

5.3 Answers

GCSE Mathematics Algebraic Fractions

5.3.1 Solutions to 5.1 Examples

Answer 1

$$\begin{aligned}2x^2 - 18 &= 2(x^2 - 9) \\&= 2(x^2 - 3^2) \\&= 2(x + 3)(x - 3)\end{aligned}$$

[2 marks]

Answer 2

$$\begin{aligned}4x^3 - x &= x(4x^2 - 1) \\&= x((2x)^2 - 1^2) \\&= x(2x + 1)(2x - 1)\end{aligned}$$

[2 marks]

Answer 3

$$\begin{aligned}\frac{2x^2 + 5x + 3}{4x^2 - 9} &= \frac{2x^2 + 5x + 3}{(2x)^2 - 3^2} \\&= \frac{(2x + 3)(x + 1)}{(2x + 3)(2x - 3)} \\&= \frac{x + 1}{2x - 3}\end{aligned}$$

[3 marks]

Answer 4

$$\begin{aligned}(3x + 1)^2 - (x + 2)^2 &= (3x + 1 + x + 2)(3x + 1 - (x + 2)) \\&= (4x + 3)(2x - 1)\end{aligned}$$

[3 marks]

5.3.2 Solutions to 5.2 Exercise

Answer 1

$$\begin{aligned}\text{(i)} \quad x^2 - 36 &= x^2 - 6^2 \\&= (x + 6)(x - 6)\end{aligned}$$

[1 mark]

$$\begin{aligned}\text{(ii)} \quad \frac{x^2 - 36}{x + 6} &= \frac{(x + 6)(x - 6)}{x + 6} \\&= x - 6\end{aligned}$$

[2 marks]

Answer 2

$$\begin{aligned} \text{(i)} \quad x^2 - 81 &= x^2 - 9^2 \\ &= (x + 9)(x - 9) \end{aligned}$$

[1 mark]

$$\begin{aligned} \text{(ii)} \quad \frac{x^2 - 81}{x - 9} &= 14 \\ \frac{(x + 9)(x - 9)}{x - 9} &= 14 \\ x + 9 &= 14 \\ x &= 5 \end{aligned}$$

[2 marks]

Answer 3

$$\begin{aligned} \frac{x + 1}{x^2 - 1} &= 5 \\ \frac{x + 1}{(x + 1)(x - 1)} &= 5 \\ \frac{1}{x - 1} &= 5 \\ 1 &= 5(x - 1) \\ \frac{1}{5} &= x - 1 \\ x &= \frac{6}{5} \end{aligned}$$

[2 marks]

Answer 4

$$\begin{aligned} \text{(i)} \quad 9x^2 - 16 &= (3x)^2 - 4^2 \\ &= (3x + 4)(3x - 4) \end{aligned}$$

[1 mark]

$$\begin{aligned} \text{(ii)} \quad \frac{3x^2 + 7x + 4}{9x^2 - 16} &= \frac{(3x + 4)(x + 1)}{(3x + 4)(3x - 4)} \\ &= \frac{x + 1}{3x - 4} \end{aligned}$$

[3 marks]

Answer 5

$$\begin{aligned} 2x^2 - 8 &= 2(x^2 - 4) \\ &= 2(x^2 - 2^2) \\ &= 2(x + 2)(x - 2) \end{aligned}$$

[2 marks]

Answer 6

$$\begin{aligned}
 5x^3 - 20x &= 5x(x^2 - 4) \\
 &= 5x(x^2 - 2^2) \\
 &= 5x(x + 2)(x - 2)
 \end{aligned}$$

[2 marks]**Answer 7**

$$\begin{aligned}
 \frac{x^2 - 1}{(x - 1)^2} &= 5 \\
 \frac{(x + 1)(x - 1)}{(x - 1)(x - 1)} &= 5 \\
 x + 1 &= 5(x - 1) \\
 x + 1 &= 5x - 5 \\
 6 &= 4x \\
 x &= \frac{6}{4} \\
 &= 1.5
 \end{aligned}$$

[3 marks]**Answer 8**

$$\begin{aligned}
 \text{(i)} \quad (5x + 3)^2 - (2x + 1)^2 &= (5x + 3 + 2x + 1)(5x + 3 - 2x - 1) \\
 &= (7x + 4)(3x + 2)
 \end{aligned}$$

[3 marks]

$$\begin{aligned}
 \text{(ii)} \quad \frac{(5x + 3)^2 - (2x + 1)^2}{(3x + 2)} &= 32 \\
 \frac{(7x + 4)(3x + 2)}{(3x + 2)} &= 32 \\
 7x + 4 &= 32 \\
 7x &= 28 \\
 x &= 4
 \end{aligned}$$

[3 marks]**Answer 9**

$$\begin{aligned}
 \frac{x^2 - 25}{2x^2 - 9x - 5} &= \frac{(x + 5)(x - 5)}{(2x + 1)(x - 5)} \\
 &= \frac{x + 5}{2x + 1}
 \end{aligned}$$

[3 marks]

Answer 10

$$\frac{3x + 1}{5} - \frac{x - 4}{3} = 2$$

Mukltiply through by 15

$$3 (3x + 1) - 5 (x - 4) = 30$$

$$9x + 3 - 5x + 20 = 30$$

$$4x = 7$$

$$x = \frac{7}{4}$$

[3 marks]

Answer 11

$$\frac{3}{x + 1} - \frac{2}{x - 1}$$

make a common denominator

$$\frac{3}{x + 1} - \frac{2}{x - 1} = \frac{3}{x + 1} \times \frac{x - 1}{x - 1} - \frac{2}{x - 1} \times \frac{x + 1}{x + 1}$$

$$= \frac{3x - 3 - 2x - 2}{(x + 1)(x - 1)}$$

$$= \frac{x - 5}{(x + 1)(x - 1)}$$

[3 marks]

Answer 12

$$(10a - b)^2 - (2a - 5b)^2 = (10a - b + 2a - 5b)(10a - b - 2a + 5b)$$

$$= (12a - 6b)(8a + 4b)$$

$$= 6 \times (2a - b) \times 4 \times (2a + b)$$

$$= 24(2a - b)(2a + b)$$

[3 marks]

Answer 13

$$3x^2 - 75y^2 = 3(x^2 - 25y^2)$$

$$= 3(x^2 - (5y)^2)$$

$$= 3(x + 5y)(x - 5y)$$

[2 marks]

Answer 14**(a)**

$$\begin{aligned}
\text{LHS} &= \frac{x+1}{2x+1} - \frac{1}{(2x+1)(x+1)} \\
&= \frac{x+1}{2x+1} \times \frac{x+1}{x+1} - \frac{1}{(2x+1)(x+1)} \\
&= \frac{x^2 + 2x + 1 - 1}{(2x+1)(x+1)} \\
&= \frac{x^2 + 2x}{(2x+1)(x+1)} \\
&= \text{RHS} \quad \square
\end{aligned}$$

[2 marks]**(b)**

$$\begin{aligned}
\frac{x+1}{2x+1} - \frac{1}{(2x+1)(x+1)} &= \frac{1}{(2x+1)(x+1)} \\
\frac{x^2 + 2x}{(2x+1)(x+1)} &= \frac{1}{(2x+1)(x+1)} \quad \text{using part (a)} \\
\therefore x^2 + 2x &= 1 \\
x^2 + 2x - 1 &= 0
\end{aligned}$$

Using the formula to solve a quadratic equation, yields,

$$\begin{aligned}
x &= \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-1)}}{2 \times 1} \\
&= \frac{-2 \pm \sqrt{8}}{2} \\
&= -1 \pm \sqrt{2}
\end{aligned}$$

[4 marks]