Lesson 10

A-Level Pure Mathematics : Year 2
Trigonometric Identities

10.1 Test

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 50

Question 1

(i) Find the four solutions to the following equation,

$$\cos\theta = \frac{\sqrt{3}}{2} \qquad \text{for } 0^\circ \leqslant \theta \leqslant 720^\circ$$

[4 marks]

(ii) Hence, or otherwise, solve,

$$\cos(2\theta + 15^\circ) = \frac{\sqrt{3}}{2} \quad , \qquad 0^\circ \leqslant \theta \leqslant 360^\circ$$

[2 marks]

Question 2

Prove the identity,

$$4\sin^2\theta\cos^2\theta + \cos^2 2\theta = 1$$

[5 marks]

Question 3

Here is a table of exact trigonometric values that most mathematicians know "off by heart";

Exact values table:

	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Not Defined

Show, using the formula for $\tan(A-B)$ and the table of exact trigonometric values that

$$\tan 15° = \frac{3-\sqrt{3}}{3+\sqrt{3}}$$

Rationalise the denominator to further show that for some integer values of a and b,

$$\tan 15° = a + b\sqrt{3}$$

Clearly state the value of a and the value of b.

[6 marks]

Question 4

Prove the identity

$$\frac{\sin\theta + \cos\theta}{\sin\theta} + \frac{\sin\theta - \cos\theta}{\cos\theta} == 2\csc 2\theta$$

[5 marks]

Question 5

(i) Express $15 \sin x + 8 \cos x$ in the form $R \sin (x + \alpha)$,
where $R > 0$ and α is the smallest positive angle possible.
Give your answer in degrees and accurate to one decimal place.

(ii) Hence, or otherwise, solve the following equation over $0° \leqslant x \leqslant 360°$

$$15 \sin x + 8 \cos x = 13$$

[8 marks]

Question 6

The depth of water, D metres, in a harbour on a particular day is modelled by the formula

$$D = 5 + 3\cos(30t)^\circ \qquad 0 \leqslant t \leqslant 24$$

where t is the number of hours after midnight.

(a) Low tide is the time at which the depth of water is at it's least.
On the day being modelled there are two low tides.
Find the times of these low tides.

[2 marks]

(b) A boat enters the harbour at Noon.
It must leave before the depth of the water in the harbour falls below 3 metres.
By what time, to the nearest minute, must the boat have left the harbour ?

[8 marks]

Question 7

(i) By expressing 3θ in the form $2\theta + \theta$, prove that,

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

(ii) Hence, or otherwise, solve the equation,

$$\sin 3\theta - \sin\theta = 0 \qquad \text{for } 0° \leqslant \theta \leqslant 360°$$

[10 marks]