10.2 Answers

Answer 1

(i) 30°, 330°, 390°, 690°

[4 marks]

(ii)

$$2\theta + 15 = 30,\ 330,\ 390,\ 690$$

$$2\theta = 15,\ 315,\ 375,\ 675$$

$$\theta = 7.5°,\ 157.5°,\ 187.5°,\ 337.5°$$

[2 marks]

Answer 2

1st Proof :

$$\begin{aligned}
\text{LHS} &= 4\sin^2\theta\cos^2\theta + \cos^2 2\theta \\
&= (\,2\sin\theta\cos\theta\,)^2 + \cos^2 2\theta \\
&= \sin^2 2\theta + \cos^2 2\theta \\
&= 1 \\
&= \text{RHS} \qquad \square
\end{aligned}$$

2nd Proof :

$$\begin{aligned}
\text{LHS} &= 4\sin^2\theta\cos^2\theta + \cos^2 2\theta \\
&= 4\sin^2\theta\cos^2\theta + \left(\cos^2\theta - \sin^2\theta\right)^2 \\
&= 4\sin^2\theta\cos^2\theta + \cos^4\theta - 2\sin^2\theta\cos^2\theta + \sin^4\theta \\
&= \cos^4\theta + 2\sin^2\theta\cos^2\theta + \sin^4\theta \\
&= \left(\cos^2\theta + \sin^2\theta\right)^2 \\
&= 1^2 \\
&= \text{RHS} \qquad \square
\end{aligned}$$

3rd Proof :

$$\begin{aligned}
\text{LHS} &= 4\sin^2\theta\cos^2\theta + \cos^2 2\theta \\
&= 4\sin^2\theta\cos^2\theta + \left(\cos^2\theta - \sin^2\theta\right)^2 \\
&= 4\sin^2\theta\cos^2\theta + \left(\cos^2\theta - \left(1 - \cos^2\theta\right)\right)^2 \\
&= 4\sin^2\theta\cos^2\theta + \left(2\cos^2\theta - 1\right)^2 \\
&= 4\sin^2\theta\cos^2\theta + 4\cos^4\theta - 4\cos^2\theta + 1 \\
&= 4\cos^2\theta\left(\sin^2\theta + \cos^2\theta - 1\right) + 1 \\
&= 4\cos^2\theta\left(1 - 1\right) + 1 \\
&= 1 \\
&= \text{RHS} \qquad \square
\end{aligned}$$

[5 marks]

Answer 3

1st Method

$$
\begin{aligned}
\text{LHS} &= \tan 15 \\
&= \tan(45 - 30) \\
&= \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} \\
&= \frac{1 - \frac{\sqrt{3}}{3}}{1 + 1 \times \frac{\sqrt{3}}{3}} \times \frac{3}{3} \\
&= \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \qquad \square \\
&= \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \times \frac{3 - \sqrt{3}}{3 - \sqrt{3}} \\
&= \frac{9 - 6\sqrt{3} + 3}{9 - 3} \\
&= \frac{12 - 6\sqrt{3}}{6} \\
&= 2 - \sqrt{3}
\end{aligned}
$$

i.e. $a = 2$ and $b = -1$

2nd Method

$$
\begin{aligned}
\text{LHS} &= \tan 15 \\
&= \tan(60 - 45) \\
&= \frac{\tan 60 - \tan 45}{1 + \tan 60 \tan 45} \\
&= \frac{\sqrt{3} - 1}{1 + \sqrt{3} \times 1} \times \frac{\sqrt{3}}{\sqrt{3}} \\
&= \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \qquad \square \\
&= \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \times \frac{3 - \sqrt{3}}{3 - \sqrt{3}} \\
&= \frac{9 - 6\sqrt{3} + 3}{9 - 3} \\
&= \frac{12 - 6\sqrt{3}}{6} \\
&= 2 - \sqrt{3}
\end{aligned}
$$

i.e. $a = 2$ and $b = -1$

[6 marks]

Answer 4

Method 1

$$
\begin{aligned}
\text{LHS} &= \frac{\sin\theta + \cos\theta}{\sin\theta} + \frac{\sin\theta - \cos\theta}{\cos\theta} \\
&= \frac{\sin\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} \\
&= 1 + \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} - 1 \\
&= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} \\
&= \frac{\cos\theta}{\sin\theta} \times \frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta} \times \frac{\sin\theta}{\sin\theta} \\
&= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} \\
&= \frac{1}{\sin\theta\ \cos\theta} \times \frac{2}{2} \\
&= \frac{2}{\sin 2\theta} \\
&= 2\csc 2\theta \\
&= \text{RHS} \qquad \square
\end{aligned}
$$

Method 2

$$
\begin{aligned}
\text{LHS} &= \frac{\sin\theta + \cos\theta}{\sin\theta} + \frac{\sin\theta - \cos\theta}{\cos\theta} \\
&= \frac{\sin\theta + \cos\theta}{\sin\theta} \times \frac{\cos\theta}{\cos\theta} + \frac{\sin\theta - \cos\theta}{\cos\theta} \times \frac{\sin\theta}{\sin\theta} \\
&= \frac{\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta - \sin\theta\cos\theta}{\sin\theta\cos\theta} \\
&= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} \\
&= \frac{1}{\sin\theta\ \cos\theta} \times \frac{2}{2} \\
&= \frac{2}{\sin 2\theta} \\
&= 2\csc 2\theta \\
&= \text{RHS} \qquad \square
\end{aligned}
$$

[5 marks]

Answer 5

(i) $$R\,sin\,(\,\theta\,+\,\alpha\,)\;=\;R\,sin\,\theta\,cos\,\alpha\,+\,R\,cos\,\theta\,sin\,\alpha$$

which is required to be $15\,sin\,\theta\,+\,8\,cos\,\theta$

$$\therefore\;R\,cos\,\alpha\;=\;15\quad\text{and}\quad R\,sin\,\alpha\;=\;8$$

Solving these two equations simultaneously by division,

$$\frac{R\,sin\,\alpha}{R\,cos\,\alpha}\;=\;\frac{8}{15}$$

$$\alpha\;=\;arctan\left(\frac{8}{15}\right)$$

$$\alpha\;=\;28.1°$$

$$R\;=\;\sqrt{15^2\,+\,8^2}$$

$$=\;17$$

Conclusion :

$$15\,sin\,\theta\,+\,8\,cos\,\theta\;=\;17\,sin\,(\,\theta\,+\,28.1°\,)$$

[4 marks]

(ii)

$$15\,sin\,\theta\,+\,8\,cos\,\theta\;=\;13$$

$$17\,sin\,(\,\theta\,+\,28.1\,)\;=\;13$$

$$sin\,(\,\theta\,+\,28.1\,)\;=\;0.7647$$

$$\theta\,+\,28.1\;=\;49.9,\;\;130.1$$

$$\theta\;=\;21.8°,\;\;102°$$

[4 marks]

Answer 6

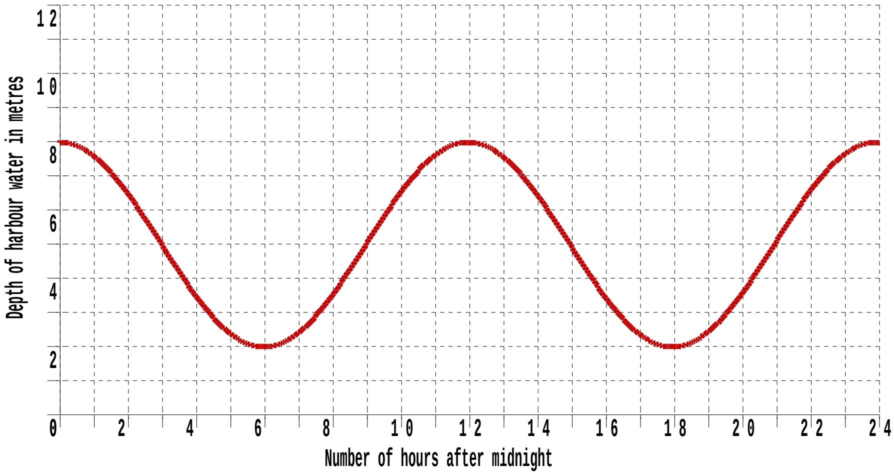
(a) For D to be a minimum, $cos\,(30t) = -1$

$$30t = 180,\ 540,\ \ldots$$

$$t = 6,\ 18,\ \ldots$$

i.e. Low tide at 6 am and 6 pm

[2 marks]



(b) To find the critical times,

$$5 + 3\,cos\,(30t) = 3$$

$$3\,cos\,(30t) = -2$$

$$cos\,(30t) = -\frac{2}{3}$$

$$30t = 131.8,\ 228.2,\ 491.8,\ 588.2$$

$$t = 4.39,\ 7.61,\ 16.39,\ 19.61\ \text{hours}$$

The boat must leave before 16 hours, 23 minutes

i.e. Before 4.23 pm

[8 marks]

Answer 7

(i)

(a)

$$
\begin{aligned}
\text{LHS} &= \sin 3\theta \\
&= \sin\,(\,2\theta + \theta\,) \\
&= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta \\
&= 2\sin\theta\cos\theta\cos\theta + \left(\cos^2\theta - \sin^2\theta\right)\sin\theta \\
&= 2\sin\theta\cos^2\theta + \cos^2\theta\sin\theta - \sin^3\theta \\
&= 3\sin\theta\cos^2\theta - \sin^3\theta \\
&= 3\sin\theta\left(1 - \sin^2\theta\right) - \sin^3\theta \\
&= 3\sin\theta - 3\sin^3\theta - \sin^3\theta \\
&= 3\sin\theta - 4\sin^3\theta \\
&= \text{RHS} \qquad \square
\end{aligned}
$$

[5 marks]

(ii)

$$
\begin{aligned}
\sin 3\theta - \sin\theta &= 0 \\
3\sin\theta - 4\sin^3\theta - \sin\theta &= 0 \\
2\sin\theta - 4\sin^3\theta &= 0 \\
2\sin\theta\left(1 - 2\sin^2\theta\right) &= 0
\end{aligned}
$$

Either $2\sin\theta = 0$ or $1 - 2\sin^2\theta = 0$

$\theta = 0°,\ 180°,\ 360°$ $\qquad$ $\sin^2\theta = \dfrac{1}{2}$

$$\sin\theta = \pm\frac{\sqrt{2}}{2}$$

$$\theta = 45°,\ 135°,\ 225°,\ 315°$$

$\therefore$ Solutions are $0°,\ 45°,\ 135°,\ 180°,\ 225°,\ 315°,\ 360°$

[5 marks]