

Classic conundrums in Mathematics

An infinite series of puzzles and amusements
so that you may become
one of the
E N L I G H T E N E D

$$\frac{A}{F} + \frac{\sqrt{L}}{R} \times \frac{G}{A^2} - \left(\frac{E}{C!} \div \frac{B}{T} \right) = \frac{R}{I} + \frac{A}{O^0} \times \sqrt{\left(\frac{I}{N} \right)} + \frac{C^{0.5}}{S}$$

*The
Curious
Mystery
Of
The
Algebraic
Fractions*

$$\frac{A}{F} + \frac{\sqrt{L}}{R} \times \frac{G}{A^2} - \left(\frac{E}{C!} \div \frac{B}{T} \right) = \frac{R}{I} + \frac{A}{O^0} \times \sqrt{\left(\frac{I}{N} \right)} + \frac{C^{0.5}}{S}$$

ALGEBRAIC FRACTIONS

Lesson 1

1.1 FOIL

Two methods of expanding brackets

METHOD 1

$$(4x + 7)(3x - 5)$$

METHOD 2

$$(4x + 7)(3x - 5)$$

[2, 2 marks]

1.2 Factorisation

This is the reverse of expanding the brackets.

GCSE Examination Question from November 2010, 3H, Q13 (Edexcel)

(a) Factorise $x^2 - 8x + 15$

[2 marks]

(b) Factorise $x^2 - 49$

[1 mark]

1.3 An Algebraic Fraction

Simplify the following algebraic expressions by first factorising the quadratics:

$$\frac{x^2 + 5x - 66}{x^2 + 2x - 48}$$

[3 marks]

1.4 Proof of “A difference of two squares”

[3 marks]

1.5 Mental Arithmetic

Without using a calculator, what is $47^2 - 43^2$?

[2 marks]

1.6 Exercise

Marks Available : 55

Question 1

Expand the brackets and simplify;

(i) $(x - 6)(x + 3)$

(ii) $(x + 4)(x - 10)$

(iii) $(x + 5)(x - 4)$

(iv) $(x - 8)(x + 3)$

(v) $(x + 11)(x - 8)$

(vi) $(x - 4)(x + 13)$

[6 marks]

Question 2

Expand the brackets and simplify;

(i) $(3x - 7)(4x + 3)$

(ii) $(4x + 3)(5x - 2)$

(iii) $(9x + 7)(x + 4)$

(iv) $(5x - 7)(3x - 4)$

(v) $(13x + 5)^2$

(vi) $(11x - 8)(9x - 2)$

[12 marks]

Question 3

Simplify the following algebraic expressions by first factorising the quadratics:

(i) $\frac{x^2 + 6x + 8}{x^2 + 7x + 10}$

[3 marks]

(ii) $\frac{x^2 + 3x - 18}{x^2 + 5x - 24}$

[3 marks]

(iii) $\frac{x^2 + x - 90}{x^2 + 2x - 80}$

[3 marks]

(iv) $\frac{x^2 + 4x - 21}{x^2 + 2x - 15}$

[3 marks]

(v) $\frac{x^2 + x - 20}{x - 4} + \frac{x^2 + 5x - 6}{x + 6}$

[3 marks]

Question 4

Simplify the following algebraic expressions by first factorising the quadratics:

(i) $\frac{x^2 + 8x - 9}{x - 1} + \frac{x^2 + x - 30}{x + 6}$

[3 marks]

(ii) $\frac{x^2 + 3x - 28}{x^2 + 5x - 14}$

[3 marks]

(iii) $\frac{x^2 + 10x - 24}{x^2 + 9x - 22}$

[3 marks]

(iv) $\frac{x^2 + 7x - 44}{x - 4} + \frac{x^2 + 10x - 39}{x - 3}$

[3 marks]

(v) $\frac{x^2 + 3x - 54}{x - 6} + \frac{x^2 + 4x - 5}{x - 1}$

[3 marks]

Question 5

GCSE Examination Question from November 2008, 4H, Q17 (Edexcel)

(a) Factorise $2x^2 + 5x + 3$

[2 marks]

(b) Factorise $4y^2 - 9$

[2 marks]

Question 6

GCSE Examination Question from June 2010, 3H, Q18 (Edexcel)

Simplify fully

$$\frac{x^2 + 6x}{x^2 - 36}$$

[3 marks]

1.7 Homework

GCSE Mathematics Algebraic Fractions

Marks Available : 30

Simplify the following algebraic expressions by first factorising the quadratics:

(i) $\frac{x^2 + 5x + 6}{x^2 + 8x + 12}$

[3 marks]

(ii) $\frac{x^2 + 6x + 9}{x^2 + 7x + 12}$

[3 marks]

(iii) $\frac{x^2 + 15x + 50}{x^2 + 9x + 20}$

[3 marks]

(iv) $\frac{x^2 + 7x + 12}{x^2 + 9x + 20}$

[3 marks]

(v) $\frac{x^2 + 13x + 22}{x + 11} + \frac{x^2 + 8x + 16}{x + 4}$

[3 marks]

$$(vi) \quad \frac{x^2 + 10x + 16}{x + 2} + \frac{x^2 + 8x + 15}{x + 5}$$

[3 marks]

$$(vii) \quad \frac{x^2 - 6x + 9}{x^2 - 7x + 12}$$

[3 marks]

$$(viii) \quad \frac{x^2 - 13x + 40}{x^2 - 10x + 25}$$

[3 marks]

$$(ix) \quad \frac{x^2 + 18x + 80}{x + 10} + \frac{x^2 - 7x + 10}{x - 2}$$

[3 marks]

$$(x) \quad \frac{x^2 + 8x + 12}{x + 2} + \frac{x^2 - 13x + 36}{x - 4}$$

[3 marks]

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1.8 Answers

GCSE Mathematics Algebraic Fractions

1.8.1

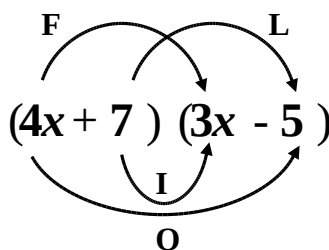
METHOD 1 (Multiplication Grid)

	$4x$	$+7$
$3x$	$12x^2$	$21x$
-5	$-20x$	-35

$$12x^2 + x - 35$$

[2 marks]

METHOD 2 (Foil)



$$12x^2 - 20x + 21x - 35$$

$$12x^2 + x - 35$$

[2 marks]

1.8.2 Factorisation

(a) $x^2 - 8x + 15 = (x - 3)(x - 5)$

[2 marks]

(b) $x^2 - 49 = x^2 - 7^2$ a difference of two squares
 $= (x + 7)(x - 7)$

[1 mark]

1.8.3 An Algebraic Fraction

$$\begin{aligned}\frac{x^2 + 5x - 66}{x^2 + 2x - 48} &= \frac{(x + 11)(x - 6)}{(x + 8)(x - 6)} \\ &= \frac{x + 11}{x + 8}\end{aligned}$$

[3 marks]

1.8.4 Proof of “A difference of two squares”

Clearly, $(x + y)(x - y) = x^2 - xy + xy - y^2$ using FOIL
 $= x^2 - y^2$

$$\therefore x^2 - y^2 = (x + y)(x - y) \quad \text{as claimed}$$

[3 marks]

1.8.5 Mental Arithmetic

$$\begin{aligned} 47^2 - 43^2 &= (47 + 43)(47 - 43) \\ &= 90 \times 4 \\ &= 360 \end{aligned}$$

[2 marks]

1.8.6 Solutions (1.6 Exercise)

Answer 1

(i) $x^2 - 3x - 18$

(ii) $x^2 - 6x - 40$

(iii) $x^2 + x - 20$

(iv) $x^2 - 5x - 24$

(v) $x^2 + 3x - 88$

(vi) $x^2 + 9x - 52$

[6 marks]

Answer 2

(i) $12x^2 - 19x - 21$

(ii) $20x^2 + 7x - 6$

(iii) $9x^2 + 43x + 28$

(iv) $15x^2 - 41x + 28$

(v) $169x^2 + 130x + 25$

(vi) $99x^2 - 94x + 16$

Answer 3

(i) $\frac{x + 4}{x + 5}$

(ii) $\frac{x + 6}{x + 8}$

(iii) $\frac{x - 9}{x - 8}$

(iv) $\frac{x + 7}{x + 5}$

(v) $2x + 4$

Answer 4

(i) $2x + 4$

(ii) $\frac{x - 4}{x - 2}$

(iii) $\frac{x + 12}{x + 11}$

(iv) $2x + 24$

(v) $2x + 14$

Answer 5

(a) $(2x + 3)(x + 1)$

(b) $(2y - 3)(2y + 3)$

Answer 6

$$\frac{x}{x - 6}$$

1.8.7 Solutions (1.7 Homework)

GCSE Mathematics Algebraic Fractions

$$(i) \quad \frac{x+3}{x+6}$$

$$(iii) \quad \frac{x+10}{x+4}$$

$$(v) \quad 2x+6$$

$$(vii) \quad \frac{x-3}{x-4}$$

$$(ix) \quad 2x+3$$

$$(ii) \quad \frac{x+3}{x+4}$$

$$(iv) \quad \frac{x+3}{x+5}$$

$$(vi) \quad 2x+11$$

$$(viii) \quad \frac{x-8}{x-5}$$

$$(x) \quad 2x-3$$

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Lesson 2

GCSE Mathematics Algebraic Fractions

2.1 Common Denominator

To add or subtract two fractions, they must first be numerically adjusted to have a common denominator.

With this in mind;

Simplify the following expression;

$$\frac{5(x + 3)}{7} + \frac{2(x + 1)}{3}$$

[3 marks]

Note

The working is easier if care is taken to get the LCM of the denominators given.

In the example, above, $\text{LCM} \{ 3, 7 \} = 21$ so both denominators were adjusted to be 21. As 3 and 7 are co-prime (no factors in common) the LCM was just 3×7 .

BUT ... The given denominators will not always be co-prime !

State the values of (i) $\text{LCM} \{ 5, 6 \}$

(ii) $\text{LCM} \{ 3, 9 \}$

(iii) $\text{LCM} \{ 8, 12 \}$

[3 marks]

2.2 Exercise

Marks Available : 35

Question 1

Express as a single fraction

(i) $\frac{2(3x + 7)}{5} + \frac{3(2x + 1)}{2}$

[3 marks]

(ii) $\frac{2(5x + 4)}{3} + \frac{3(3x - 2)}{4}$

[3 marks]

(iii) $\frac{4 (x + 1)}{7} + x$

Hint: $x = \frac{x}{1}$

[3 marks]

(iv) $\frac{8 (2x + 1)}{3} + \frac{5 (3x + 2)}{6}$

Hint: $\text{LCM} \{ 3, 6 \} = 6$

[3 marks]

(v) $\frac{2 (4x + 5)}{3} - \frac{5 (2x - 3)}{4}$

Careful: *double minus* !

[3 marks]

(vi) $\frac{3}{4} - \frac{3 - 2x}{6}$

Nasty !

[3 marks]

Question 2

GCSE Examination Question from June 2010, 4H, Q11 (Edexcel)

Simplify fully, $\frac{x}{6} + \frac{3x}{4}$

[3 marks]

Question 3

GCSE Examination Question from May 2008, 4H, Q23 (a)

Simplify, $\frac{x^2 - 9}{x^2 + 3x}$

[3 marks]

Question 4

GCSE Examination Question from November 2006, 4H, Q23

Simplify fully, $\frac{2x^2 - 5x - 12}{4x^2 - 9}$

[3 marks]

Question 5

GCSE Examination Question from May 2009, 3H, Q18

Simplify fully

$$\frac{5x^2 + 14x - 3}{50x^2 - 2}$$

[4 marks]

Question 6

GCSE Examination Question from November 2010, 4H, Q22 (Edexcel)

Simplify fully, $1 + \frac{x^2 + x - 6}{(x + 4)(x - 2)}$

[4 marks]

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2.3 Homework

GCSE Mathematics Algebraic Fractions

Marks Available: 14

Question 1

Express as a single fraction, $\frac{5(4x + 1)}{2} + \frac{3(2x + 3)}{7}$

[3 marks]

Question 2

Express as a single fraction; $\frac{2}{3} - \frac{2 - 3x}{6}$

[3 marks]

Question 3

GCSE Examination Question from May 2007, 4H, Q12 (Edexcel)

Simplify

(a) $\frac{a^3 \times a^4}{a^2}$

[2 marks]

(b) $(\sqrt{x})^6$

[1 mark]

(c) $\frac{3(x+1)^2}{6(x+1)}$

[2 marks]

Question 4

GCSE Examination Question from January 2017, 3H, Q17 (Edexcel)

Simplify fully, $\frac{4x^2 - 25}{6x^2 + 13x - 5}$

[3 marks]

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2.4 Answers

GCSE Mathematics Algebraic Fractions

2.4.1 Solutions (2.1 Common Denominator)

$$\frac{29x + 59}{21}$$

[3 marks]

- (i) LCM { 5, 6 } = 30
(ii) LCM { 3, 9 } = 9
(iii) LCM { 8, 12 } = 24

[3 marks]

2.4.2 Solutions (2.2 Exercise)

Answer 1

(i) $\frac{42x + 43}{10}$

(ii) $\frac{67x + 14}{12}$

(iii) $\frac{11x + 4}{7}$

(iv) $\frac{47x + 26}{6}$

(v) $\frac{2x + 85}{12}$

(vi) $\frac{3 + 4x}{12}$

[18 marks]

Answer 2

$$\frac{11x}{12}$$

Answer 3

$$\frac{x - 3}{x}$$

Answer 4

$$\frac{x - 4}{2x - 3}$$

[3, 3, 3 marks]

Answer 5

$$\frac{x + 3}{2 (5x + 1)}$$

Answer 6

$$\frac{2x + 7}{x + 4}$$

[4, 4 marks]

2.4.3 Solutions (2.3 Homework)

GCSE Mathematics Algebraic Fractions

Answer 1

$$\frac{152x + 53}{14}$$

Answer 2

$$\frac{3x + 2}{6}$$

[3, 3 marks]

Answer 3

(a) a^5

(b) x^3

(c) $0.5 (x + 1)$

[2, 1, 2 marks]

Answer 4

$$\frac{2x - 5}{3x - 1}$$

[3 marks]

Lesson 3

GCSE Mathematics Algebraic Fractions

3.1 Algebra in the Denominator

As previously observed, to add or subtract two fractions, they must first be numerically adjusted to have a common denominator. When the denominator is algebraic, rather than numeric, the adjustment required is also algebraic.

3.2 Example

Simplify the following expression;

$$\frac{3}{(x + 1)} + \frac{5}{(x + 2)}$$

[4 marks]

3.3 Try

Try to simplify the following expression.

Once done, take a look at the answer, over the page.

$$\frac{7}{(x + 4)} + \frac{3}{(x + 3)}$$

[4 marks]

3.4 Try Answer

$$\begin{aligned} & \frac{7}{(x+4)} + \frac{3}{(x+3)} \\ = & \frac{(x+3)}{(x+3)} \times \frac{7}{(x+4)} + \frac{3}{(x+3)} \times \frac{(x+4)}{(x+4)} \\ = & \frac{7(x+3) + 3(x+4)}{(x+3)(x+4)} \\ = & \frac{7x + 21 + 3x + 12}{(x+3)(x+4)} \\ = & \frac{10x + 33}{(x+3)(x+4)} \end{aligned}$$

Note : A mathematician would not expand the brackets in the concluding denominator
[4 marks]

3.5 Exercise

Marks Available: 64

Question 1

Simplify the following expression;

$$\frac{4}{(x+5)} + \frac{3}{(x+7)}$$

[4 marks]

Question 2

Simplify the following expression;

$$\frac{5}{(2x + 3)} + \frac{3}{(x + 5)}$$

[4 marks]

Question 3

Show that;

$$\frac{5}{(x + 4)} - \frac{3}{(x + 8)} = \frac{2(x + 14)}{(x + 4)(x + 8)}$$

[4 marks]

Question 4

Show that,

$$\frac{8}{(x+4)} + \frac{3}{(x+1)} = \frac{11x+20}{(x+4)(x+1)}$$

[4 marks]

Question 5

Simplify fully the following expression;

$$\frac{x}{x+2} + \frac{1}{x}$$

[4 marks]

Question 6

GCSE Examination Question from January 2007, 3H, Q16 (Edexcel)

Simplify

(a) $\frac{x^2 - 3x}{2x - 6}$

[3 marks]

(b) $\frac{2}{x - 1} - \frac{3}{x}$

[3 marks]

Question 7

Show that

$$\frac{4}{x} + \frac{x}{x + 1} = \frac{(x + 2)^2}{x(x + 1)}$$

[4 marks]

Question 8

GCSE Examination Question from January 2012, 3H, Q20 (Edexcel)

Simplify fully

$$\frac{4}{x} + \frac{3}{2 - x}$$

[3 marks]

Question 9

GCSE Examination Question from November 2009, 3H, Q19 (Edexcel)

(a) Simplify, $\frac{x^2}{x^2 - 2x}$

[2 marks]

(b) Simplify, $\frac{2}{2x - 1} - \frac{1}{x + 1}$

[4 marks]

Question 10

Expand the brackets;

$$(x - 7)(x + 5)$$

[2 marks]

Question 11

Solve these equations without using a calculator;

(i) $8x + 1 = 5$

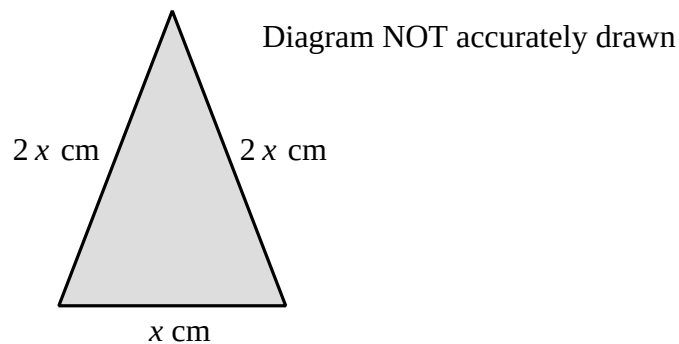
(ii) $3 + 5x = 17$

[2, 2 marks]

Question 12

GCSE Examination Question from November 2007, 3H, Q3 (Edexcel)

A triangle has two equal sides of length $2x$ cm and one side of length x cm.



The perimeter of this triangle is 12 cm.

(i) Use this information to write down an equation in x .

(ii) Solve your equation to find the value of x .

[3 marks]

Question 13

Expand the brackets

$$(5x + 8)(3x + 7)$$

[2 marks]

Question 14

A rectangle has a length, in cm, of $x + 7$ and a height of 6 cm.

It has a perimeter is 32 cm.

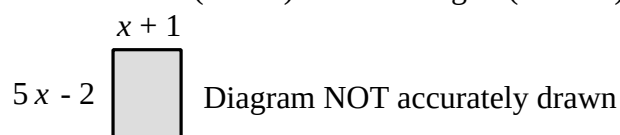
- (i) Calculate the value of x .
- (ii) Draw the rectangle full size.
- (iii) What is the area of the rectangle ?

[4 marks]

Question 15

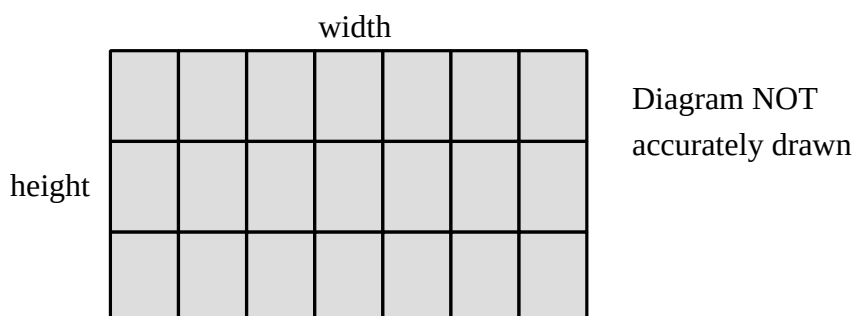
Expand the brackets

$$(2x + 9)(5x - 6)$$

[2 marks]**Question 16***GCSE Examination Question from June 2010, 4H, Q7 (Edexcel)*Rectangular tiles have width $(x + 1)$ cm and height $(5x - 2)$ cm.

Some of these tiles are used to form a large rectangle.

The large rectangle is 7 tiles wide and 3 tiles high.



The perimeter of the large rectangle is 68 cm.

(a) Write down an equation in x .**[3 marks]****(b)** Solve this equation to find the value of x .**[3 marks]**

Question 17

Expand the brackets

$$(3x + 5)^2$$

[2 marks]

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3.5 Answers

GCSE Mathematics Algebraic Fractions

3.5.1 Solutions (3.2 Example)

$$\begin{aligned} & \frac{3}{(x+1)} + \frac{5}{(x+2)} \\ = & \frac{(x+2)}{(x+2)} \times \frac{3}{(x+1)} + \frac{5}{(x+2)} \times \frac{(x+1)}{(x+1)} \\ = & \frac{3(x+2) + 5(x+1)}{(x+1)(x+2)} \\ = & \frac{3x+6+5x+5}{(x+1)(x+2)} \\ = & \frac{8x+11}{(x+1)(x+2)} \end{aligned}$$

[4 marks]

3.5.2 Solutions (3.4 Exercise)

Answer 1

$$\frac{7x+43}{(x+5)(x+7)}$$

Answer 2

$$\frac{11x+34}{(2x+3)(x+5)}$$

Answer 5

$$\frac{x^2+x+2}{x(x+2)}$$

[4, 4, 4 marks]

Answer 6

$$\frac{x}{2} \quad \frac{3-x}{x(x-1)}$$

Answer 8

$$\frac{8-x}{x(2-x)}$$

Answer 9

$$\frac{x}{x-2} \quad \frac{3}{(x+1)(2x-1)}$$

[3, 3, 3, 2, 4 marks]

Answer 10

$$x^2 - 2x - 35$$

Answer 11

- (i) 0.5
(ii) 2.8

Answer 12

- (i) $5x = 12$
(ii) $x = 2.4$ cm

[2, 4, 3 marks]

Answer 13

$$15x^2 + 59x + 56$$

Answer 14

- (i) 3 cm
(ii) Rectangle 6×10 cm
(iii) 60 cm^2

Answer 15

$$10x^2 + 33x - 54$$

[2, 4, 2 marks]

Answer 16

- (a) $68 = 14(x+1) + 6(5x-2)$
 $68 = 44x + 2$
(b) $66 = 44x$
 $x = 1.5$

Answer 17

$$9x^2 + 30x + 25$$

[6, 2 marks]

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Lesson 4

GCSE Mathematics Algebraic Fractions

4.1 Equations involving fractions

In our work with adding and subtracting fractions we have seen that the LCM, the lowest common multiple, is an essential ingredient. The following example illustrates another situation where the key to an efficient solution is the LCM.

4.2 Numerical Denominators Example

Solve, $\frac{x-1}{3} = \frac{x+2}{4}$

The key solution technique is to multiply through by the LCM of the denominators

[3 marks]

4.3 Algebraic Denominators Example

Solve, $\frac{x}{x+2} - \frac{1}{x} = 1$

The key solution technique is to multiply through by the LCM of the denominators

[4 marks]

4.4 Exercise

Marks Available:

Question 1

Solve, $\frac{2x + 1}{4} = \frac{x + 1}{3}$

[3 marks]

Question 2

Solve, $\frac{3x - 2}{3} = \frac{x - 3}{5}$

[3 marks]

Question 3

Solve, $\frac{2(x - 3)}{3} = \frac{3(x + 4)}{8}$

[3 marks]

Question 4

Solve, $\frac{x + 3}{7} - \frac{3(x - 2)}{14} = 1$

[3 marks]

Question 5

GCSE Examination Question from May 2012, 3H, Q9 (Edexcel)

- (a) Solve, $3(2x - 1) = 6$
 Show clear algebraic working.

[3 marks]

- (b) Solve, $\frac{2y + 1}{3} = \frac{y - 2}{4}$
 Show clear algebraic working.

[4 marks]

Question 6

GCSE Examination Question from June 2011, 3H, Q13 (Edexcel)

Solve, $\frac{2x - 1}{4} + \frac{x - 1}{5} = 2$

[4 marks]

Question 7

Solve, $\frac{x}{x - 1} + \frac{1}{x} = 1$

[4 marks]

Question 8

Simplify, $\frac{5x + 15}{x + 3}$

[2 marks]

Question 9

Beginning “LHS = ” show that, $\frac{x - 2}{3} - \frac{x}{4} = \frac{x - 8}{12}$

[3 marks]

Question 10

Solve the equation, $\frac{x}{3} = \frac{x - 2}{5}$

[3 marks]

Question 11

Find the two solutions to the equation, $\frac{4}{x-1} - \frac{2}{x+2} = 1$

[5 marks]

Question 12

GCSE Examination Question from May 2012, 4H, Q21 (Edexcel)

Solve, $\frac{5}{(x+2)} + \frac{9}{(x-2)} = 2$

Show clear algebraic working.

[5 marks]

Question 13

Beginning “LHS = ”, show that, $\frac{3}{x-1} - \frac{2}{x+1} = \frac{x+8}{x^2-1}$

[5 marks]

Question 14

Find both solutions to the equation, $\frac{3}{x-1} - \frac{8}{x+2} = 1$

[5 marks]

Question 15

GCSE Examination Question from November 2008, 3H, Q18 (Edexcel)

Simplify fully, $\frac{2}{x+2} + \frac{x}{x^2+5x+6}$

[5 marks]

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4.5 Answers

GCSE Mathematics Algebraic Fractions

4.5.1 Solutions (4.2 Numerical Denominators Example)

$$\begin{aligned}\frac{x-1}{3} &= \frac{x+2}{4} && \text{LCM is 12} \\ \frac{12(x-1)}{3} &= \frac{12(x+2)}{4} \\ 4(x-1) &= 3(x+2) \\ 4x-4 &= 3x+6 \\ x &= 10\end{aligned}$$

[3 marks]

4.5.2 Solutions (4.3 Algebraic denominators Example)

$$\begin{aligned}\frac{x}{x+2} - \frac{1}{x} &= 1 && \text{LCM is } x(x+2) \\ \frac{x(x+2)x}{x+2} - \frac{x(x+2)}{x} &= x(x+2) \\ x^2 - (x+2) &= x^2 + 2x \\ -x - 2 &= 2x \\ -2 &= 3x \\ x &= -\frac{2}{3}\end{aligned}$$

[4 marks]

4.5.3 Solutions (4.4 Exercise)

Answer 1

$$\frac{1}{2}$$

Answer 2

$$\frac{1}{12}$$

Answer 3

$$12$$

Answer 4

$$-2$$

[3, 3, 3, 3 marks]

Answer 5

(a) 1.5

Answer 6

3.5

Answer 7

0.5

Answer 8

5

(b) -2

[7, 4, 4, 2 marks]

Answer 10

$$-3$$

Answer 11

$$-3, 4$$

Answer 12

$$-1, 8$$

Answer 14

$$-8, 2$$

[3, 5, 5, 5 marks]

Answer 15

$$\frac{3}{x+3}$$

[5 marks]

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Lesson 5

GCSE Mathematics Algebraic Fractions

5.1 Difference Of Two Squares

The Theory

$$x^2 - y^2 = (x + y)(x - y)$$

Example 1

Factorise completely, $2x^2 - 18$

[2 marks]

Example 2

Factorise

Factorise completely, $4x^3 - x$

[2 marks]

Example 3

Simplify, $\frac{2x^2 + 5x + 3}{4x^2 - 9} \quad x \neq \pm \frac{3}{2}$

[3 marks]

Example 4

Factorise completely, $(3x + 1)^2 - (x + 2)^2$

[3 marks]

5.2 Exercise

Marks Available : 45

Question 1

(i) Factorise, $x^2 - 36$

[1 mark]

(ii) Hence, or otherwise, simplify $\frac{x^2 - 36}{x + 6}$ $x \neq -6$

[2 marks]

Question 2

(i) Factorise, $x^2 - 81$

[1 mark]

(ii) Hence, or otherwise, solve $\frac{x^2 - 81}{x - 9} = 14$ $x \neq 9$

[2 marks]

Question 3

Solve $\frac{x + 1}{x^2 - 1} = 5$ $x \neq \pm 1$

[2 marks]

Question 4

(i) Factorise $9x^2 - 16$

[1 mark]

(ii) Hence, or otherwise, simplify $\frac{3x^2 + 7x + 4}{9x^2 - 16}$ $x \neq \pm \frac{4}{3}$

[3 marks]

Question 5

Factorise, $2x^2 - 8$

[2 marks]

Question 6

Factorise $5x^3 - 20x$

[2 marks]

Question 7

Solve $\frac{x^2 - 1}{(x - 1)^2} = 5$

[3 marks]

Question 8

(i) Factorise $(5x + 3)^2 - (2x + 1)^2$

[3 marks]

(ii) Hence, or otherwise, solve $\frac{(5x + 3)^2 - (2x + 1)^2}{(3x + 2)} = 32$

[3 marks]

Question 9

GCSE Examination Question, 9th June 2016, Paper 4H, Q14 (Edexcel)

Simplify $\frac{x^2 - 25}{2x^2 - 9x - 5}$

[3 marks]

Question 10

GCSE Examination Question, January 2017, Paper 3H (R), Q16 (a) (Edexcel)

Solve $\frac{3x + 1}{5} - \frac{x - 4}{3} = 2$

Show clear algebraic working

[3 marks]

Question 11

GCSE Examination Question, January 2017, Paper 4H (R), Q15 (c) (Edexcel)

Simplify fully $\frac{3}{x + 1} - \frac{2}{x - 1}$

[3 marks]

Question 12

GCSE Examination Question, January 2018, Paper 3H, Q21 (Edexcel)

Factorise completely, $(10a - b)^2 - (2a - 5b)^2$

[3 marks]

Question 13

GCSE Examination Question, January 2018, Paper 4H, Q11(d) (Edexcel)

Factorise completely $3x^2 - 75y^2$

[2 marks]

Question 14

GCSE Examination Question, January 2018, Paper 3HR, Q17 (Edexcel)

(a) Show that $\frac{x + 1}{2x + 1} - \frac{1}{(2x + 1)(x + 1)} = \frac{x^2 + 2x}{(2x + 1)(x + 1)}$

[2 marks]

(b) Hence solve

$$\frac{x + 1}{2x + 1} - \frac{1}{(2x + 1)(x + 1)} = \frac{1}{(2x + 1)(x + 1)}$$

Hint: The equation $ax^2 + bx + c = 0$ has solutions given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

[4 marks]

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5.3 Answers

GCSE Mathematics Algebraic Fractions

5.3.1 Solutions to 5.1 Examples

Answer 1

$$\begin{aligned}2x^2 - 18 &= 2(x^2 - 9) \\&= 2(x^2 - 3^2) \\&= 2(x + 3)(x - 3)\end{aligned}$$

[2 marks]

Answer 2

$$\begin{aligned}4x^3 - x &= x(4x^2 - 1) \\&= x((2x)^2 - 1^2) \\&= x(2x + 1)(2x - 1)\end{aligned}$$

[2 marks]

Answer 3

$$\begin{aligned}\frac{2x^2 + 5x + 3}{4x^2 - 9} &= \frac{2x^2 + 5x + 3}{(2x)^2 - 3^2} \\&= \frac{(2x + 3)(x + 1)}{(2x + 3)(2x - 3)} \\&= \frac{x + 1}{2x - 3}\end{aligned}$$

[3 marks]

Answer 4

$$\begin{aligned}(3x + 1)^2 - (x + 2)^2 &= (3x + 1 + x + 2)(3x + 1 - (x + 2)) \\&= (4x + 3)(2x - 1)\end{aligned}$$

[3 marks]

5.3.2 Solutions to 5.2 Exercise

Answer 1

(i)
$$\begin{aligned}x^2 - 36 &= x^2 - 6^2 \\&= (x + 6)(x - 6)\end{aligned}$$

[1 mark]

(ii)
$$\begin{aligned}\frac{x^2 - 36}{x + 6} &= \frac{(x + 6)(x - 6)}{x + 6} \\&= x - 6\end{aligned}$$

[2 marks]

Answer 2

$$\begin{aligned} \text{(i)} \quad x^2 - 81 &= x^2 - 9^2 \\ &= (x + 9)(x - 9) \end{aligned}$$

[1 mark]

$$\begin{aligned} \text{(ii)} \quad \frac{x^2 - 81}{x - 9} &= 14 \\ \frac{(x + 9)(x - 9)}{x - 9} &= 14 \\ x + 9 &= 14 \\ x &= 5 \end{aligned}$$

[2 marks]

Answer 3

$$\begin{aligned} \frac{x + 1}{x^2 - 1} &= 5 \\ \frac{x + 1}{(x + 1)(x - 1)} &= 5 \\ \frac{1}{x - 1} &= 5 \\ 1 &= 5(x - 1) \\ \frac{1}{5} &= x - 1 \\ x &= \frac{6}{5} \end{aligned}$$

[2 marks]

Answer 4

$$\begin{aligned} \text{(i)} \quad 9x^2 - 16 &= (3x)^2 - 4^2 \\ &= (3x + 4)(3x - 4) \end{aligned}$$

[1 mark]

$$\begin{aligned} \text{(ii)} \quad \frac{3x^2 + 7x + 4}{9x^2 - 16} &= \frac{(3x + 4)(x + 1)}{(3x + 4)(3x - 4)} \\ &= \frac{x + 1}{3x - 4} \end{aligned}$$

[3 marks]

Answer 5

$$\begin{aligned} 2x^2 - 8 &= 2(x^2 - 4) \\ &= 2(x^2 - 2^2) \\ &= 2(x + 2)(x - 2) \end{aligned}$$

[2 marks]

Answer 6

$$\begin{aligned}
 5x^3 - 20x &= 5x(x^2 - 4) \\
 &= 5x(x^2 - 2^2) \\
 &= 5x(x + 2)(x - 2)
 \end{aligned}$$

[2 marks]**Answer 7**

$$\begin{aligned}
 \frac{x^2 - 1}{(x - 1)^2} &= 5 \\
 \frac{(x + 1)(x - 1)}{(x - 1)(x - 1)} &= 5 \\
 x + 1 &= 5(x - 1) \\
 x + 1 &= 5x - 5 \\
 6 &= 4x \\
 x &= \frac{6}{4} \\
 &= 1.5
 \end{aligned}$$

[3 marks]**Answer 8**

$$\begin{aligned}
 \text{(i)} \quad (5x + 3)^2 - (2x + 1)^2 &= (5x + 3 + 2x + 1)(5x + 3 - 2x - 1) \\
 &= (7x + 4)(3x + 2)
 \end{aligned}$$

[3 marks]

$$\begin{aligned}
 \text{(ii)} \quad \frac{(5x + 3)^2 - (2x + 1)^2}{(3x + 2)} &= 32 \\
 \frac{(7x + 4)(3x + 2)}{(3x + 2)} &= 32 \\
 7x + 4 &= 32 \\
 7x &= 28 \\
 x &= 4
 \end{aligned}$$

[3 marks]**Answer 9**

$$\begin{aligned}
 \frac{x^2 - 25}{2x^2 - 9x - 5} &= \frac{(x + 5)(x - 5)}{(2x + 1)(x - 5)} \\
 &= \frac{x + 5}{2x + 1}
 \end{aligned}$$

[3 marks]

Answer 10

$$\frac{3x + 1}{5} - \frac{x - 4}{3} = 2$$

Mukltiply through by 15

$$3 (3x + 1) - 5 (x - 4) = 30$$

$$9x + 3 - 5x + 20 = 30$$

$$4x = 7$$

$$x = \frac{7}{4}$$

[3 marks]

Answer 11

$$\frac{3}{x + 1} - \frac{2}{x - 1}$$

make a common denominator

$$\begin{aligned} \frac{3}{x + 1} - \frac{2}{x - 1} &= \frac{3}{x + 1} \times \frac{x - 1}{x - 1} - \frac{2}{x - 1} \times \frac{x + 1}{x + 1} \\ &= \frac{3x - 3 - 2x - 2}{(x + 1)(x - 1)} \\ &= \frac{x - 5}{(x + 1)(x - 1)} \end{aligned}$$

[3 marks]

Answer 12

$$\begin{aligned} (10a - b)^2 - (2a - 5b)^2 &= (10a - b + 2a - 5b)(10a - b - 2a + 5b) \\ &= (12a - 6b)(8a + 4b) \\ &= 6 \times (2a - b) \times 4 \times (2a + b) \\ &= 24(2a - b)(2a + b) \end{aligned}$$

[3 marks]

Answer 13

$$\begin{aligned} 3x^2 - 75y^2 &= 3(x^2 - 25y^2) \\ &= 3(x^2 - (5y)^2) \\ &= 3(x + 5y)(x - 5y) \end{aligned}$$

[2 marks]

Answer 14**(a)**

$$\begin{aligned}
\text{LHS} &= \frac{x+1}{2x+1} - \frac{1}{(2x+1)(x+1)} \\
&= \frac{x+1}{2x+1} \times \frac{x+1}{x+1} - \frac{1}{(2x+1)(x+1)} \\
&= \frac{x^2 + 2x + 1 - 1}{(2x+1)(x+1)} \\
&= \frac{x^2 + 2x}{(2x+1)(x+1)} \\
&= \text{RHS} \quad \square
\end{aligned}$$

[2 marks]**(b)**

$$\begin{aligned}
\frac{x+1}{2x+1} - \frac{1}{(2x+1)(x+1)} &= \frac{1}{(2x+1)(x+1)} \\
\frac{x^2 + 2x}{(2x+1)(x+1)} &= \frac{1}{(2x+1)(x+1)} \quad \text{using part (a)} \\
\therefore x^2 + 2x &= 1 \\
x^2 + 2x - 1 &= 0
\end{aligned}$$

Using the formula to solve a quadratic equation, yields,

$$\begin{aligned}
x &= \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-1)}}{2 \times 1} \\
&= \frac{-2 \pm \sqrt{8}}{2} \\
&= -1 \pm \sqrt{2}
\end{aligned}$$

[4 marks]

Lesson 6

GCSE Mathematics Algebraic Fractions

6.1 Revision for The Test

Marks Available: 60

Question 1

GCSE Examination Question from November 2008, 3H, Q2 (Edexcel)

(a) Factorise $7p - 21$

[1 mark]

(b) Solve $4(x + 5) = 12$

You must show sufficient working.

[2 marks]

Question 2

GCSE Examination Question from June 2011, 3H, Q5 (Edexcel)

Show that $\frac{5}{6} - \frac{3}{4} = \frac{1}{12}$

[2 marks]

Question 3

GCSE Examination Question from November 2008, 3H, Q8 (Edexcel)

(a) Simplify

(i) $p^5 \times p$

(ii) $\frac{q^5}{q^3}$

[2 marks]

(b) Expand and simplify $3(4x - 1) - 4(2x - 3)$

[2 marks]

(c) Expand and simplify $(y + 3)(y + 5)$

[2 marks]

Question 4

GCSE Examination Question from May 2012, 3H, Q3 (Edexcel)

- (a) Write as a single power of 2,

$$2^3 \times 2^6$$

[1 mark]

- (b) Write as a single power of 3,

$$\frac{3^9}{3^4}$$

[1 mark]

- (c) $\frac{5^n}{5^4 \times 5^6} = 5^3$
Find the value of n .

[2 marks]

Question 5

GCSE Examination Question from November 2008, 3H, Q14 (Edexcel)

- (a) Factorise completely $9ab - 12b^2$

[2 marks]

- (b) Simplify $(2ab^2)^3$

[2 marks]

Question 6

GCSE Examination Question from May 2012, 3H, Q17 (Edexcel)

(a) Simplify $(3a^2b)^4$

[2 marks]

(b) Simplify $(9c^8)^{\frac{1}{2}}$

[2 marks]

Question 7

(a) Factorise $x^2 - 8x + 12$

[2 marks]

(b) Factorise $x^2 - 81$

[1 mark]

Question 8

Simplify the following algebraic expressions by first factorising the quadratics:

$$\frac{x^2 + 3x - 88}{x^2 - 2x - 48}$$

[2 marks]

Question 9

Simplify fully $\frac{x^2 + 5x}{x^2 - 25}$

[3 marks]

Question 10

Express as a single fraction

(i) $\frac{2 (2x + 7)}{5} + \frac{5 (4x + 1)}{3}$

[3 marks]

(ii) $\frac{2 (5x + 4)}{3} - \frac{3 (3x - 2)}{4}$

[3 marks]

Question 11

Simplify the following expression;

$$\frac{7}{(x + 4)} + \frac{5}{(x + 3)}$$

[3 marks]

Question 12

Beginning "LHS = " show that;

$$\frac{3}{(x + 6)} + \frac{5}{(x - 10)} = \frac{8x}{(x + 6)(x - 10)}$$

[3 marks]

Question 13

(a) Simplify, $\frac{x^2}{x^2 - 5x}$

[2 marks]

(b) Simplify $\frac{6}{2x - 9} - \frac{2}{2x + 3}$

[4 marks]

Question 14

Solve

$$\frac{x - 3}{2} + \frac{x - 5}{3} = 6$$

[4 marks]

Question 15

Solve the equation;

$$\frac{x}{2} = \frac{2(x - 2)}{7}$$

[3 marks]

Question 16

Find the two solutions to the equation;

$$\frac{x}{x + 3} = \frac{10}{x - 3}$$

[4 marks]

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6.2 Answers (Revision for The Test)

GCSE Mathematics Algebraic Fractions

Answer 1

(a) $7(p - 3)$ (b) -2

[1, 2 marks]

Answer 2

$$\begin{aligned}\text{LHS} &= \frac{5}{6} - \frac{3}{4} & \text{LCM}\{6, 4\} &= 12 \\ &= \frac{2}{2} \times \frac{5}{6} - \frac{3}{4} \times \frac{3}{3} \\ &= \frac{10}{12} - \frac{9}{12} \\ &= \frac{1}{12} \\ &= \text{RHS} \quad \square\end{aligned}$$

[2 marks]

Answer 3

(a) (i) p^6 (ii) q^2

[1, 1 marks]

(b) $4x + 9$

[2 marks]

(c) $y^2 + 8y + 15$

[2 marks]

Answer 4

(a) 2^9 (b) 3^5 (c) $n = 13$

[1, 1, 2 marks]

Answer 5

(a) $3b(3a - 4b)$ (b) $8a^3b^6$

[2, 2 marks]

Answer 6

(a) $81a^8b^4$ (b) $3c^4$

[2, 2 marks]

Answer 7

(a) $(x - 2)(x - 6)$ (b) $(x + 9)(x - 9)$

[2, 1 marks]

Answer 8

$$\frac{x + 11}{x + 6}$$

[2 marks]

Answer 9

$$\frac{x}{x-5}$$

[3 marks]

Answer 10

$$(i) \quad \frac{112x + 67}{15}$$

$$(ii) \quad \frac{13x + 50}{12}$$

[3, 3 marks]

Answer 11

$$\frac{12x + 41}{(x+4)(x+3)}$$

[3 marks]

Answer 12

$$\begin{aligned} \text{LHS} &= \frac{3}{(x+6)} + \frac{5}{(x-10)} \\ &= \frac{(x-10)}{(x-10)} \times \frac{3}{(x+6)} + \frac{5}{(x-10)} \times \frac{(x+6)}{(x+6)} \\ &= \frac{3x - 30 + 5x + 30}{(x+6)(x-10)} \\ &= \frac{8x}{(x+6)(x-10)} \\ &= \text{RHS} \end{aligned}$$

□

[3 marks]

Answer 13

$$(a) \quad \frac{x}{x-5}$$

$$(b) \quad \frac{8x + 36}{(2x+3)(2x-9)}$$

[2, 4 marks]

Answer 14

$$x = 11$$

[4 marks]

Answer 15

$$x = -\frac{8}{3} \quad (-2.67)$$

[3 marks]

Answer 16

$$x = -2, 15$$

[4 marks]

Lesson 7

GCSE Mathematics Algebraic Fractions

7.1 The Test

Marks Available: 75

Question 1

GCSE Examination Question from November 2007, 4H, Q2 (Edexcel)

(a) Factorise $5x - 20$

[1 mark]

(b) Factorise $y^2 + 6y$

[2 marks]

Question 2

GCSE Examination Question from May 2007, 3H, Q9 (Edexcel)

(a) Solve $5x - 4 = 2x + 7$

[2 marks]

(b) Solve $\frac{7 - 2y}{4} = 2y + 3$

[4 marks]

Question 3

GCSE Examination Question from January 2017, 4H, Q8 (Edexcel)

(a) Factorise $n^2 + 8n$

[2 marks]

(b) Expand and simplify $3(2x - 5) - 4(x + 3)$

[2 marks]

(c) Expand and simplify $(y + 7)(y + 2)$

[2 marks]

Question 4

GCSE Examination Question from May 2008, 4H, Q6 (Edexcel)

Show that $\frac{2}{3} + \frac{1}{4} = \frac{11}{12}$

[2 marks]

Question 5

GCSE Examination Question from May 2007, 3H, Q5

(a) Simplify, leaving your answers in index form

(i) $7^5 \times 7^3$ (ii) $5^9 \div 5^3$

[2 marks]

(b) Solve $\frac{2^9 \times 2^4}{2^n} = 2^8$

[2 marks]

Question 6

GCSE Examination Question from May 2008, 3H, Q14 (Edexcel)

(a) Factorise $10y - 15$

[1 mark]

(b) Factorise completely $9p^2q + 12pq^2$

[2 marks]

(c) (i) Factorise $x^2 + 6x - 16$

(ii) Solve $x^2 + 6x - 16 = 0$

[3 marks]

Question 7

GCSE Examination Question from January 2017, 4H, Q15 (Edexcel)

(a) Simplify $\frac{5x^5y^6}{x^2y^4}$

[2 marks]

(b) Simplify $(2n^4)^3$

[2 marks]

Question 8

GCSE Examination Question from May 2006, 3H, Q13 (a) (c) (Edexcel)

(a) Expand and simplify $(3x - 5)(4x + 7)$

[2 marks]

(b) Simplify $(64y^6)^{\frac{2}{3}}$

[2 marks]

Question 9

Simplify the following algebraic expressions by first factorising the quadratics:

$$\frac{x^2 + 2x - 24}{x^2 - 3x - 54}$$

[2 marks]

Question 10

GCSE Examination Question from May 2004, 3H, Q16 (Edexcel)

Express this algebraic fraction as simply as possible

$$\frac{2x^2 - 3x - 20}{x^2 - 16}$$

[3 marks]

Question 11

Express as a single fraction

(i) $\frac{(3x + 7)}{5} + \frac{(7x - 4)}{3}$

[3 marks]

(ii) $\frac{5(4x + 1)}{2} - \frac{3(7x - 2)}{5}$

[3 marks]

Question 12

Simplify the following expression;

$$\frac{1}{(x + 4)} + \frac{4}{(x + 5)}$$

[3 marks]

Question 13

Beginning “LHS = ” show that;

$$\frac{8}{(x - 4)} + \frac{2}{(x + 6)} = \frac{10(x + 4)}{(x - 4)(x + 6)}$$

[3 marks]

Question 14

GCSE Examination Question from May 2006, 4H, Q4 (Edexcel)

Arul had x sweets.

Nikos had four times as many sweets as Arul.

(a) Write down an expression, in terms of x , for the number of sweets Nikos had.

[1 mark]

Nikos gave 6 of his sweets to Arul.

Now they both have the same number of sweets.

(b) Use this information to form an equation in x .

[2 marks]

(c) Solve your equation to find the number of sweets that Arul had at the start.

[2 marks]

Question 15

(a) Simplify $\frac{x^2 - 7x}{x^2}$

[2 marks]

(b) Simplify $\frac{2}{x + 1} - \frac{4}{2x + 3}$

[4 marks]

Question 16

Solve $\frac{x-3}{2} + \frac{x-2}{5} = 10$

[4 marks]

Question 17

GCSE Examination Question from January 2017, 4H, Q15 (Edexcel)

Solve

$$\frac{2}{5x-2} = \frac{3}{6x+1}$$

[4 marks]

Question 18

Find the two solutions to the equation;

$$\frac{x}{x+5} = \frac{2}{x-7}$$

[4 marks]

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7.2 Answers (The Test)

GCSE Mathematics Algebraic Fractions

Answer 1

(a) $5(x - 4)$

(b) $y(y + 6)$

Answer 2

(a) $\frac{11}{3}$

(b) -0.5

Answer 3

(a) $n(n + 8)$

(b) $2x - 27$

(c) $y^2 + 9y + 14$

[3, 6, 6 marks]

Answer 4

$$\begin{aligned}\text{LHS} &= \frac{2}{3} + \frac{1}{4} \\ &= \frac{4}{4} \times \frac{2}{3} + \frac{1}{4} \times \frac{3}{3} \\ &= \frac{8}{12} + \frac{3}{12} \\ &= \frac{11}{12} \\ &= \text{RHS}\end{aligned}$$

$$\text{LCM}\{3, 4\} = 12$$

□

[2 marks]

Answer 5

(a) (i) 7^8

(b) $n = 5$

(ii) 5^6

[2, 2 marks]

Answer 6

(a) $5(2y - 3)$

(b) $3pq(3p + 4q)$

[1, 2 marks]

(c) (i) $(x + 8)(x - 2)$

(ii) $x = -8$ or $x = 2$

[3 marks]

Answer 7

(a) $5x^3y^2$

(b) $8n^{12}$

[2, 2 marks]

Answer 8

(a) $12x^2 + x - 35$

(b) $16y^4$

[2, 2 marks]

Answer 9

$$\frac{x - 4}{x - 9}$$

Answer 10

$$\frac{2x + 5}{x + 4}$$

[2, 3 marks]

Answer 11

(i) $\frac{44x + 1}{15}$

(ii) $\frac{58x + 37}{10}$

[3, 3 marks]

Answer 12

$$\frac{5x + 21}{(x + 5)(x + 4)}$$

[3 marks]

Answer 13

$$\begin{aligned}
\text{LHS} &= \frac{8}{(x - 6)} + \frac{2}{(x + 6)} \\
&= \frac{(x + 6)}{(x + 6)} \times \frac{8}{(x - 4)} + \frac{2}{(x + 6)} \times \frac{(x - 4)}{(x - 4)} \\
&= \frac{8x + 48 + 2x - 8}{(x + 6)(x - 4)} \\
&= \frac{10x + 40}{(x - 4)(x + 6)} \\
&= \frac{10(x + 4)}{(x - 4)(x + 6)} \\
&= \text{RHS}
\end{aligned}$$

□

[3 marks]

Answer 14

(a) $4x$

(b) $x + 6 = 4x - 6$

(c) $x = 4$

[1, 2, 2 marks]

Answer 15

(a) $\frac{x - 7}{x}$

(b) $\frac{2}{(2x + 3)(x + 1)}$

[2, 4 marks]

Answer 16

$x = 17$

Answer 17

$x = 2\frac{2}{3}$

Answer 18

$x = -1, 10$

[4, 4, 4 marks]