5.3 Answers

5.3.1 Solutions to the 5.1 Example

Time taken (minutes)	Number of runners
20 - 24	3
25 - 29	5
30 - 34	2

Make a fresh table to organise the process,

Time taken, t (minutes)	Number of runners, f	Mid-interval value, x	$f\,x$	x^2	$f\,x^2$
$19.5 \leqslant t < 24.5$	3	22	66	484	1452
$24.5 \leqslant t < 29.5$	5	27	135	729	3645
$29.5 \leqslant t < 34.5$	2	32	64	1024	2048
	10		265		7145

$$\mu = \frac{\sum fx}{\sum f} = \frac{265}{10} = 26.5 \quad \text{(exact)}$$

That is, 26 minutes 30 seconds

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$= \sqrt{\frac{7145}{10} - 26.5^2} \qquad \text{(using the exact value of the mean)}$$

$$= 3.5 \text{ minutes}$$

That is, 3 minutes 30 seconds

In examination questions, candidates are often told the numerical values of

$$\sum fx \qquad \text{and} \qquad \sum f\,x^2$$

5.3.2 Solutions to 5.2 Exercise

Answer 1

Time in Shop (minutes)	Number of shoppers
10 - 16	8
17 - 23	5
24 - 30	2

Make a fresh table to organise the process,

Time in shop, t (minutes)	Number of shoppers, f	Mid-interval value, x	$x f$	x^2	$x^2 f$
$9.5 \leqslant t < 16.5$	8	13	104	169	1352
$16.5 \leqslant t < 23.5$	5	20	100	400	2000
$23.5 \leqslant t < 30.5$	2	27	54	729	1458
	15		258		4810

$$\mu = \frac{\sum fx}{\sum f}$$

$$= \frac{258}{15}$$

$$= 17.2 \quad \text{(exact)}$$

That is, 17 minutes 12 seconds

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$= \sqrt{\frac{4810}{15} - 17.2^2} \qquad \text{(using the exact value of the mean)}$$

$$= 4.9826\ldots \text{ minutes}$$

That is, 4 minutes 59 seconds

Answer 2

$$\mu = \frac{\Sigma fx}{\Sigma f}$$
$$= \frac{3740}{80}$$
$$= 46.75 \quad \text{(exact)}$$

That is, 46 minutes 45 seconds

$$\sigma = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$
$$= \sqrt{\frac{183\,040}{80} - 46.75^2} \qquad \text{(using the exact value of the mean)}$$
$$= 10.12114124\ldots \text{ minutes}$$

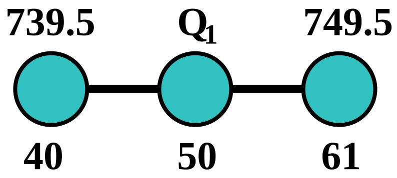
That is, 10 minutes 7 seconds

Answer 3

Lifetime, x (to nearest 100 hours)	Number of light-bulbs, f	Cumulative frequency
700 - 719	10	10
720 - 729	14	24
730 - 739	16	40
740 - 749	21	61
750 - 754	35	96
755 - 759	41	137
760 - 764	38	175
765 - 769	15	190
770 - 779	7	197
780 - 799	3	200

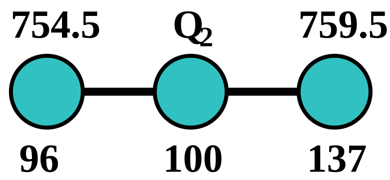
(i)

For Q_1 : $\dfrac{200 \times 1}{4} = 50$ which is in the class 740 - 749



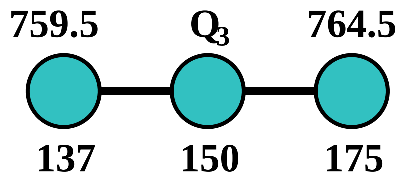
$$\frac{Q_1 - 739.5}{749.5 - 739.5} = \frac{50 - 40}{61 - 40} \Rightarrow Q_1 = 744.3 \text{ hundred hours}$$

For Q_2 : $\dfrac{200 \times 2}{4} = 100$ which is in the class 755 - 759



$$\frac{Q_2 - 754.5}{759.5 - 754.5} = \frac{100 - 96}{137 - 96} \Rightarrow Q_2 = 755.0 \text{ hundred hours}$$

For Q_3 : $\dfrac{200 \times 3}{4} = 150$ which is in the class 760 - 764



$$\frac{Q_3 - 759.5}{764.5 - 759.5.5} = \frac{150 - 137}{175 - 137} \Rightarrow Q_3 = 761.2 \text{ humdred hours}$$

(ii)

$$\mu = \frac{\sum fx}{\sum f}$$
$$= \frac{150\,232.5}{200}$$
$$= 751.1625 \quad \text{(exact)} \quad \text{hundred hours}$$

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$
$$= \sqrt{\frac{112\,899\,573.8}{200} - 751.1625^2} \qquad \text{(using the exact mean)}$$
$$= 15.8987 \quad \text{hundred hours}$$

(iii)

$$skew = \frac{3\ (mean - median)}{standard\ deviation}$$
$$= \frac{3\ (751.1625 - 755)}{15.8987}$$
$$= -0.724 \Rightarrow \textit{Negative skew}$$

(iv)

$$Q_3 - Q_2 = 761.2 - 755 = 6.2 \ : \ Q_2 - Q_1 = 755 - 744.3 = 10.7$$

$$\therefore\ Q_3 - Q_2 < Q_2 - Q_1 \Rightarrow \textit{Negative skew}$$

Thus, COMPATIBLE as both methods indicate negative skew

Answer 4

(a)

$$mean = \frac{2757}{12}$$
$$= 229.75 \ \ \text{(exact)}$$

$$standard\ deviation = \sqrt{\frac{724\,961}{12} - 229.75^2} \qquad \text{(using exact mean)}$$
$$= 87.34045$$

(b) The list needs to be put into order,

125, 160, 169, 171, 175, 186, 210, 243, 250, 258, 390, 420

For Q_1 : $\frac{12 \times 1}{4} = 3 \Rightarrow$ Integer $\Rightarrow \frac{169 + 171}{2} = 170$

For Q_2 : $\frac{12 \times 2}{4} = 6 \Rightarrow$ Integer $\Rightarrow \frac{186 + 210}{2} = 198$

For Q_3 : $\frac{12 \times 3}{4} = 9 \Rightarrow$ Integer $\Rightarrow \frac{250 + 258}{2} = 254$

(c)

$$Q_3 + 1.5 \times \left(Q_3 - Q_1 \right) = 254 + 1.5 \times (254 - 170)$$
$$= 380$$

Patient *B* on Cotinine level 390, and patient *F* on Cotinine level 420 are thus outliers

(d)

$$skew = \frac{(Q_1 - 2Q_2 + Q_3)}{(Q_3 - Q_1)}$$
$$= \frac{(170 - 2 \times 198 + 254)}{(254 - 170)}$$
$$= \frac{28}{84}$$
$$= \frac{1}{3}$$

The positive skew means that more patents are piled up at the lower end of the distribution than at the upper end.