

4.3 Answers

A-level Statistics : Year 1 Partitioning Data

4.3.1 Solution to *Fortune Smiles on Lucky Lucy*

Data in order is

1p 1p 2p 2p 5p 10p 200p

- (i) Median : 7 items of data
 $7 / 2 = 3.5^{\text{th}}$
 Not an integer
 $\therefore$ Round up (ARU) to 4^{th} which gives the median as 2p

- (ii) Mean:

$$\begin{aligned}\mu &= \frac{1 + 1 + 2 + 2 + 5 + 10 + 200}{7} \\ &= \frac{221}{7} \\ &= 31.6 \text{ pence}\end{aligned}$$

- (iii) Standard deviation:

$$\begin{aligned}\sigma^2 &= \frac{1^2 + 1^2 + 2^2 + 2^2 + 5^2 + 10^2 + 200^2}{7} - \left(\frac{221}{7} \right)^2 \\ &= \frac{1 + 1 + 4 + 4 + 25 + 100 + 40000}{7} - \left(\frac{221}{7} \right)^2 \\ &= \frac{40135}{7} - \frac{48841}{49} \\ &= \frac{280945 - 48841}{49} \\ &= \frac{232104}{49} \\ \sigma &= 68.8 \text{ pence}\end{aligned}$$

- (iv) Skew :

$$\begin{aligned}\text{skew} &= \frac{3 (31.6 - 2)}{68.8} \\ &= 1.3\end{aligned}$$

- (v) Strong positive skew means that Lucy's finds are all piled up at the small coin values. The £2 find was an outlier (unusual)

4.3.2 Solutions to 4.2 Exercise

Answer 1

(a) mode = 56

(b)

For Q_1 : $\frac{27 \times 1}{4} = 6.75 \Rightarrow \text{ARU} \Rightarrow 7^{\text{th}}$ which is 35

For Q_2 : $\frac{27 \times 2}{4} = 13.5 \Rightarrow \text{ARU} \Rightarrow 14^{\text{th}}$ which is 52

For Q_3 : $\frac{27 \times 3}{4} = 20.25 \Rightarrow \text{ARU} \Rightarrow 21^{\text{st}}$ which is 60

(c)

$$\begin{aligned}\mu &= \frac{\sum x}{\sum f} \\&= \frac{1335}{27} \\&= \frac{445}{9} \quad (\text{exact}) \\&= 49.4\end{aligned}$$

$$\begin{aligned}\sigma &= \sqrt{\frac{\sum x^2}{\sum f} - \mu^2} \\&= \sqrt{\frac{71\,801}{27} - \left(\frac{445}{9}\right)^2} \\&= 14.6472936... \\&= 14.6\end{aligned}$$

(d)

$$\begin{aligned}\text{skew} &= \frac{\text{mean} - \text{mode}}{\text{standard deviation}} \\&= \frac{49.4 - 56}{14.6} \\&= -0.452 \quad \text{That is, negative skew} \quad \square\end{aligned}$$

(e) mean < median < mode because $49.4 < 52 < 56$
 $\therefore$ skew is negative

$Q_3 - Q_2 < Q_2 - Q_1$ because $60 - 52 < 52 - 35$
That is, $8 < 17$
 $\therefore$ skew is negative

Answer 2

For the Balmoral Hotel,

(a) mode = 50

(b)

$$\begin{aligned}\text{For } Q_1 : \frac{28 \times 1}{4} &= 7 \Rightarrow \text{Integer} \\ \Rightarrow \frac{7^{\text{th}} + 8^{\text{th}}}{2} &= \frac{45 + 45}{2} \text{ which is } 45\end{aligned}$$

$$\begin{aligned}\text{For } Q_2 : \frac{28 \times 2}{4} &= 14 \Rightarrow \text{Integer} \\ \Rightarrow \frac{14^{\text{th}} + 15^{\text{th}}}{2} &= \frac{50 + 51}{2} \text{ which is } 50.5\end{aligned}$$

$$\begin{aligned}\text{For } Q_3 : \frac{28 \times 3}{4} &= 21 \Rightarrow \text{Integer} \\ \Rightarrow \frac{21^{\text{st}} + 22^{\text{nd}}}{2} &= \frac{63 + 63}{2} \text{ which is } 63\end{aligned}$$

(c) (i)

$$\begin{aligned}\text{mean} &= \frac{1469}{28} \text{ (exact)} \\ &= 52.5\end{aligned}$$

(ii)

$$\begin{aligned}\text{standard deviation} &= \sqrt{\frac{81\,213}{28} - \left(\frac{1469}{28}\right)^2} \\ &= 12.164\,004\,69 \\ &= 12.2\end{aligned}$$

(d)

$$\begin{aligned}\text{skew} &= \frac{\text{mean} - \text{mode}}{\text{standard deviation}} \\ &= \frac{52.5 - 50}{12.2} \\ &= 0.2049\end{aligned}$$

(e) The average age at the Balmoral is higher than at the Abbey
(When using either mean, median or mode)

So older residents on average at the Balmoral Hotel

Lower standard deviation at Balmoral of 12.2 implies there is less variation in the ages of the residents there compared with the Addey, which has 12.7

Positive skew at Balmoral implies bunching at the younger end of their age range, whereas the opposite is true at the Abbey with its negative skew.

Answer 3**(a)** For Greenslax,

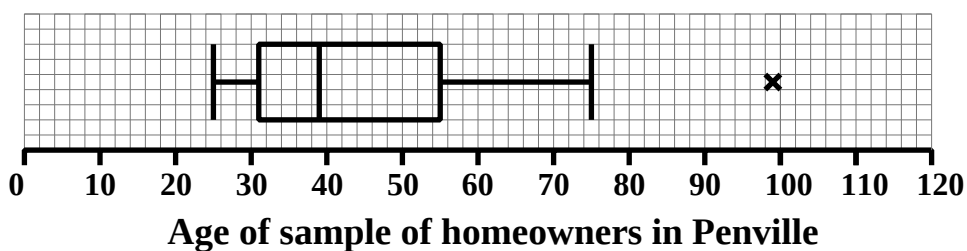
$$\text{For } Q_1 : \frac{35 \times 1}{4} = 8.75 \Rightarrow \text{ARU} \Rightarrow 9^{\text{th}} \text{ which gives } a = 44$$

$$\text{For } Q_3 : \frac{35 \times 3}{4} = 26.25 \Rightarrow \text{ARU} \Rightarrow 27^{\text{th}} \text{ which gives } b = 76$$

(b) For Penville, calculating the “cut offs”,

$$\text{high end} : 55 + 1.5 \times (55 - 31) = 91$$

$$\text{low end} : 31 - 1.5 \times (55 - 31) = -5$$



A whisker that goes up to the “cut off” at 91 is also acceptable

(c) From the information given the most convenient measure of skew to use is that based on the quartiles.

For Greenslax,

$$Q_3 - Q_2 = 76 - 64 = 12$$

$$Q_2 - Q_1 = 64 - 44 = 20$$

$$Q_3 - Q_2 < Q_2 - Q_1 \text{ indicates negative skew}$$

For Penville,

$$Q_3 - Q_2 = 55 - 39 = 16$$

$$Q_2 - Q_1 = 39 - 31 = 8$$

$$Q_3 - Q_2 > Q_2 - Q_1 \text{ indicates positive skew}$$