that your calculation predicts.

[2 marks]

(ii) $f'(1)$ and check that the ***gradient*** has the up. flat or down slope at the point $(1, f(1))$ on the graph that your calculation predicts

[2 marks]

(iii) $f'(-1)$ and check that the ***gradient*** has the up, flat or down slope at the point $(-1, f(-1))$ on the graph that your calculation predicts

[2 marks]

(d) Differentiate, $f(x) = x^6 - 3x^4 + 2x^2 + x$ a second time to obtain the ***bend*** function of the curve, $f''(x)$

[2 marks]

(e) Work out,

(i) $f''(0)$ and check on the graph that the ***bend*** is curving at the point $(0,\ f(0))$ as your calculation predicts.

[2 marks]

(ii) $f''(1)$ and hence state if the graph is bending anticlockwise or clockwise at $(1,\ f(1))$ as it's hard to tell from the graph !

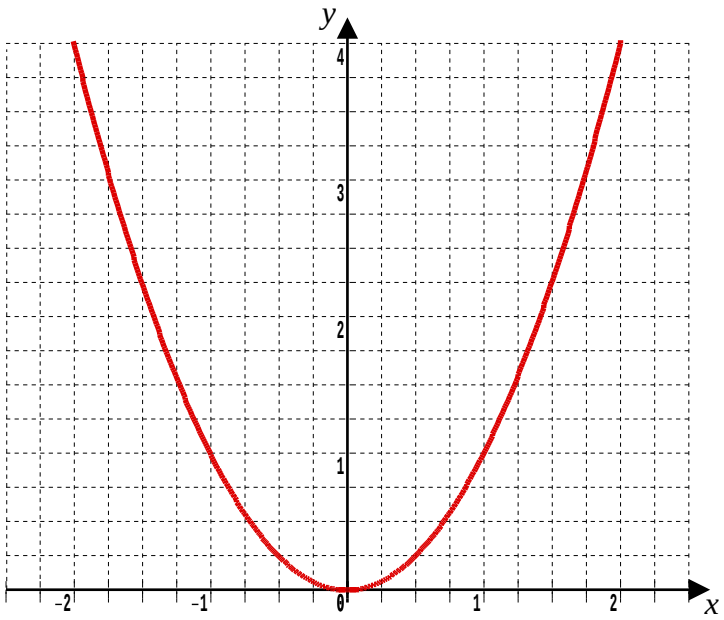
[2 marks]

(iii) $f''(-1)$ and hence state if the graph is bending anticlockwise or clockwise at $(-1,\ f(-1))$ as it's hard to tell from the graph !

[2 marks]

Question 3

The graph of the function $g(x) = x^2$ is given below (again),



(a) Work out

(i) $g(-2)$ and indicate where the point $(-2,\ g(-2))$ is on the graph

[2 marks]

(ii) $g\left(\frac{1}{2}\right)$ and indicate where $\left(\frac{1}{2},\ g\left(\frac{1}{2}\right)\right)$ is on the graph

[2 marks]

(iii) $g\left(\frac{3}{2}\right)$ and indicate where $\left(\frac{3}{2},\ g\left(\frac{3}{2}\right)\right)$ is on the graph

[2 marks]

(b) Differentiate, $g(x) = x^2$ to obtain the GRADIENT function of the curve $g'(x)$

[2 marks]

(c) Work out,

(i) $g'(-2)$ and check that the gradient has the up, flat or down slope at the point $(-2,\ g(-2))$ on the graph that your calculation predicts.

[2 marks]

(ii) $g'\left(\frac{1}{2}\right)$ and check that the gradient has the up, flat or down slope at the point $\left(\frac{1}{2},\ g\left(\frac{1}{2}\right)\right)$ on the graph that your calculation predicts.

[2 marks]

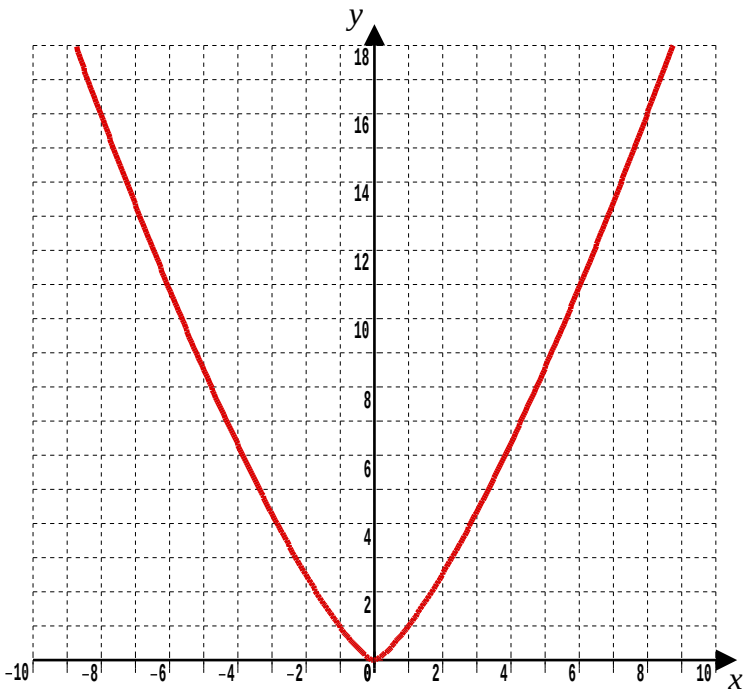
(iii) $g'\left(\frac{3}{2}\right)$ and check that the gradient has the up, flat or down slope at the point $\left(\frac{3}{2},\ g\left(\frac{3}{2}\right)\right)$ on the graph that your calculation predicts.

[2 marks]

Question 4

The graph shown below is of the function,

$$h(x) = x^{\frac{4}{3}}$$



(a) Work out

(i) $h(0)$ and indicate where the point $(0, h(0))$ is on the graph.

[2 marks]

(ii) $h(1)$ and indicate where the point $(1, h(1))$ is on the graph.

[2 marks]

(iii) $h(8)$ and indicate where the point $(8, h(8))$ is on the graph.

[2 marks]

(b) Differentiate, $h(x) = x^{\frac{4}{3}}$ to obtain the GRADIENT function of the curve, $h'(x)$

[2 marks]

(c) Work out,

(i) $h'(0)$ and check that the gradient has the down, flat or up slope at the point $(0, h(0))$ on the graph that your calculation predicts.

[2 marks]

(ii) $h'(1)$ and check that the gradient has the down, flat or up slope at the point $(1, h(1))$ on the graph that your calculation predicts.

[2 marks]

(iii) $h'(8)$ and check that the gradient has the down, flat or up slope at the point $(8, h(8))$ on the graph that your calculation predicts.

[2 marks]

(d) Differentiate, $h(x) = x^{\frac{4}{3}}$ a second time to obtain the BEND function of the curve, $h''(x)$

[2 marks]

(e) **(i)** Show that $h''(0) = 0$. This result is a #FAIL of the method. It does not tell you what the bend is doing at $(0, h(0))$.

[2 marks]

(ii) Work out $h''(1)$ and check on the graph that the bend is curving at the point $(1, h(1))$ as your calculations predict

[2 marks]

(iii) Work out $h''(8)$ and check on the graph that the bend is curving at the point $(8, h(8))$ as your calculations predict

[2 marks]

(f) By looking at the graph, or otherwise, try to find the value of x at which this curve has a 45° slope. i.e. Where is the gradient equal to 1 ?

[3 marks]

Question 5

Differentiate the following;

(i) $y = 7x^3 + 4x^2$

(ii) $y = 13x$

(iii) $y = 2.5x^{10}$

(iv) $y = 19$

(v) $y = \frac{5}{2}x^4$

(vi) $y = \frac{7}{3}x^9$

(vii) $y = 1.5x^3$

(viii) $y = 3.2x$

(ix) $y = 5x^4 + 2x^3 + 7$

(x) $y = 4x^5 + 7x + 1.2$

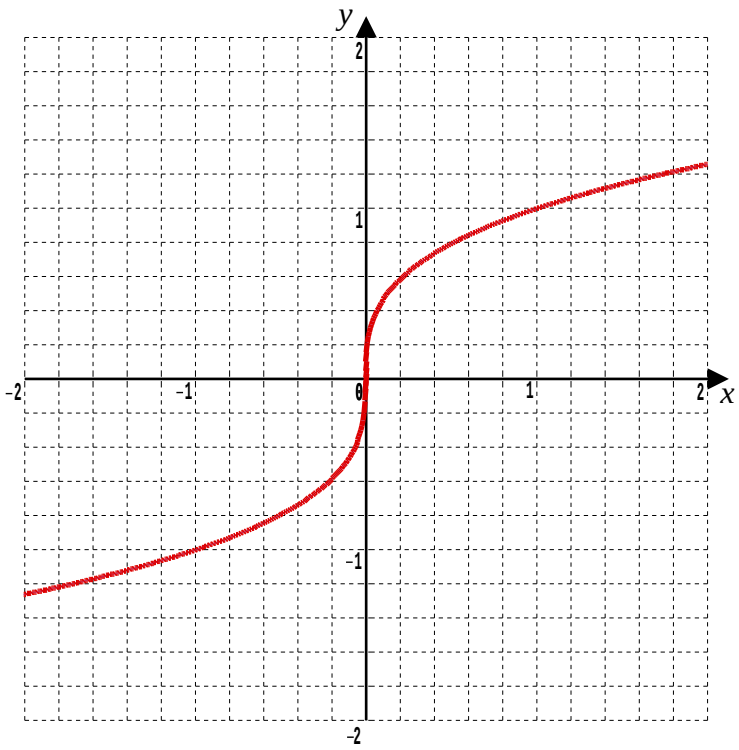
(xi) $y = (4x + 3)(4x + 3)$

(xii) $y = (3x + 5)^2$

[12 marks]

Question 6 * Hard Question

The graph is of the function $m(x) = x^{\frac{1}{3}}$



(a) Work out,

(i) $m\left(\frac{1}{8}\right)$ and indicate where the point $\left(\frac{1}{8},\ m\left(\frac{1}{8}\right)\right)$ is on the graph.

[2 marks]

(ii) $m(1)$ and indicate where the point $(1,\ m(1))$ is on the graph

[2 marks]

(iii) $m\left(\frac{216}{125}\right)$ and indicate where the point $\left(\frac{216}{125},\ m\left(\frac{216}{125}\right)\right)$ is.

[2 marks]

(b) Differentiate, $m(x) = x^{\frac{1}{3}}$ to obtain the gradient function, $m'(x)$

[2 marks]

(c) Work out,

(i) $m'\left(\frac{1}{8}\right)$ and check that the gradient has the down, flat or up slope at the point $\left(\frac{1}{8},\ m\left(\frac{1}{8}\right)\right)$ on the graph that your calculation predicts.

[2 marks]

(ii) $m'(1)$ and check that the gradient has the down, flat or up slope at the point $(1,\ m(1))$ on the graph that your calculation predicts.

[2 marks]

(iii) $m'\left(\frac{216}{125}\right)$ and check that the gradient has the down, flat or up slope at the point $\left(\frac{216}{125},\ m\left(\frac{216}{125}\right)\right)$ that your calculation predicts.

[2 marks]

(d) By looking at the graph, or otherwise, try to find the value of x at which this curve has a 45° slope. i.e. Where is the gradient equal to 1 ?

[3 marks]

4.4 Answers to 4.2 Exercise

4.4.1 Solution to 4.2 Example (Teaching Video)

$$k(x) = 2x^9 - 3x^7 + x^5 - 3x^3 + 2x$$

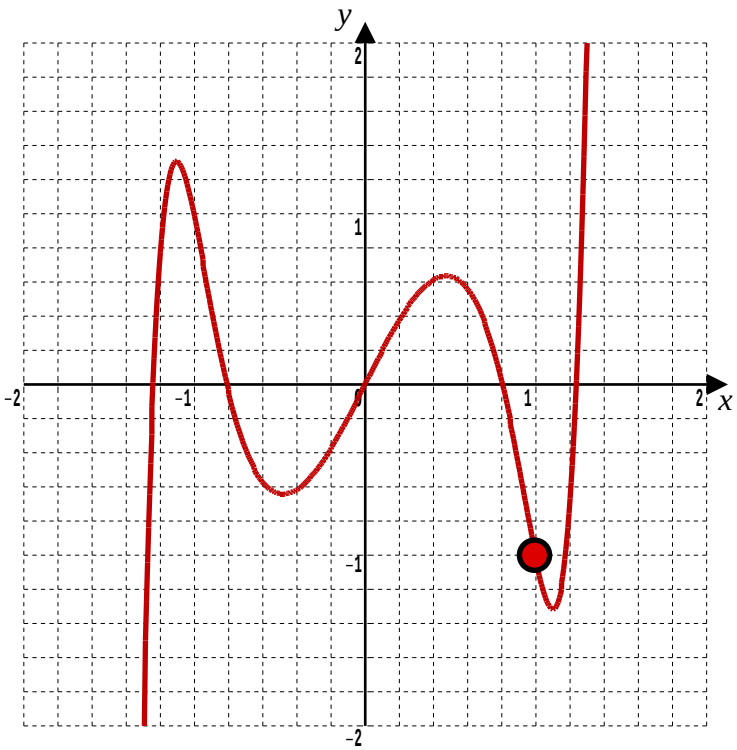
(i) $k'(x) = 18x^8 - 21x^6 + 5x^4 - 9x^2 + 2$

(ii) $k''(x) = 144x^7 - 126x^5 + 20x^3 - 18x$

(iii) $k(1) = 2 - 3 + 1 - 3 + 2$

$= -1$

Thus $(1,\ k(1)) = (1, -1)$



(iv) $k'(1) = 18 - 21 + 5 - 9 + 2$

$= -5$ $\therefore$ Sloping downwards at $(1, -1)$

(v) $k''(1) = 144 - 126 + 20 - 18$

$= 20$ As this is positive curve is bending anticlockwise at $(1, -1)$

[10 marks]

4.4.2 Solutions to 4.3 Exercise

Answer 1

$p(x) = 0.02x^3 + 0.5$

(i) $p'(x) = 0.06x^2$

[2 marks]

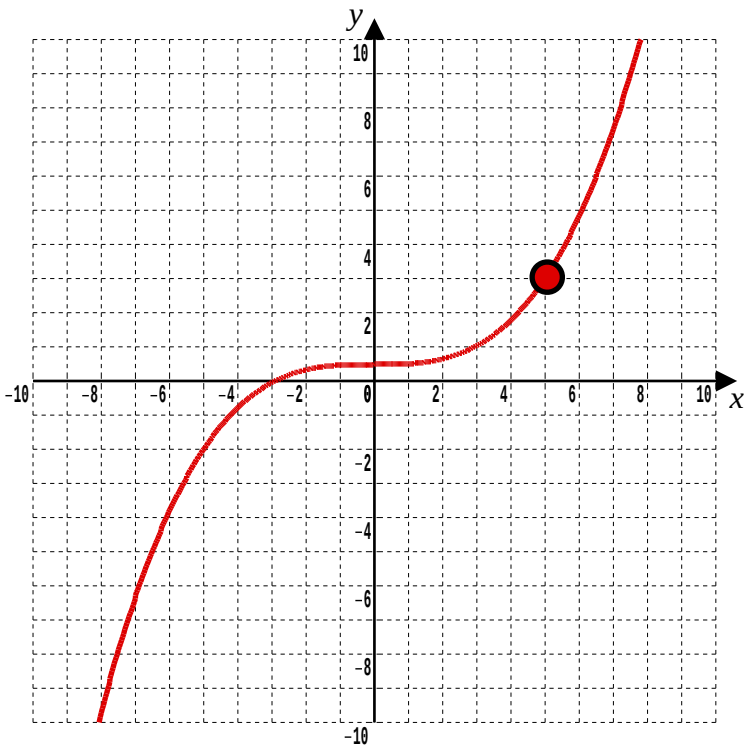
(ii) $p''(x) = 0.12x$

[2 marks]

(iii) $p(5) = 3$

Thus $(5,\ p(5)) = (5,\ 3)$

[2 marks]



(iv) $p'(5) = 1.5$ $\therefore$ Sloping upwards at $(5,\ 3)$

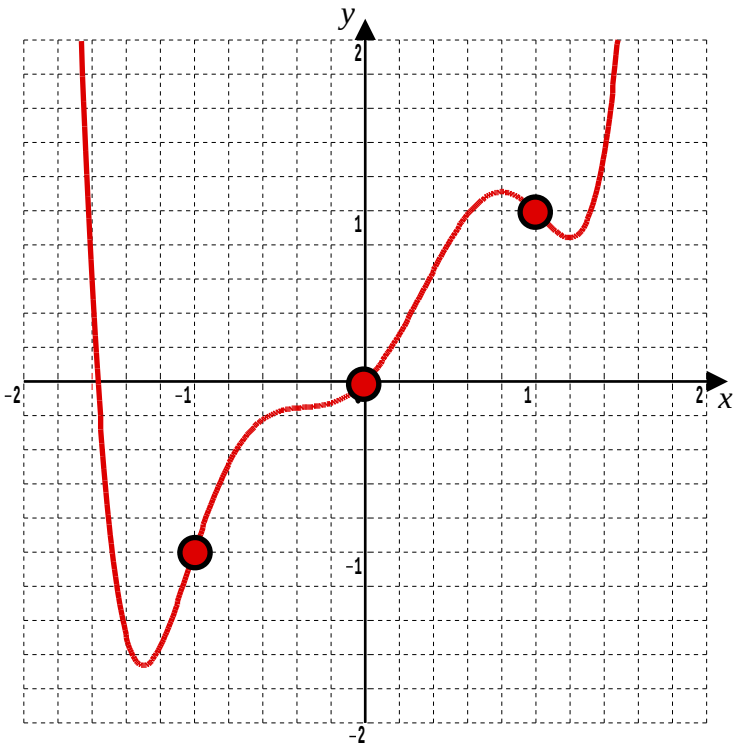
[2 marks]

(v) $p''(5) = 0.6$ As this is positive curve is bending anticlockwise at $(5,\ 3)$

[2 marks]

Answer 2

(a) **(i)** (0, 0) **(ii)** (1, 1) **(iii)** $(-1, -1)$



[6 marks]

(b) $f'(x) = 6x^5 - 12x^3 + 4x + 1$

[2 marks]

(c) **(i)** $f'(0) = 1$ Yes, graph slopes upwards at (0, 0)

(ii) $f'(1) = -1$ Yes, graph slopes downward at (1, 1)

(iii) $f'(-1) = 3$ Yes, graph slopes upward at $(-1, -1)$

[6 marks]

(d) $f''(x) = 30x^4 - 36x^2 + 4$

[2 marks]

(e) **(i)** $f''(0) = 4$ Yes, graph bending anticlockwise at (0, 0)

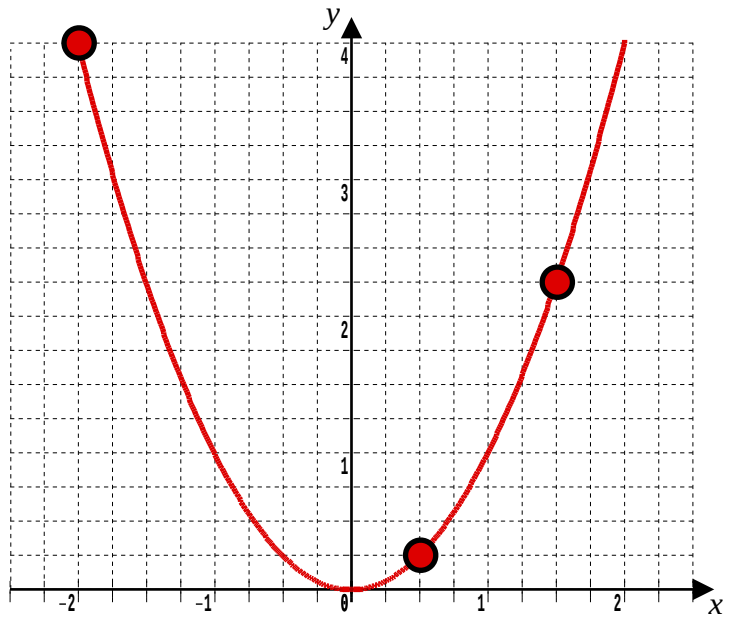
(ii) $f''(1) = -2$ $\therefore$ Graph bending clockwise at (1, 1)

(iii) $f''(-1) = -2$ $\therefore$ Graph bending clockwise at $(-1, -1)$

[6 marks]

Answer 3

(a) **(i)** $(-2,\ 4)$ **(ii)** $\left(\frac{1}{2},\ \frac{1}{4}\right)$ **(iii)** $\left(\frac{3}{2},\ \frac{9}{4}\right)$



[6 marks]

(b) $g'(x) = 2x$

[2 marks]

(c) **(i)** $g'(-2) = 4$

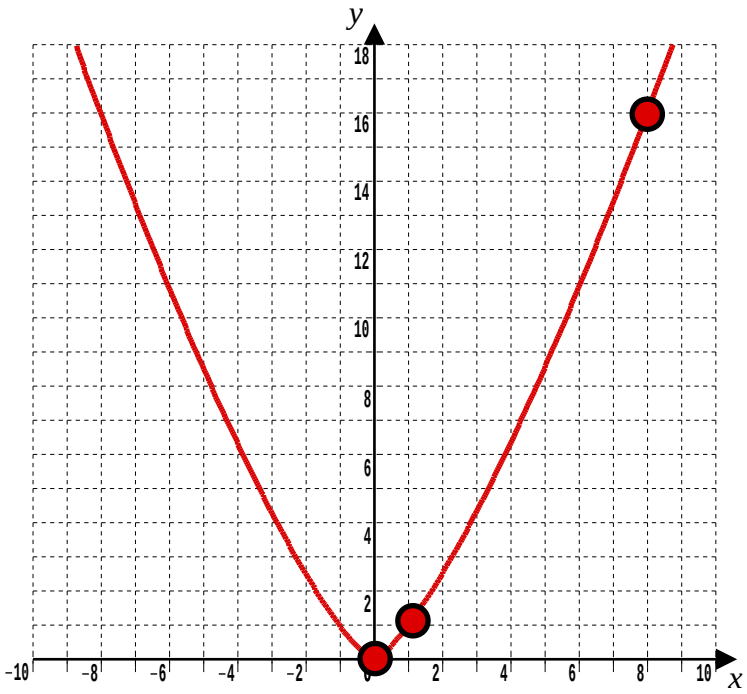
(ii) $g'\left(\frac{1}{2}\right) = 1$

(iii) $g'\left(\frac{3}{2}\right) = 3$

[6 marks]

Answer 4

(a) **(i)** (0, 0) **(ii)** (1, 1) **(iii)** (8, 16)



[6 marks]

(b) $h'(x) = \frac{4}{3}x^{\frac{1}{3}}$

[2 marks]

(c) **(i)** $h'(0) = 0$ Yes, graph flat at (0, 0)

(ii) $h'(1) = \frac{4}{3}$ Yes, graph sloping upwards at (1, 1)

(iii) $h'(8) = \frac{8}{3}$ Yes, graph sloping upwards at (8, 16)

[6 marks]

(d) $h''(x) = \frac{4}{9}x^{-\frac{2}{3}}$

[2 marks]

(e) **(i)** $h''(0) = \frac{4}{9} \times 0^{-\frac{2}{3}} = 0$ □

(ii) $h''(1) = \frac{4}{9}$ Yes, graph bending anticlockwise at (1, 1)

(iii) $h''(8) = \frac{8}{3}$ Yes, graph bending anticlockwise at (8, 16)

[6 marks]

(f) Require $\frac{4}{3}x^{\frac{1}{3}} = 0 \Rightarrow x^{\frac{1}{3}} = \frac{3}{4} \Rightarrow x = \frac{27}{64} \Rightarrow x = 0.421875$

[3 marks]

Answer 5

(i) $\frac{dy}{dx} = 21x^2 + 8x$

(ii) $\frac{dy}{dx} = 13$

(iii) $\frac{dy}{dx} = 25x^9$

(iv) $\frac{dy}{dx} = 0$

(v) $\frac{dy}{dx} = 10x^3$

(vi) $\frac{dy}{dx} = 21x^8$

(vii) $\frac{dy}{dx} = 4.5x^2$

(viii) $\frac{dy}{dx} = 3.2$

(ix) $y = 5x^4 + 2x^3 + 7$

$\frac{dy}{dx} = 20x^3 + 6x^2$

(x) $y = 4x^5 + 7x + 1.2$

$\frac{dy}{dx} = 20x^4 + 7$

(xi) $y = 16x^2 + 24x + 9$

$\frac{dy}{dx} = 32x + 24$

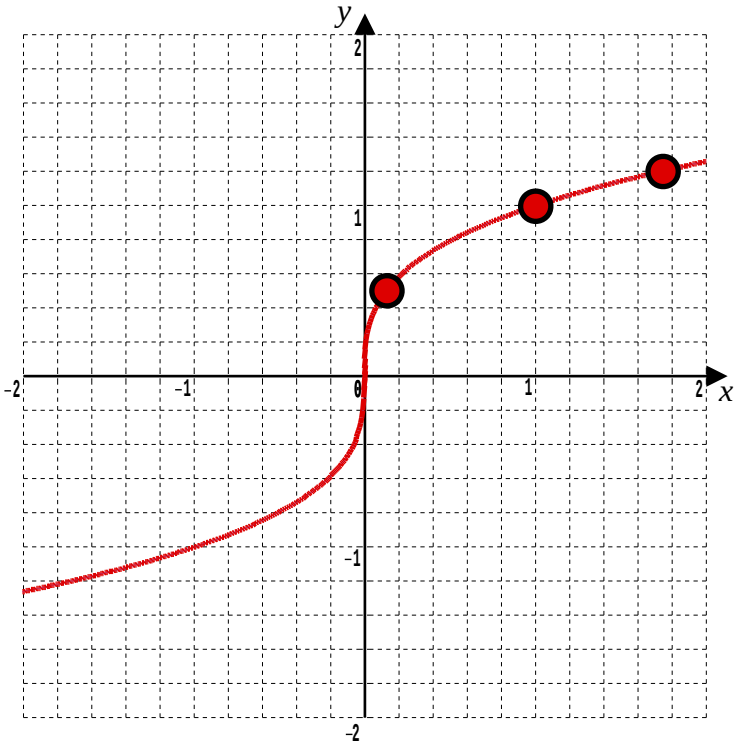
(xii) $y = 9x^2 + 30x + 25$

$\frac{dy}{dx} = 18x + 30$

[12 marks]

Answer 6

(a) **(i)** $\left(\frac{1}{8},\ \frac{1}{2}\right)$ **(ii)** $(1,\ 1)$ **(iii)** $\left(\frac{216}{125},\ \frac{6}{5}\right)$



[6 marks]

(b) $m'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3x^{\frac{2}{3}}}$

[2 marks]

(c) **(i)** $m'\left(\frac{1}{8}\right) = \frac{1}{3} \times 8^{\frac{2}{3}} = \frac{4}{3}$ Yes, sloping up at $\left(\frac{1}{8},\ \frac{1}{2}\right)$

(ii) $m'(1) = \frac{1}{3}$ Yes, sloping up at (1, 1)

(iii) $m'\left(\frac{216}{125}\right) = \frac{1}{3} \times \left(\frac{125}{216}\right)^{\frac{2}{3}} = \frac{25}{108}$ which is about 0.231

Yes, sloping up at $\left(\frac{216}{125},\ \frac{6}{5}\right)$

[6 marks]

(d) $\frac{1}{3}x^{-\frac{2}{3}} = 1 \Leftrightarrow \frac{1}{x^{-\frac{2}{3}}} = 3 \Leftrightarrow x^{\frac{2}{3}} = \frac{1}{3} \Leftrightarrow x = \left(\frac{1}{3}\right)^{\frac{3}{2}}$

Which is about 0.19245...

[3 marks]

Lesson 5

GCSE Mathematics
Differentiation I

5.1 Homework (Consolidation)

Marks Available : 76

Question 1
Write down the exact value of each of the following:

(i) 8^2 **(ii)** $(-6)^3$ **(iii)** $\left(\frac{1}{2}\right)^5$

(iv) $100^{\frac{1}{2}}$ **(v)** $8^{\frac{1}{3}}$ **(vi)** $(-1)^{97}$

(vii) $\left(\frac{\pi}{2}\right)^0$ **(viii)** 0^{67} **(ix)** $\left(\frac{5}{9}\right)^2$

[9 marks]

Question 2
Consider the curve, $y = x^3 - x$
Write down the points on the curve that have the x part as given;

(i) (0, _____) **(ii)** (1, _____) **(iii)** (2, _____)

(iv) (4, _____) **(v)** (10, _____) **(vi)** (−10, _____)

[6 marks]

Question 3
Write down the exact value of the following:

(i) 3^{-2} **(ii)** $\sqrt{\left(\frac{16}{49}\right)}$

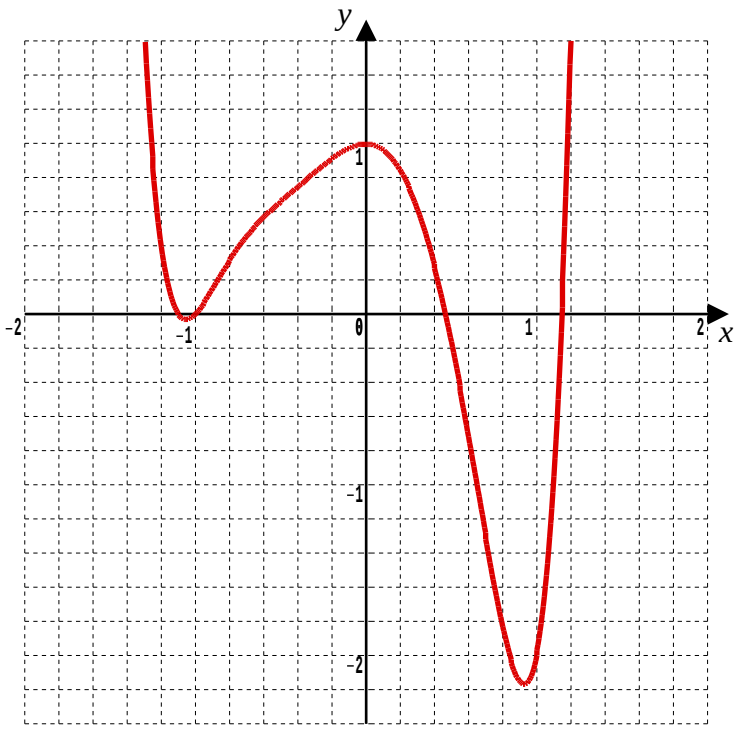
(iii) $\sqrt{0.16}$ **(iv)** $(-1)^{1001}$

[4 marks]

Question 4

The graph is of the function

$$w(x) = x^8 + 3x^5 - 4x^3 - 3x^2 + 1$$



(i) Write down the gradient function, $w'(x)$

[2 marks]

(ii) Write down the bend detector function, $w''(x)$

[2 marks]

(iii) Use the appropriate function to find the point on this curve where $x = 1$

[2 marks]

(iv) Use the appropriate function to find the gradient of this curve when $x = 1$

[2 marks]

(v) Determine if the curve is bending anticlockwise or clockwise when $x = 1$

[2 marks]

Question 5

Differentiate the following taking care to write "$y =$" or "$\frac{dy}{dx} =$" as appropriate;

(i) $y = 24x^5$ **(ii)** $y = 4x^{-3}$ **(iii)** $y = 8x + 3$

[1, 1, 1 marks]

(iv) $y = (2x + 3)^2$ Hint : Begin by expanding the brackets

[2 marks]

(v) $y = \sqrt{x}$ Hint : Begin by writing it in the form $y = x^n$

[2 marks]

(vi) $y = \frac{1}{x^4}$ Hint : Begin by writing it in the form $y = x^n$

[2 marks]

Question 6

Write down the exact value of the following:

(i) $\left(\frac{11}{5}\right)^{-2}$ **(ii)** 0.04^3

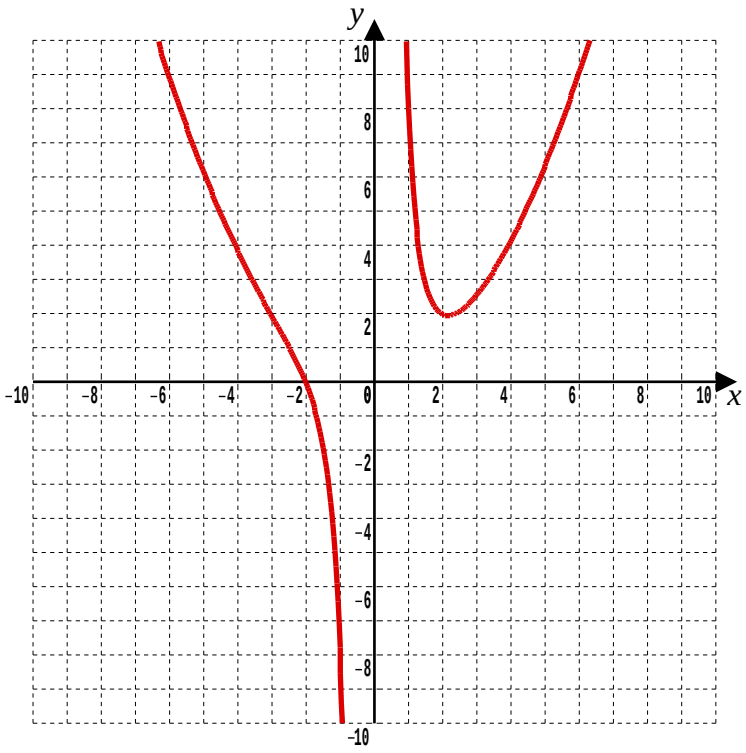
[2, 2 marks]

(iii) $\left(1 + \frac{9}{16}\right)^{\frac{1}{2}}$ **(iv)** $\left(-\frac{3}{4}\right)^{-3}$

[2, 2 marks]

Question 7

The graph is of the function $s(x) = \frac{8}{x^3} + \frac{x^2}{4}$



(i) Write down the gradient function, $s'(x)$

[3 marks]

(ii) Write down the bend detector function, $s''(x)$

[3 marks]

(iii) Use the appropriate function to find the point on this curve where $x = 2$

[2 marks]

(iv) Use the appropriate function to find the gradient of this curve when $x = 2$

[2 marks]

(v) Determine if the curve is bending anticlockwise or clockwise when $x = 2$

[2 marks]

Question 8

Differentiate the following taking care to write "$y =$" or "$\frac{dy}{dx} =$" as appropriate;

(i) $y = 9x^4 - 8x^{-2}$

(ii) $y = 22x^4 + \frac{12}{x^4}$

(iii) $y = x^8\left(4x^3 + 7x^2\right)$

(iv) $y = \frac{1}{5x^2}$

(v) $y = \frac{7x^3}{11}$

(vi) $y = \frac{x^9 + 6x^5}{2x^3}$

[18 marks]

5.2 Answers

GCSE Mathematics
Differentiation I

Answer 1

(i) 64 **(ii)** -216 **(iii)** $\frac{1}{32}$

(iv) 10 **(v)** 2 **(vi)** -1

(vii) 1 **(viii)** 0 **(ix)** $\frac{25}{81}$

[9 marks]

Answer 2

(i) $(0,\ 0)$ **(ii)** $(1,\ 0)$ **(iii)** $(2,\ 6)$

(iv) $(4,\ 60)$ **(v)** $(10,\ 990)$ **(vi)** $(-10, -990)$

[6 marks]

Answer 3

(i) $\frac{1}{9}$ **(ii)** $\frac{4}{7}$

(iii) $\frac{4}{10} = \frac{2}{5}$ **(iv)** -1

[4 marks]

Answer 4

$w(x) = x^8 + 3x^5 - 4x^3 - 3x^2 + 1$

(i) $w'(x) = 8x^7 + 15x^4 - 12x^2 - 6x$

[2 marks]

(ii) $w''(x) = 56x^6 + 60x^3 - 24x - 6$

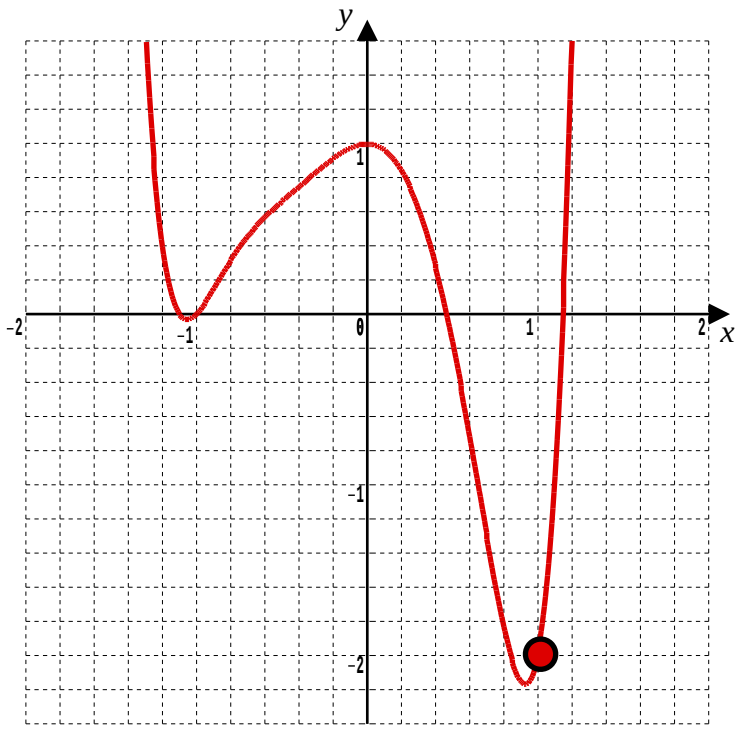
[2 marks]

(iii) $w(1) = 1 + 3 - 4 - 3 + 1$

$= -2$

Thus $(1,\ w(1)) = (1, -2)$

[2 marks]



(iv) $w'(1) = 8 + 15 - 12 - 6$

$= 5$ $\therefore$ Sloping upwards at $(1, -2)$

[2 marks]

(v) $w''(1) = 56 + 60 - 24 - 6$

$= 86$

As this is positive the curve is bending anticlockwise at $(1, -2)$

[2 marks]

Answer 5

(i) $y = 24x^5$ $\therefore$ $\frac{dy}{dx} = 120x^4$

[1 mark]

(ii) $y = 4x^{-3}$ $\therefore$ $\frac{dy}{dx} = -12x^{-4}$

which means $\frac{dy}{dx} = -\frac{12}{x^4}$

[1 mark]

(iii) $y = 8x + 3$ $\therefore$ $\frac{dy}{dx} = 8$

[1 mark]

(iv) $y = (2x + 3)^2$

$y = 4x^2 + 12x + 9$ $\therefore$ $\frac{dy}{dx} = 8x + 12$

[2 marks]

(v) $y = \sqrt{x}$

$y = x^{\frac{1}{2}}$ $\therefore$ $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

which means $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

[2 marks]

(vi) $y = \frac{1}{x^4}$

$y = x^{-4}$ $\therefore$ $\frac{dy}{dx} = -4x^{-5}$

which means $\frac{dy}{dx} = -\frac{4}{x^5}$

[2 marks]

Answer 6

(i) $\frac{25}{121}$ **(ii)** $\frac{64}{1000000}$ or 0.000064

[2, 2 marks]

(iii) $\frac{5}{4}$ **(iv)** $-\frac{64}{27}$

[2, 2 marks]

Answer 7

$$s(x) = \frac{8}{x^3} + \frac{x^2}{4} = 8x^{-3} + \frac{x^2}{4}$$

(i) $s'(x) = -24x^{-4} + \frac{x}{2}$

[3 marks]

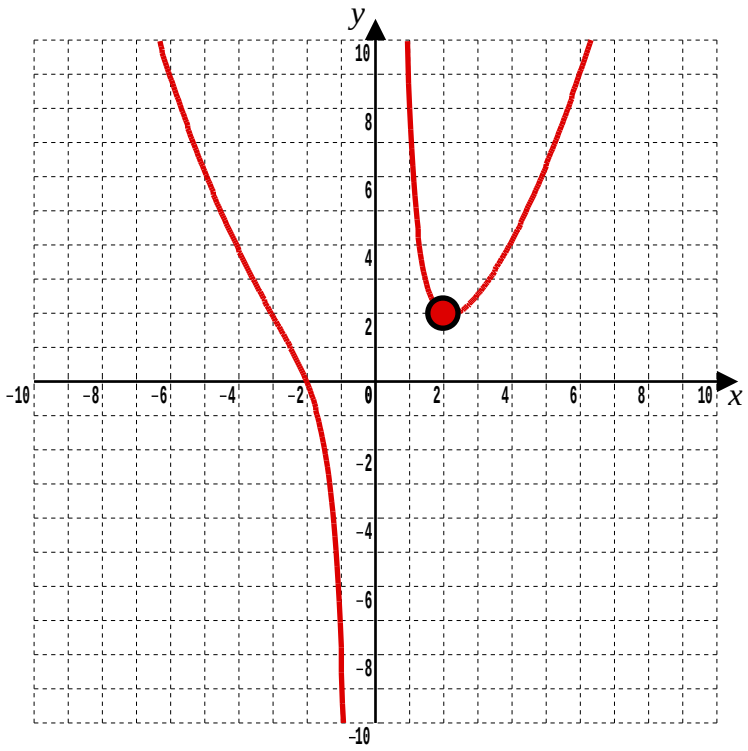
(ii) $s''(x) = 96x^{-5} + \frac{1}{2}$

[3 marks]

(iii) $s(2) = \frac{8}{2^3} + \frac{2^2}{4}$

$= 2$ Thus $(2,\ w(2)) = (2,\ 2)$

[2 marks]



(iv) $s'(2) = -\frac{24}{2^4} + \frac{2}{2}$

$= -\frac{1}{2}$ $\therefore$ Sloping downwards at $(2,\ 2)$

[2 marks]

(v) $s''(1) = \frac{96}{2^5} + \frac{1}{2}$

$= 3.5$

As this is positive the curve is bending anticlockwise at $(2,\ 2)$

[2 marks]

Answer 8

(i) $y = 9x^4 - 8x^{-2}$ $\therefore$ $\dfrac{dy}{dx} = 36x^3 + 16x^{-3}$

which means $\dfrac{dy}{dx} = 36x^3 + \dfrac{16}{x^3}$

[3 marks]

(ii) $y = 22x^4 + \dfrac{12}{x^4}$

$y = 22x^4 + 12x^{-4}$ $\therefore$ $\dfrac{dy}{dx} = 88x^3 - 48x^{-5}$

which means $\dfrac{dy}{dx} = 88x^3 - \dfrac{48}{x^5}$

[3 marks]

(iii) $y = x^8\left(4x^3 + 7x^2\right)$

$y = 4x^{11} + 7x^{10}$ $\therefore$ $\dfrac{dy}{dx} = 44x^{10} + 70x^9$

[3 marks]

(iv) $y = \dfrac{1}{5x^2}$

$y = \dfrac{1}{5}x^{-2}$ $\therefore$ $\dfrac{dy}{dx} = -\dfrac{2}{5}x^{-3}$

which means $\dfrac{dy}{dx} = -\dfrac{2}{5x^3}$

[3 marks]

(v) $y = \dfrac{7x^3}{11}$ $\therefore$ $\dfrac{dy}{dx} = \dfrac{21x^2}{11}$

[3 marks]

(vi) $y = \dfrac{x^9 + 6x^5}{2x^3}$

$y = \dfrac{x^9}{2x^3} + \dfrac{6x^5}{2x^3}$

$y = \dfrac{x^6}{2} + 3x^2$ $\therefore$ $\dfrac{dy}{dx} = 3x^5 + 6x$

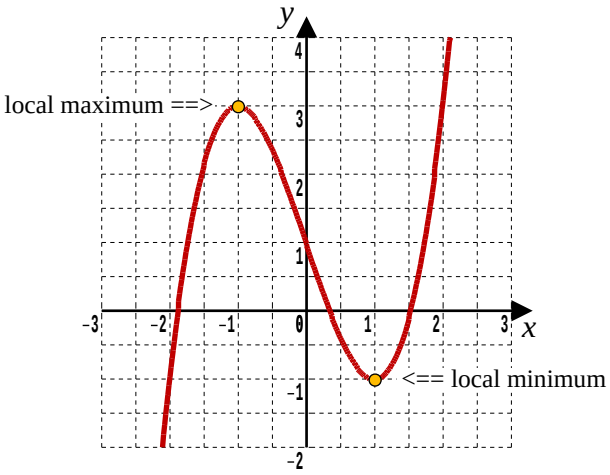
[3 marks]

Lesson 6

GCSE Mathematics
Differentiation I

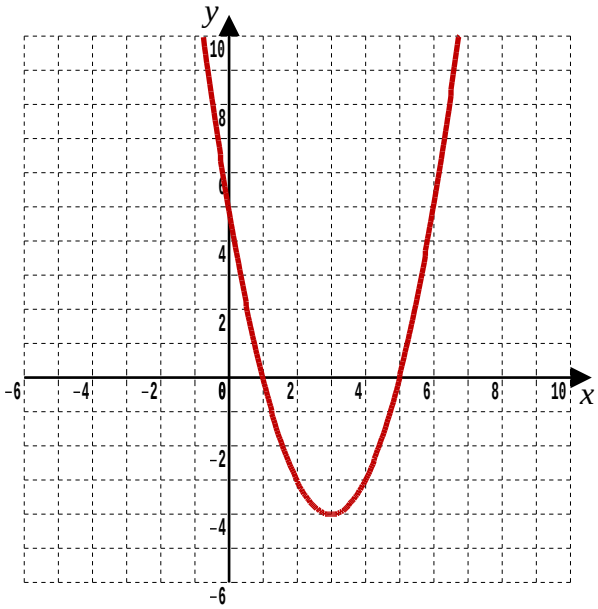
6.1 Local Minimum & Local Maximum

Differentiation is used to find the optimal solutions to problems.
On a graph, such 'best' solutions are often found where there is either a **local maximum** or a **local minimum**.



6.2 The Graphical Method

The graph below is of the equation $y = x^2 - 6x + 5$



By looking at the graph, write down the integer coordinates of the local minimum.

☞

[1 mark]

As mathematicians, we don't want to have to go to the bother of plotting the graph to find this important point.

6.3 The Mathematical Method

Teaching Video : http://www.NumberWonder.co.uk/v9036/6.mp4

The teaching video will talk you through the following method of finding all local minima and local maxima of a function.

Finding Local Minima and Local Maxima

STEP 1 : Differentiate the **POINTS** equation to get its **GRADIENT** equation.
STEP 2 : Set the **GRADIENT** equation equal to zero and solve.
STEP 3 : Put the solution(s) from STEP 2 back into the **POINTS** equation.

Example

Find all local maxima and minima (if any) on the curve with equation;

$$y = x^2 - 6x + 5$$

[3 marks]

6.4 Exercise

Marks Available : 50

Question 1

Find the coordinates of the **local minimum** point on the following quadratic curve;

$$y = x^2 - 8x + 9$$

[3 marks]

Question 2

Find the coordinates of the **local maximum** point on the following parabola,

$$y = 6x + 14 - x^2$$

[3 marks]

Question 3

Find the coordinates of the **local minimum** point on the following parabola,

$$y = 2x^2 - 20x + 52$$

[3 marks]

Question 4

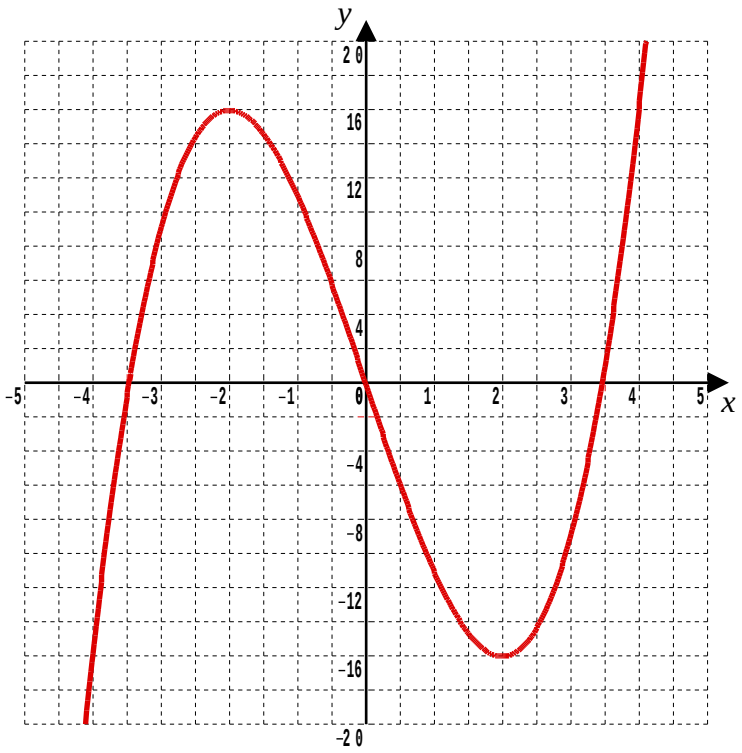
Find the coordinates of the **local maximum** point on the following parabola

$$y = 12x - 7 - 3x^2$$

[3 marks]

Question 5

Consider the equation, $y = x^3 - 12x$



(a) From looking at the curve,

(i) write down the coordinates of the **local maximum** point.

[1 mark]

(ii) write down the coordinates of the **local minimum** point.

[1 mark]

(**b)** Use the mathematical method to obtain the same answers.

[4 marks]

Question 6

Find the coordinates of any **local minimum** or **local maximum** point on;

(i) $y = x^3 - 27x$

[4 marks]

(ii) $y = (x + 7)(x + 1)$

[4 marks]

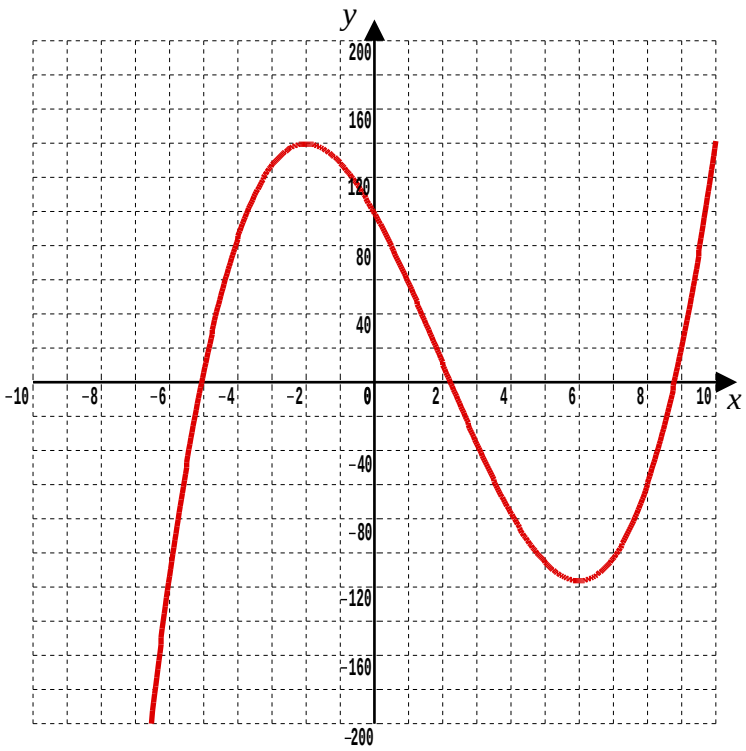
(iii) $y = x^4 - 256x$

[4 marks]

Question 7

Use mathematics to find the local minimum and local maximum of the curve,

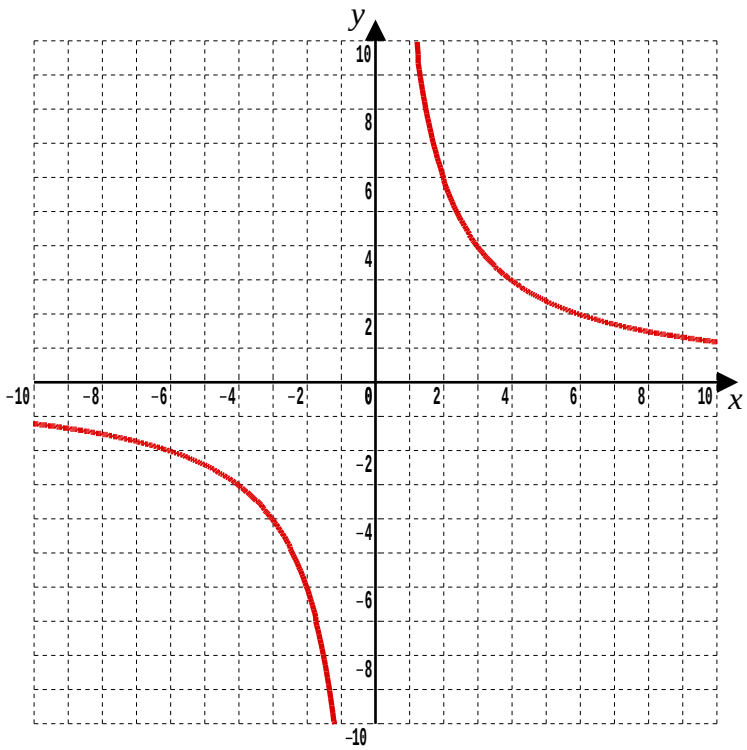
$$y = x^3 - 6x^2 - 36x + 100$$



[6 marks]

Question 8

The graph is of the "inverse proportion" function $f(x) = \frac{12}{x}$



(i) Write down the gradient function, $f'(x)$

[2 marks]

(ii) Write down the bend detector function, $f''(x)$

[2 marks]

(iii) Use the appropriate function to find the point on this curve where $x = 2$

[2 marks]

(iv) Use the appropriate function to find the gradient of this curve when $x = 2$

[2 marks]

(v) Determine if the curve is bending anticlockwise or clockwise when $x = 2$

[2 marks]

Question 9

The curve $y = x^3 + 12x$ has no turning points

Show that this is the case by trying to find them via the mathematical method.

What goes “wrong” ?

[4 marks]

6.5 Answers

6.5.1 Solution to 6.3 Example (Teaching Video)

STEP 1 $y = x^2 - 6x + 5$

$$\frac{dy}{dx} = 2x - 6$$

STEP 2 $2x - 6 = 0$

$x = 3$

STEP 3 $y = 3^2 - 6 \times 3 + 5$

$y = -4$

$\therefore$ Minimum at $(3, -4)$ as the coefficient of x^2 in parabola's equation is positive

[3 marks]

6.5.2 Solution to 6.4 Exercise

Answer 1

STEP 1 $y = x^2 - 8x + 9$

$$\frac{dy}{dx} = 2x - 8$$

STEP 2 $2x - 8 = 0$

$x = 4$

STEP 3 $y = 4^2 - 8 \times 4 + 9$

$y = -7$

$\therefore$ Minimum at $(4, -7)$ as the coefficient of x^2 in parabola's equation is positive

[3 marks]

Answer 2

STEP 1 $y = 6x + 14 - x^2$

$$\frac{dy}{dx} = 6 - 2x$$

STEP 2 $6 - 2x = 0$

$x = 3$

STEP 3 $y = 6 \times 3 + 14 - 3^2$

$y = 23$

$\therefore$ Maximum at $(3, 23)$ as the coefficient of x^2 in parabola's equation is negative

[3 marks]

Answer 3

STEP 1 $y = 2x^2 - 20x + 52$

$$\frac{dy}{dx} = 4x - 20$$

STEP 2 $4x - 20 = 0$

$x = 5$

STEP 3 $y = 2 \times 5^2 - 20 \times 5 + 52$

$y = 2$

$\therefore$ Minimum at $(5,\ 2)$ as the coefficient of x^2 in parabola's equation is positive

[3 marks]

Answer 4

STEP 1 $y = 12x - 7 - 3x^2$

$$\frac{dy}{dx} = 12 - 6x$$

STEP 2 $12 - 6x = 0$

$x = 2$

STEP 3 $y = 12 \times 2 - 7 - 3 \times 2^2$

$y = 5$

$\therefore$ Maximum at $(2,\ 5)$ as the coefficient of x^2 in parabola's equation is negative

[3 marks]

Answer 5

(a) **(i)** $(-2,\ 16)$ **(ii)** $(2, -16)$

[1, 1 mark]

(b)

STEP 1 $y = x^3 - 12x$

$$\frac{dy}{dx} = 3x^2 - 12$$

STEP 2 $3x^2 - 12 = 0$

$x^2 = 4$

$x = \pm 2$

STEP 3 $y = (-2)^3 - 12 \times (-2)$ $\quad y = 2^3 - 12 \times 2$

$y = 16$ $\quad y = -16$

$\therefore$ Maximum at $(-2,\ 16)$ and a minimum $(2, -16)$

as coefficient of x^3 in cubic's equation is positive

[4 marks]

Answer 6

(i) STEP 1 $y = x^3 - 27x$

$$\frac{dy}{dx} = 3x^2 - 27$$

STEP 2 $3x^2 - 27 = 0$

$x^2 = 9$

$x = \pm 3$

STEP 3 $y = (-3)^3 - 27 \times (-3)$ $\quad y = 3^3 - 27 \times 3$

$y = 54$ $\quad y = -54$

$\therefore$ Maximum at $(-3,\ 54)$ and a minimum $(3, -54)$

as coefficient of x^3 in cubic's equation is positive

[4 marks]

(ii) STEP 1 $y = (x + 7)(x + 1)$

$= x^2 + 8x + 7$

$$\frac{dy}{dx} = 2x + 8$$

STEP 2 $2x + 8 = 0$

$x = -4$

STEP 3 $y = (-4 + 7)(-4 + 1)$

$y = -9$

$\therefore$ Minimum at $(-4, -9)$ as the coefficient of x^2 in parabola's equation is positive

[4 marks]

(iii) STEP 1 $y = x^4 - 256x$

$$\frac{dy}{dx} = 4x^3 - 256$$

STEP 2 $4x^3 - 256 = 0$

$x^3 = 64$

$x = 4$

STEP 3 $y = 4^4 - 256 \times 4$

$y = -768$

$\therefore$ Stationary point at $(4, -768)$

To show it's a minimum, plot graph or use second derivative

[4 marks]

Answer 7

STEP 1 $y = x^3 - 6x^2 - 36x + 100$

$$\frac{dy}{dx} = 3x^2 - 12x - 36$$

STEP 2 $3x^2 - 12x - 36 = 0$

Divide through by 3

$$x^2 - 4x - 12 = 0$$

$$(x + 2)(x - 6) = 0$$

Either $x = -2$ or $x = 6$

STEP 3 For $x = -2$, $y = (-2)^3 - 6 \times (-2)^2 - 36 \times (-2) + 100$

$y = 140$

For $x = 6$, $y = 6^3 - 6 \times 6^2 - 36 \times 6 + 100$

$y = -116$

$\therefore$ Maximum at $(-2,\ 140)$ and a minimum $(6, -116)$

as coefficient of x^3 in cubic's equation is positive

[6 marks]

Answer 8 $f(x) = \dfrac{12}{x} = 12x^{-1}$

(i) $f'(x) = -12x^{-2}$ which can be written $f'(x) = -\dfrac{12}{x^2}$

[2 marks]

(ii) $f''(x) = 24x^{-3}$ which can be written $f''(x) = \dfrac{24}{x^3}$

[2 marks]

(iii) $(2,\ 6)$ **(iv)** -3 **(v)** Anticlockwise

[2, 2, 2 marks]

Answer 9

STEP 1 $y = x^3 + 12x$

$$\frac{dy}{dx} = 3x^2 + 12$$

STEP 2 $3x^2 + 12 = 0$

$x^2 = -4$ which has no real solutions

$\therefore$ No turning points $\square$

7.1 From the Exam

Marks Available : 52

Question 1

GCSE Examination Question from June 2019, Paper 2H, Q18

The diagram shows a solid cuboid

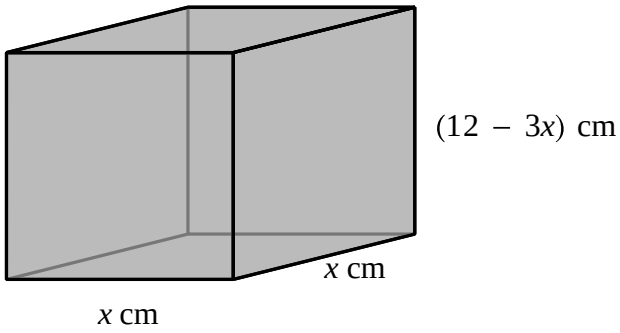


Diagram NOT accurately drawn

The total surface area of the cuboid is A cm^2
Find the maximum value of A

[5 marks]

Question 2

GCSE Examination Question from June 2009, 4H, Q17

A curve has equation

$$y = x^2 + 3x$$

(a) Find $\frac{dy}{dx}$

[2 marks]

(b) Find the gradient of the curve at the point where $x = -4$

[1 mark]

(c) The curve has a minimum point.
Find the coordinates of this minimum point.

[3 marks]

Question 3

GCSE Examination Question from November 2010, 3H, Q16

A curve has equation

$$y = x^3 + 3x^2 - 24x$$

(a) Find $\frac{dy}{dx}$

[3 marks]

(b) Find the coordinates of the turning points of the curve.

[5 marks]

Question 4

GCSE Examination Question from January 2019, Paper 1HR, Q13

A curve C has equation $y = x^3 - x^2 - 8x + 12$

(a) Find $\frac{dy}{dx}$

[2 marks]

The curve C has two turning points

(b) Work out the x coordinates of the two turning points
Show your working clearly

[3 marks]

(c) Show that the x-axis is a tangent to the curve, C

[2 marks]

Question 5

GCSE Examination Question from January 2013, 3H, Q18

(a) Differentiate with respect to x

(i) $8x^2$

(ii) $\frac{2}{x}$

(b) The curve with equation $y = 8x^2 + \frac{2}{x}$ has one turning point

Find the coordinates of this turning point.

Show your working clearly.

[4 marks]

Question 6

GCSE Examination Question from May 2007, 3H, Q17

A curve has equation $y = x^2 + \frac{16}{x}$

The curve has one turning point.

Find $\frac{dy}{dx}$ and use your answer to find the coordinates of this turning point

[4 marks]

Question 7

GCSE Examination Question from January 2020, Paper 1H, Q15

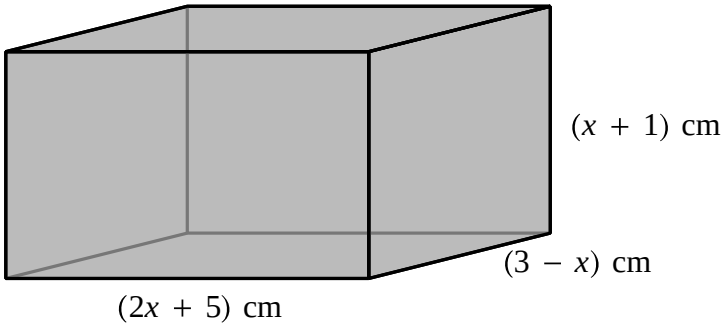


Diagram NOT accurately drawn

The diagram shows a cuboid of volume V cm^3

(a) Show that $V = 15 + 16x - x^2 - 2x^3$

[3 marks]

There is a value of x for which the volume of the cuboid is a maximum.

(b) Find this value of x

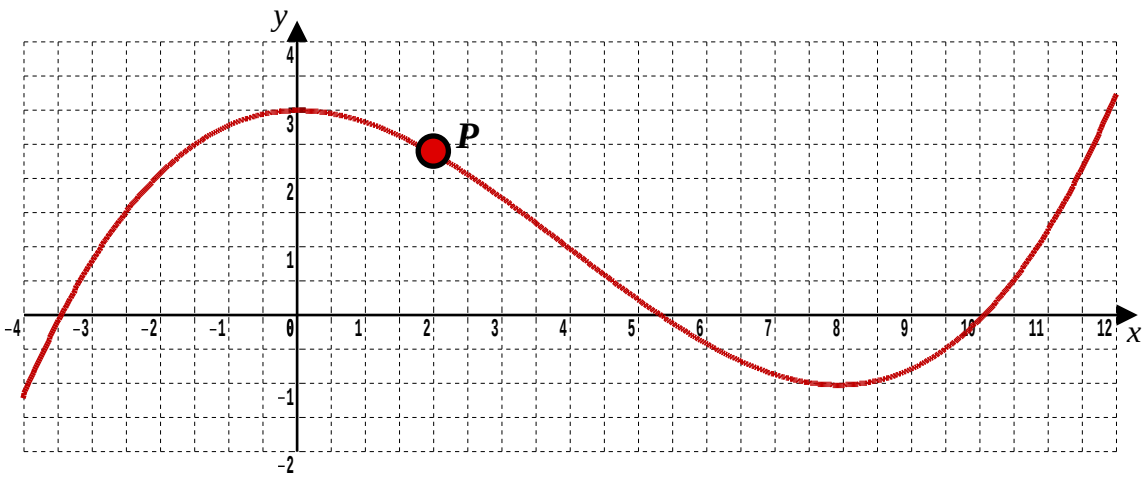
Show your working clearly

Give your answer correct to 3 significant figures

[5 marks]

Question 8

GCSE Examination Question from January 2020, Paper 1HR, Q18



The diagram shows the graph of $y = f(x)$ for $-4 \leqslant x \leqslant 12$

The point P on the curve has x coordinate 2

(a) **(i)** Use the graph to find an estimate for the gradient of the curve at P

[3 marks]

(ii) Hence find an equation of the tangent to the curve at P

Give your answer in the form $y = mx + c$

[2 marks]

The equation $f(x) = k$ has exactly two different solutions for $-4 \leqslant x \leqslant 12$

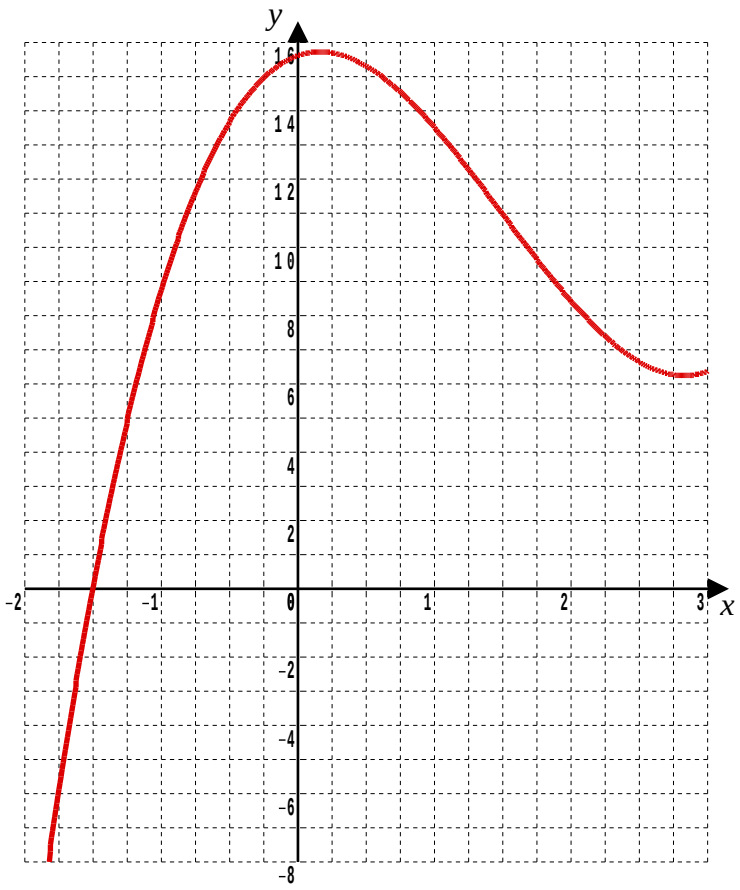
(b) Use the graph to find the two possible values of k

[2 marks]

Question 9

GCSE Examination Question from June 2019, Paper 2HR, Q18(d)

Part of the curve with equation $y = h(x)$ is shown on the grid



Find an estimate for the gradient of the curve at the point where $x = -0.5$
Show your working clearly.

[3 marks]

7.2 Answers

GCSE Mathematics
Differentiation I

Answers 1

$$\begin{aligned}
\text{Total surface area} &= 2x^2 + 4x(12 - 3x) \\
&= 2x^2 + 48x - 12x^2 \\
&= 48x - 10x^2 \\
\frac{dA}{dx} &= 48 - 20x \\
48 - 20x &= 0 \\
x &= 2.4 \text{ cm} \\
A_{MAX} &= 48 \times 2.4 - 10 \times 2.4^2 \\
&= 115.2 - 57.6 \\
&= 57.6 \text{ cm}^2
\end{aligned}$$

$\therefore$ Maximum at $(2.4,\ 57.6)$ as the coefficient of x^2 in the parabola's equation is negative.

[5 marks]

Answer 2

$$y = x^2 + 3x$$

(a) $\dfrac{dy}{dx} = 2x + 3$

[2 marks]

(b) When $x = -4$, $\dfrac{dy}{dx} = -5$

[2 marks]

(c) $2x + 3 = 0 \Rightarrow x = -\dfrac{3}{2}$

$$\begin{aligned}
y &= \left(-\frac{3}{2}\right)^2 + 3\left(-\frac{3}{2}\right) \\
&= -\frac{9}{4}
\end{aligned}$$

$\therefore$ Minimum point is $\left(-\dfrac{3}{2}, -\dfrac{9}{4}\right)$

[3 marks]

Answer 3

$$y = x^3 + 3x^2 - 24x$$

(a) $\dfrac{dy}{dx} = 3x^2 + 6x - 24$

[3 marks]

(b) $3x^2 + 6x - 24 = 0$

Divide through by 3

$$x^2 + 2x - 8 = 0$$

$$(x + 4)(x - 2) = 0$$

Either $x = -4$ or $x = 2$

For $x = -4$, $y = (-4)^3 + 3 \times (-4)^2 - 24 \times (-4)$

$= -64 + 48 + 96$

$= 80$

For $x = 2$, $y = 2^3 + 3 \times 2^2 - 24 \times 2$

$= 8 + 12 - 48$

$= -28$

$\therefore$ Turning points are $(-4,\ 80)$, $(2, -28)$

[5 marks]

Answer 4

$$y = x^3 - x^2 - 8x + 12$$

(a) $\dfrac{dy}{dx} = 3x^2 - 2x - 8$

[2 marks]

(b) $3x^2 - 2x - 8 = 0$

$$(3x + 4)(x - 2) = 0$$

Either $x = -\dfrac{4}{3}$ or $x = 2$

[3 marks]

(c) For $x = -\dfrac{4}{3}$, $y = \left(-\dfrac{4}{3}\right)^3 - \left(-\dfrac{4}{3}\right)^2 - 8 \times \left(-\dfrac{4}{3}\right) + 12$

$= \left(\dfrac{500}{27}\right)$ $\therefore$ Tangent is not at $\left(-\dfrac{4}{3},\ \dfrac{500}{27}\right)$

For $x = 2$, $y = 2^3 - 2^2 - 8 \times 2 + 12$

$= 0$ $\therefore$ Tangent is at $(2,\ 0)$

That is, the x-axis is a tangent to the curve, C

[2 marks]

Answer 5

(a) **(i)** $\dfrac{dy}{dx} = 16x$

(ii) $y = 2x^{-1} \Rightarrow \dfrac{dy}{dx} = -2x^{-2}$

(b) $y = 8x^2 + \dfrac{2}{x} \Rightarrow \dfrac{dy}{dx} = 16x - \dfrac{2}{x^2}$

$$16x - \frac{2}{x^2} = 0$$

Multiply through by x^2

$$16x^3 - 2 = 0$$

$$x^3 = \frac{1}{8}$$

$$x = \frac{1}{2}$$

$$y = 8 \times \left(\frac{1}{2}\right)^2 + 2 \div \left(\frac{1}{2}\right)$$

$$= 8 \times \frac{1}{4} + 2 \times 2$$

$$= 6$$

$\therefore$ Turning point is at $\left(\dfrac{1}{2},\ 6\right)$

[4 marks]

Answer 6

$$y = x^2 + \frac{16}{x} \Rightarrow \frac{dy}{dx} = 2x - \frac{16}{x^2}$$

$$2x - \frac{16}{x^2} = 0$$

Multiply through by x^2

$$2x^3 - 16 = 0$$

$$x^3 = 8$$

$$x = 2$$

$$y = 2^2 + \frac{16}{2}$$

$$= 12$$

$\therefore$ Turning point is at $(2,\ 12)$

[4 marks]

Answer 7

(a) $V = (3 - x)(2x + 5)(x + 1)$

$= (3 - x)\left(2x^2 + 7x + 5\right)$

Multiplication grid :

	$2x^2$	$7x$	5
3	$6x^2$	$21x$	15
$-x$	$-2x^3$	$-7x^2$	$-5x$

$= 15 + 16x - x^2 - 2x^3$ □

[3 marks]

(b) $\dfrac{dV}{dx} = 16 - 2x - 6x^2$

$16 - 2x - 6x^2 = 0$

Divide through by -2

$3x^2 + x - 8 = 0$

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 3 \times (-8)}}{2 \times 3}$$

$$= \frac{-1 \pm \sqrt{97}}{6}$$

Either $x = -1.808$ or $x = 1.475$

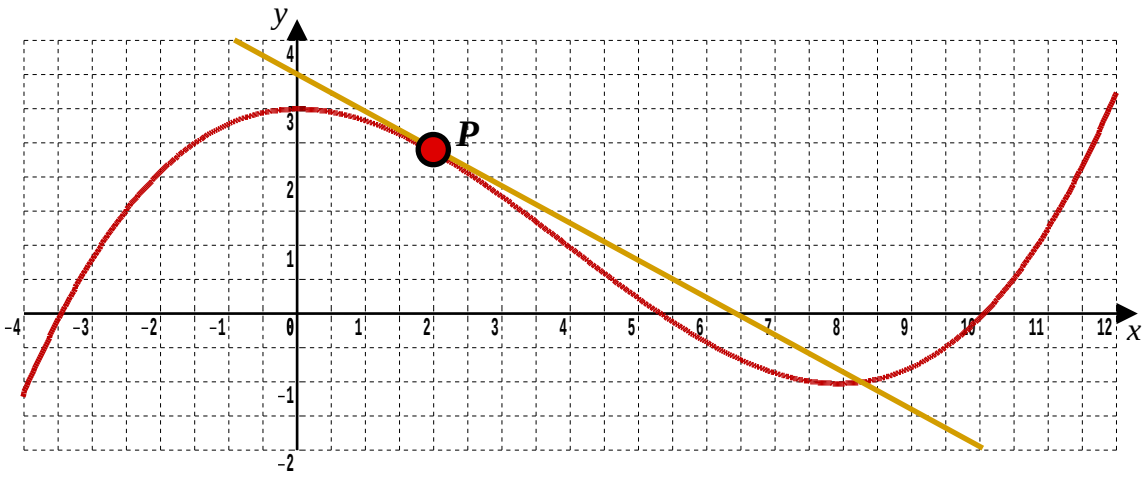
The negative value of x does not make sense in terms of the physical problem as the length $(x + 1)$ cm would then be negative.

$\therefore$ The maximum must correspond to $x = 1.475$

[5 marks]

Answer 8

(a) **(i)**



For the tangent drawn between $(0,\ 3.6)$ and $(10, -1.8)$

$$m = \frac{\Delta Y}{\Delta X}$$

$$= \frac{3.6 - (-1.8)}{0 - 10}$$

$$= -0.54 \qquad \text{Accept } (-0.4) \leqslant m \leqslant (-0.7)$$

[3 marks]

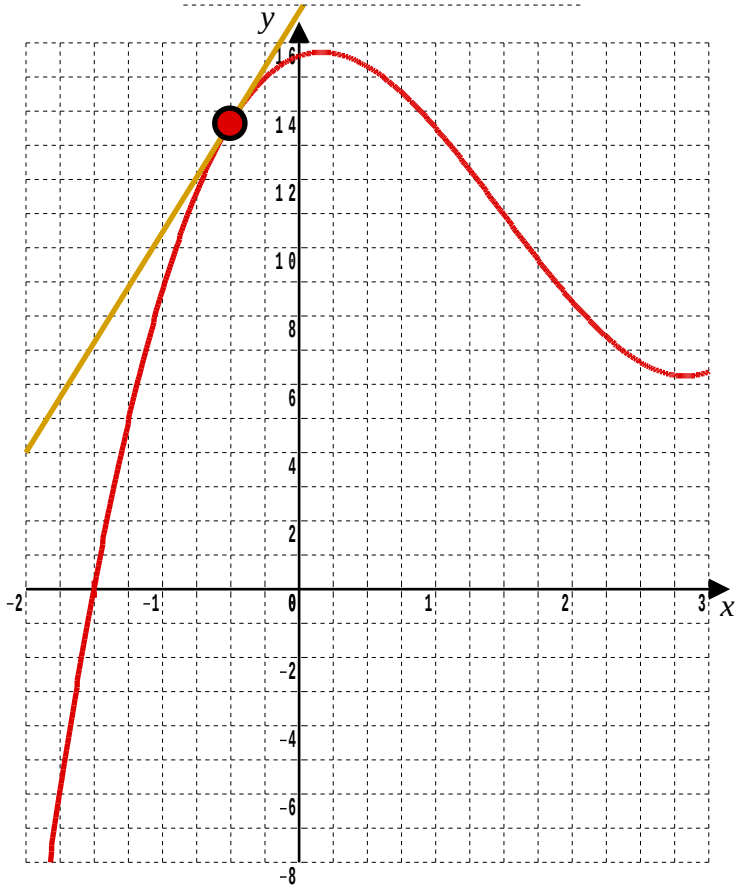
(ii) $y = -0.54x + 3.6$

[2 marks]

(b) $k = -1,\ 3,$

[2 marks]

Answer 9



For the tangent drawn between $(0,\ 17)$ and $(-2,\ 4)$

$$m = \frac{\Delta Y}{\Delta X}$$

$$= \frac{17 - 4}{0 - (-2)}$$

$$= 6.5 \qquad \text{Accept } 5 \leqslant m \leqslant 8$$

[3 marks]

Lesson 8

GCSE Mathematics
Differentiation I

8.1 REVISION for the TEST

Marks Available : 64

Question 1

Write down the exact value of each of the following:

(i) 9^2 **(ii)** $(-2)^3$ **(iii)** $\left(\frac{1}{3}\right)^3$

(iv) $25^{\frac{1}{2}}$ **(v)** $27^{\frac{1}{3}}$ **(vi)** $(-1)^{18}$

(vii) $\left(\frac{2}{\pi}\right)^0$ **(viii)** 0^7 **(ix)** $\left(\frac{8}{11}\right)^2$

[9 marks]

Question 2

Consider the curve, $y = x^3 - 3x$

Write down the points on the curve that have the x part as given;

(i) (2, _____) **(ii)** (10, _____) **(iii)** (– 10, _____)

[3 marks]

Question 3

A quintic curve has equation, $y = 5x^5 - 7x^3$

(i) Write down the gradient equation of the polynomial curve

[2 marks]

(ii) Write down the bend detector equation of the polynomial curve

[1 mark]

(iii) Use the appropriate equation to find the point on the curve when $x = 1$

[1 mark]

(iv) Use the appropriate equation to find the gradient of the curve when $x = 1$

[1 mark]

(v) Determine, when $x = 1$, if the curve is bending anticlockwise or clockwise

[2 marks]

Question 4

Write down the exact value of the following:

(i) 4^{-2} **(ii)** $8^{\frac{2}{3}}$

[2 marks]

Question 5

A curve has equation, $y = x^3 - 3x$

(i) Find $\frac{dy}{dx}$

[2 marks]

(ii) Find the gradient of the curve at the point where $x = -4$

[1 mark]

(iii) The curve has two turning points.
Find the coordinates of the two turning points.

[4 marks]

Question 6

Differentiate the following;

(i) $y = 5x^{-3}$

[2 marks]

(ii) $y = (5x + 3)^2$

[3 marks]

Some Theory of Mechanics

Starting with a displacement, *s*

Differentiate the displacement to get the velocity...

$$velocity = \frac{ds}{dt}$$

Differentiate the velocity to get the acceleration...

$$acceleration = \frac{d^2s}{dt^2}$$

Question 7

GCSE Examination question from May 2008, 4H, Q19

A particle moves in a straight line through a fixed point *O*.

The displacement of the particle from *O* at time *t* seconds is *s* metres, where

$$s = t^2 - 6t + 10$$

(a) Find $\frac{ds}{dt}$

[2 marks]

(b) Find the velocity of the particle when $t = 5$ s

[2 marks]

(c) Find the acceleration of the particle.

[2 marks]

Question 8

GCSE Examination question from November 2007, 4H, Q20.

A curve has equation, $y = x^3 - 5x^2 + 8x - 7$

(a) Find the gradient of the curve at $(2, -3)$

[4 marks]

(b) What does your answer to part (a) tell you about the point $(2, -3)$?

[1 mark]

Question 9

(i) Find the gradient equation of the curve,

$$y = \frac{16}{x^3}$$

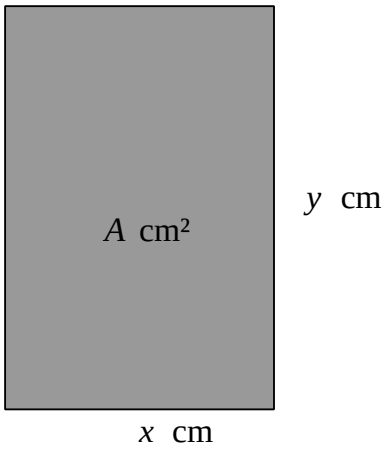
[3 marks]

(ii) Find the gradient of the curve when $x = -2$

[2 marks]

Question 10

GCSE Examination question from January 2012, 3H, Q14



The diagram shows a rectangular photo frame of area A cm^2
The width of the photo frame is x cm
The height of the photo frame is y cm
The perimeter of the photo frame is 72 cm

(a) Show that $A = 36x - x^2$

[3 marks]

(b) Find $\frac{dA}{dx}$

[2 marks]

(c) Find the maximum value of A

[3 marks]

Question 11

GCSE Examination question from November 2009, 4H, Q19.

A particle moves in a straight line through a fixed point *O*.
The displacement, *s* metres, of the particle from *O* at time *t* seconds is given by

$$s = t^3 - 5t^2 + 8$$

(a) Find an expression for the velocity, *v* ms^{-1}, of the particle after *t* seconds.

[2 marks]

(b) Find the time at which the acceleration of the particle is 20 ms^{-2}

[2 marks]

Question 12

Find the derivative of, $y = 18\sqrt{x}$

[3 marks]

8.2 Answers (Revision for TEST)

GCSE Mathematics
Differentiation I

Answer 1

(i) 81 **(ii)** -8 **(iii)** $\frac{1}{27}$
(iv) 5 **(v)** 3 **(vi)** 1
(vii) 1 **(viii)** 0 **(ix)** $\frac{64}{121}$

[9 marks]

Answer 2

(i) (2, 2) **(ii)** (10, 970) **(iii)** ($-10.$ $-$ 970)

[3 marks]

Answer 3

(i) $\frac{dy}{dx} = 25x^4 - 21x^2$ **(ii)** $\frac{d^2y}{dx^2} = 100x^3 - 42x$

[2, 1 marks]

(iii) (1, -2) **(iv)** 4 **(v)** 58, i.e. +ve $\therefore$ Anticlockwise

[1, 1, 2 marks]

Answer 4

(i) $\frac{1}{16}$ **(ii)** 4

[2 marks]

Answer 5

(i) $\frac{dy}{dx} = 3x^2 - 3$ **(ii)** 45 **(iii)** (1, -2), (-1, 2)

[2, 1, 4 marks]

Answer 6

(i) $\frac{dy}{dx} = -15x^{-4}$ **(ii)** $y = 25x^2 + 30x + 9 \Rightarrow \frac{dy}{dx} = 50x + 30$

[2, 3 marks]

Answer 7

(a) $\frac{ds}{dt} = 2t - 6$ **(b)** 4 ms^{-1} **(c)** 2 ms^{-2}

[2, 2, 2 marks]

Answer 8

(a) 0 **(b)** It's a turning point

[4, 1 marks]

Answer 9

(i) $y = 16x^{-3}$ **(ii)** -4

$\Rightarrow \frac{dy}{dx} = -48x^{-4}$

[3, 2 marks]

Answer 10

(a) Consider Perimeter

$$2x + 2y = 72$$

$$x + y = 36$$

$$\therefore\ y = 36 - x$$

Consider Area

$$A = x \times y$$

$$A = x\,(36 - x)$$

$$A = 36x - x^2$$

[3 marks]

(b) $36 - 2x$

[2 marks]

(c) $x = 18$ cm $\Rightarrow$ $A = 324$ cm^2

[3 marks]

Answer 11

(a) $v = \dfrac{ds}{dt} = 3t^2 - 10t$

[2 marks]

(b) $a = \dfrac{d^2s}{dt^2} = 6t - 10 \qquad \therefore t = 5$ seconds

[2 marks]

Answer 12

$$y = 18x^{\frac{1}{2}}$$

$$\Rightarrow \frac{dy}{dx} = 9x^{-\frac{1}{2}}$$

[3 marks]

Lesson 9

GCSE Mathematics
Differentiation I

9.1 Homework REVISION for the TEST

Marks Available : 64

Question 1

Write down the exact value of each of the following:

(i) 12^2 **(ii)** $(-4)^3$ **(iii)** $\left(\frac{1}{5}\right)^3$

(iv) $100^{\frac{1}{2}}$ **(v)** $64^{\frac{1}{3}}$ **(vi)** $(-1)^{51}$

(vii) $\left(\frac{\sqrt{7\sin(60)}}{\pi}\right)^0$ **(viii)** $0^{0.5}$ **(ix)** $\left(\frac{4}{9}\right)^2$

[9 marks]

Question 2

Consider the curve, $y = x^3 - 5x$

Write down the points on the curve that have the x part as given;

(i) (2, _____) **(ii)** (10, _____) **(iii)** (– 10, _____)

[3 marks]

Question 3

Consider the heptic equation, $y = 5x^6 - 7x^4$

(i) Write down the gradient equation of the polynomial curve.

[2 marks]

(ii) Write down the bend detector equation of the polynomial curve.

[1 mark]

(iii) Use the appropriate equation to find the point on the curve when $x = 1$

[1 mark]

(iv) Use the appropriate equation to find the gradient of the curve when $x = 1$

[1 mark]

(v) Determine, when $x = 1$, if the curve is bending anticlockwise or clockwise

[2 marks]

Question 4

Write down the exact value of the following:

(i) 7^{-2} **(ii)** $16^{\frac{3}{2}}$

[2 marks]

Question 5

A curve has equation, $y = x^3 - 12x$

(i) Find $\frac{dy}{dx}$

[2 marks]

(ii) Find the gradient of the curve at the point where $x = -3$

[1 mark]

(iii) The curve has two turning points.
Find the coordinates of the two turning points.

[4 marks]

Question 6

Differentiate the following;

(i) $y = 5x^{-2}$

[2 marks]

(ii) $y = (7x + 3)^2$

[3 marks]

Some Theory of Mechanics

Starting with a displacement, s

Differentiate the displacement to get the velocity...

$$velocity = \frac{ds}{dt}$$

Differentiate the velocity to get the acceleration...

$$acceleration = \frac{d^2s}{dt^2}$$

Question 7

A particle moves in a straight line through a fixed point O.
The displacement of the particle from O at time t seconds is s metres, where

$$s = t^2 - 9t + 13$$

(a) Find $\frac{ds}{dt}$

[2 marks]

(b) Find the velocity of the particle when $t = 5$ seconds

[2 marks]

(c) Find the acceleration of the particle.

[2 marks]

Question 8

A curve has equation, $y = x^3 - 8x^2 + 21x - 21$

(a) Find the gradient of the curve at $(3, -3)$

[4 marks]

(b) What does your answer to part **(a)** tell you about the point $(3, -3)$?

[1 mark]

Question 9

(i) Find the gradient equation of the curve,

$$y = \frac{16}{x^4}$$

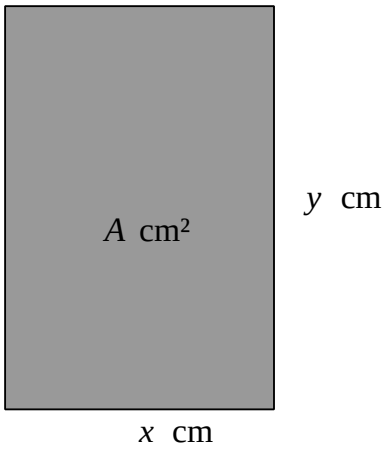
[3 marks]

(ii) Find the gradient of the curve when $x = -2$

[2 marks]

Question 10

GCSE Examination question from January 2012, 3H, Q14.



The diagram shows a rectangular photo frame of area A cm^2
The width of the photo frame is x cm
The height of the photo frame is y cm
The perimeter of the photo frame is 84 cm

(a) Show that $A = 42x - x^2$

[3 marks]

(b) Find $\frac{dA}{dx}$

[2 marks]

(c) Find the maximum value of A.

[3 marks]

Question 11

A particle moves in a straight line through a fixed point O.
The displacement, s metres, of the particle from O at time t seconds is given by

$$s = t^3 - 3t^2 - 2t + 12$$

(a) Find an expression for the velocity, v ms^{-1}, of the particle after t seconds.

[2 marks]

(b) Find the time at which the acceleration of the particle is 30 ms^{-2}

[2 marks]

Question 12

Find the derivative of,

$$y = 24\sqrt{x}$$

[3 marks]

9.2 Answers (Homework Revision for TEST)

GCSE Mathematics
Differentiation I

Answer 1

(i) 144 **(ii)** -64 **(iii)** $\frac{1}{125}$
(iv) 10 **(v)** 4 **(vi)** -1
(vii) 1 **(viii)** 0 **(ix)** $\frac{16}{81}$

[9 marks]

Answer 2

(i) $(2, -2)$ **(ii)** $(10, 950)$ **(iii)** $\sqrt{}(-10, -950)$

[3 marks]

Answer 3

(i) $\frac{dy}{dx} = 30x^5 - 28x^3$ (ii) $\frac{d^2y}{dx^2} = 150x^4 - 84x^2$

[2, 1 marks]

(iii) $(1, -2)$ **(iv)** 2 **(v)** 66, i.e. +ve $\therefore$ Anticlockwise

[1, 1, 2 marks]

Answer 4

(i) $\frac{1}{49}$ **(ii)** 64

[2 marks]

Answer 5

(i) $\frac{dy}{dx} = 3x^2 - 12$ **(ii)** 15 **(iii)** $(2, -16)$, $(-2, 16)$

[2, 1, 4 marks]

Answer 6

(i) $\frac{dy}{dx} = -10x^{-3}$ **(ii)** $y = 49x^2 + 42x + 9 \Rightarrow \frac{dy}{dx} = 98x + 42$

[2, 3 marks]

Answer 7

(a) $v = \frac{ds}{dt} = 2t - 9$ **(b)** $1\ \text{ms}^{-1}$ **(c)** $2\ \text{ms}^{-2}$

[2, 2, 2 marks]

Answer 8

(a) 0 **(b)** It's a turning point

[4, 1 marks]

Answer 9

(i) $y = 16x^{-4}$ **(ii)** 2

$\Rightarrow \frac{dy}{dx} = -64x^{-5}$

[3, 2 marks]

Answer 10

(a) Consider perimeter,

$$2x + 2y = 84$$

$$x + y = 42$$

$$\therefore\ y = 42 - x$$

Consider Area,

$$A = x \times y$$

$$A = x\,(42 - x)$$

$$A = 42x - x^2$$

[3 marks]

(b) $42 - 2x$

[2 marks]

(c) $x = 21$ cm $\Rightarrow$ $A = 441$ cm^2

[3 marks]

Answer 11

(a) $v = \dfrac{ds}{dt} = 3t^2 - 6t - 2$

[2 marks]

(b) $a = \dfrac{d^2s}{dt^2} = 6t - 6 \qquad \therefore\ t = 6$ seconds

[2 marks]

Answer 12

$$y = 24x^{\frac{1}{2}}$$

$$\Rightarrow \frac{dy}{dx} = 12x^{-\frac{1}{2}}$$

[3 marks]

Lesson 10

GCSE Mathematics
Differentiation I

10.1 TEST

Marks Available : 72

Question 1

Write down the exact value of each of the following:

(i) 5^3 **(ii)** $(-7)^2$ **(iii)** $\left(\frac{1}{2}\right)^4$

(iv) $64^{\frac{1}{2}}$ **(v)** $64^{\frac{1}{3}}$ **(vi)** $(-1)^{19}$

(vii) $\left(\frac{2\sqrt{\pi}}{7^2-5}\right)^0$ **(viii)** 0^8 **(ix)** $\left(\frac{9}{11}\right)^2$

[9 marks]

Question 2

Consider the curve, $y = x^3 - 5x$

Write down the points on the curve that have the x part as given;

(i) (2, _____) **(ii)** (10, _____) **(iii)** (– 10, _____)

[3 marks]

Question 3

Consider the quartic curve, $y = 3x^4 - 7x^2$

(i) Write down the gradient equation of the polynomial curve.

[2 marks]

(ii) Write down the bend detector equation of the polynomial curve.

[1 mark]

(iii) Use the appropriate equation to find the point on the curve when $x = 1$

[1 mark]

(iv) Use the appropriate equation to find the gradient of the curve when $x = 1$

[1 mark]

(v) Determine, when $x = 1$, if the curve is bending anticlockwise or clockwise.

[2 marks]

Question 4

Write down the exact value of the following:

(i) 4^{-2} **(ii)** $27^{\frac{2}{3}}$

[2 marks]

Question 5

A curve has equation, $y = x^3 - 12x$

(i) Find $\frac{dy}{dx}$

[2 marks]

(ii) Find the gradient of the curve at the point where $x = -3$

[1 mark]

(iii) The curve has two turning points.
Find the coordinates of the two turning points.

[4 marks]

Question 6

Differentiate the following,

(i) $y = 6x^{-4}$

[2 marks]

(ii) $y = (3x + 5)^2$

[3 marks]

Some Theory of Mechanics

Starting with a displacement, s

Differentiate the displacement to get the velocity...

$$velocity = \frac{ds}{dt}$$

Differentiate the velocity to get the acceleration...

$$acceleration = \frac{d^2s}{dt^2}$$

Question 7

GCSE Examination question from May 2016, 3H, Q14

A particle is moving along a straight line.
The fixed point O lies on this line.

The displacement of the particle from O at time t seconds is s metres, where

$$s = 2t^3 - 12t^2 + 7t$$

(a) Find an expression for the velocity, v ms^{-1}, of the particle at time t seconds.

[2 marks]

(b) Find the time at which the acceleration of the particle is instantaneously zero.

[2 marks]

Question 8

A curve has equation, $y = x^3 - 27x + 55$

(a) Find the gradient of the curve at (3, 1)

[4 marks]

(b) What does your answer to part **(a)** tell you about the point (3, 1) ?

[1 mark]

Question 9

(i) Find the gradient equation of the curve,

$$y = \frac{25}{x^5}$$

[3 marks]

(ii) Find the gradient of the curve when $x = -2$

[2 marks]

Question 10

GCSE Examination question from November 2006, 3H, Q14

(a) For the equation $y = 5000x - 625x^2$, find $\frac{dy}{dx}$

[2 marks]

(b) Find the coordinates of the turning point on the graph of,

$$y = 5000x - 625x^2$$

[3 marks]

(c) **(i)** State whether this turning point is a maximum or a minimum.

(ii) Give a reason for your answer.

[2 marks]

(d) A publisher has to set the price for a new book.
The profit, £y, depends on the price of the book, £x, where

$$y = 5000x - 625x^2$$

(i) What price would you advise the publisher to set for the book ?

(ii) Give a reason for your answer.

[2 marks]

Question 11

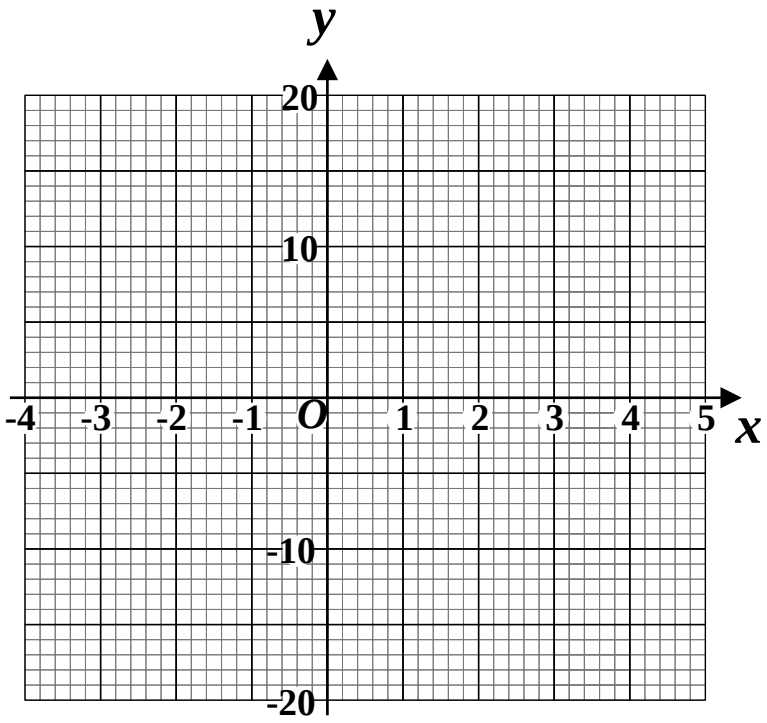
GCSE Examination question from June 2010, 3H, Q12

(a) Complete the table of values for, $y = x^3 - 12x + 2$

x	-3	-2	-1	0	1	2	3	4
y	11						-7	18

[2 marks]

(b) On the grid, draw the graph of $y = x^3 - 12x + 2$ for values of x from (-3) to 4



[2 marks]

(c) For the curve with equation $y = x^3 - 12x + 2$

(i) find $\frac{dy}{dx}$

(ii) find the gradient of the curve at the point where $x = 5$

[4 marks]

Question 12

GCSE Examination question from June 2011, 3H, Q21

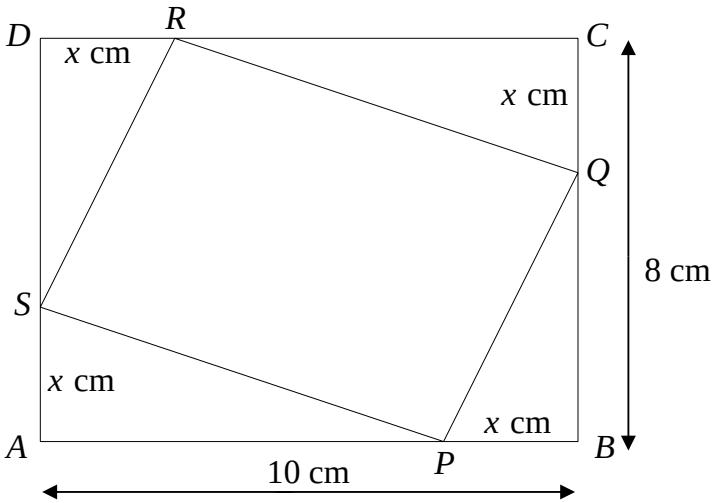


Diagram NOT accurately drawn

ABCD is a rectangle.
AB = 10 cm.
BC = 8 cm.
P, *Q*, *R* and *S* are points on the sides of the rectangle.
BP = *CQ* = *DR* = *AS* = *x* cm.

(a) Show that the area, *A* cm^2, of the quadrilateral *PQRS* is given by the formula

$$A = 2x^2 - 18x + 80$$

[3 marks]

(b) For $A = 2x^2 - 18x + 80$

(i) find $\frac{dA}{dx}$

(ii) find the value of x for which A is a minimum.

(iii) Explain how you know that A is a minimum for this value of x.

[5 marks]

10.2 Answers (TEST)

GCSE Mathematics
Differentiation I

Answer 1

(i) 125 **(ii)** 49 **(iii)** $\frac{1}{16}$
(iv) 8 **(v)** 4 **(vi)** -1
(vii) 1 **(viii)** 0 **(ix)** $\frac{81}{121}$

[9 marks]

Answer 2

(i) $(2, -2)$ **(ii)** $(10, 950)$ **(iii)** $(-10, -950)$

[3 marks]

Answer 3

(i) $\frac{dy}{dx} = 12x^3 - 14x$ **(ii)** $\frac{d^2y}{dx^2} = 36x^2 - 14$

[2, 1 marks]

(iii) $(1, -4)$ **(iv)** -2 **(v)** 22, i.e. +ve $\therefore$ Anticlockwise

[1, 1, 2 marks]

Answer 4

(i) $\frac{1}{16}$ **(ii)** 9

[2 marks]

Answer 5

(i) $\frac{dy}{dx} = 3x^2 - 12$ **(ii)** 15 **(iii)** $(2, -16), (-2, 16)$

[2, 1, 4 marks]

Answer 6

(i) $\frac{dy}{dx} = -24x^{-5}$ **(ii)** $y = 9x^2 + 30x + 25 \Rightarrow \frac{dy}{dx} = 18x + 30$

[2, 3 marks]

Answer 7

(a) $v = \frac{ds}{dt} = 6t^2 - 24t + 7$ **(b)** 2 seconds

[2, 2 marks]

Answer 8

(a) 0 **(b)** It's a turning point

[4, 1 marks]

Answer 9

(i) $y = 25x^{-5}$ (ii) $-\frac{125}{64}$ or about $-1.953125...$

$\Rightarrow \frac{dy}{dx} = -125x^{-6}$

[3, 2 marks]

Answer 10

(a) $\frac{dy}{dx} = 5000 - 1250x$

[2 marks]

(b) (4, 10 000)

[3 marks]

(c) **(i)** Maximum

(ii) The curve is an upside down quadratic equation, i.e. ∩ because of the negative in front of the x^2 and so only has a local maximum

[2 marks]

(d) **(i)** £4

(ii) It maximises his profit

[2 marks]

Answer 11

(a)

x	-3	-2	-1	0	1	2	3	4
y	11	**18**	**13**	**2**	**– 9**	**– 14**	–7	18

[2 marks]

(b) Points plotted to give cubic curve

[2 marks]

(c) **(i)** $\frac{dy}{dx} = 3x^2 - 12$

(ii) 63

[4 marks]

Answer 12

(a)

$$A = \text{area rectangle-area four triangles}$$

$$= 10 \times 8 - \frac{1}{2}(10 - x) \times 2 - \frac{1}{2}(8 - x)(x) \times 2$$

$$= 80 - x(10 - x) - x(8 - x)$$

$$= 80 - 10x + x^2 - 8x + x^2$$

$$= 2x^2 - 18x + 80$$

[3 marks]

(b) **(i)** $4x - 18$

(ii) 4.5

(iii) Second derivative is 4 which is +ve ∴ AntiClockwise ∴ Min

[5 marks]