1.3 Answers

1.3.1 Solutions 1.1 Trigonometric "Exact Values"

	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Not Defined

1.3.2 Solutions to 1.2 Exercise

Answer 1

$h = 5\sqrt{2}$ cm

[2 marks]

Answer 2

$$30^2 = b^2 + b^2$$
$$900 = 2b^2$$
$$b^2 = 450$$
$$b = 15\sqrt{2}$$

[3 marks]

Answer 3

(i)

$$h^2 = b^2 + b^2$$
$$= 2b^2$$
$$h = \sqrt{2}\,b$$

[2 marks]

(ii)

$$b = \frac{h}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{\sqrt{2}\,h}{2}$$

[2 marks]

(iii) $h = \sqrt{2}\left(1 + \sqrt{5}\right)$

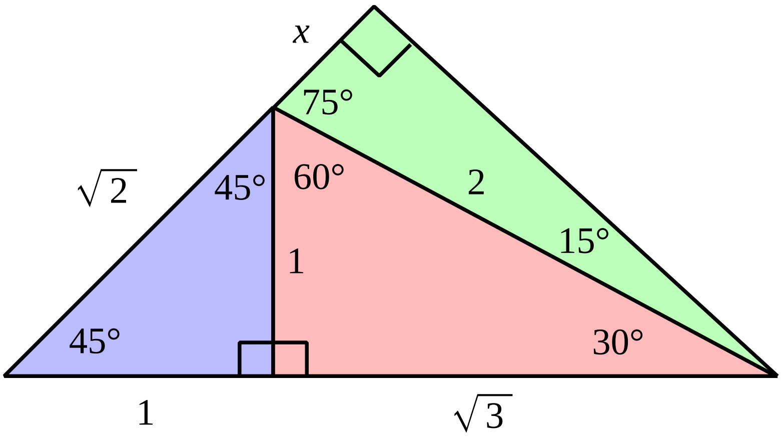
[1 mark]

(iv) $b = \dfrac{\sqrt{2}\,(1 + \sqrt{3})}{2}$

[2 mark]

Answer 4

(i)



[1 mark]

(ii) $\dfrac{\sqrt{2}\,(1+\sqrt{3})}{2} = \dfrac{\sqrt{6}+\sqrt{2}}{2}$

[1 mark]

(iii) It can now be deduced that,

$$\begin{aligned}\sqrt{2} + x &= \frac{\sqrt{2}\,(1+\sqrt{3})}{2} \\ x &= \frac{\sqrt{2}\,(1+\sqrt{3})}{2} - \sqrt{2} \\ &= \frac{\sqrt{2}+\sqrt{6}-2\sqrt{2}}{2} \\ &= \frac{\sqrt{6}-\sqrt{2}}{2}\end{aligned}$$

[3 marks]

(iv)

$$\begin{aligned}sin\,15 &= \frac{x}{2} \\ &= \frac{\sqrt{6}-\sqrt{2}}{4}\end{aligned}$$

[2 marks]

(v)

$$\cos 15^\circ = \sin 75^\circ = \left(\frac{\sqrt{6} + \sqrt{2}}{2} \right) \div 2$$
$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

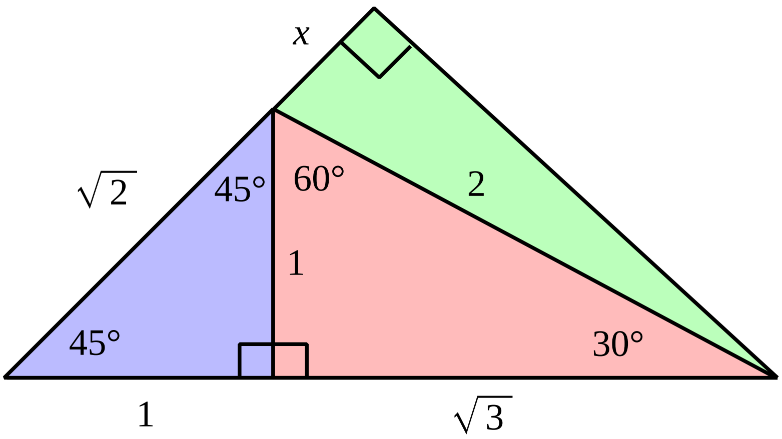
$$\tan 15 = \left(\frac{\sqrt{6} - \sqrt{2}}{2} \right) \div \left(\frac{\sqrt{6} + \sqrt{2}}{2} \right)$$
$$= \frac{\sqrt{6} - \sqrt{2}}{2} \times \frac{2}{\sqrt{6} + \sqrt{2}}$$
$$= \frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}} \times \frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} - \sqrt{2}}$$
$$= \frac{6 - \sqrt{12} - \sqrt{12} + 2}{6 - 2}$$
$$= \frac{8 - 4\sqrt{3}}{4}$$
$$= 2 - \sqrt{3}$$

$$\tan 75 = \frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$$
$$= \frac{2 + \sqrt{3}}{4 - 3}$$
$$= 2 + \sqrt{3}$$

	15°	75°
$\sin\theta$	$\frac{\sqrt{6} - \sqrt{2}}{4}$	$\frac{\sqrt{6} + \sqrt{2}}{4}$
$\cos\theta$	$\frac{\sqrt{6} + \sqrt{2}}{4}$	$\frac{\sqrt{6} - \sqrt{2}}{4}$
$\tan\theta$	$2 - \sqrt{3}$	$2 + \sqrt{3}$

[5 marks]

(vi) The 105° is the (45 + 60)° angle in the diagram



$$\cos 105° = \frac{2^2 + (\sqrt{2})^2 - (1 + \sqrt{3})^2}{2 \times 2 \times \sqrt{2}}$$

$$= \frac{4 + 2 - (1 + 2\sqrt{3} + 3)}{4\sqrt{2}}$$

$$= \frac{4 + 2 - 1 - 2\sqrt{3} - 3}{4\sqrt{2}}$$

$$= \frac{2 - 2\sqrt{3}}{4\sqrt{2}}$$

$$= \frac{1 - \sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{2} - \sqrt{6}}{4}$$

[3 marks]

Answer 5

(i) Looking at the Length Scale Factor moving from the small (pink) triangle to the big purple triangle,

$$\frac{1}{x} = \frac{x - 1}{1}$$

Multiply through by x

$$1 = x^2 - x$$

$$x^2 - x - 1 = 0$$

$$\left[x^2 - x \right] - 1 = 0$$

$$\left[\left(x - \frac{1}{2} \right)^2 - \frac{1}{4} \right] - 1 = 0$$

$$\left(x - \frac{1}{2} \right)^2 = \frac{5}{4}$$

$$x - \frac{1}{2} = \pm \frac{\sqrt{5}}{2}$$

$$x = \frac{\sqrt{5} + 1}{2}$$

This is ϕ, the golden ratio

[5 marks]

(ii) $cos\, 36 = \dfrac{x}{2} = \dfrac{\sqrt{5} + 1}{4}$

[1 mark]

(iii) Using the Theorem of Pythagoras, the fact that $\dfrac{x}{2} = \dfrac{\sqrt{5} + 1}{4}$

and the trigonometric identity that $cos^2\,\theta + sin^2\,\theta = 1$,

$$\begin{aligned} sin\, 36° &= \sqrt{1 - cos^2\, 36} \\ &= \sqrt{1 - \left(\frac{\sqrt{5} + 1}{4} \right)^2} \\ &= \sqrt{\frac{16 - 5 - 2\sqrt{5} - 1}{16}} \\ &= \sqrt{\frac{10 - 2\sqrt{5}}{16}} \\ &= \frac{\sqrt{10 - 2\sqrt{5}}}{4} \qquad \square \end{aligned}$$

[3 marks]

Answer 6

(i) Many counter examples are suitable.

For example let $A = 30°$ and $B = 60°$ in which case,

$$\sin 30° = \frac{1}{2} \quad \text{and} \quad \sin 60° = \frac{\sqrt{3}}{2} \quad \text{and} \quad \sin 90° = 1$$

$$\text{but} \quad \frac{1}{2} + \frac{\sqrt{3}}{2} \neq 1$$

$$\text{so} \quad \sin 30° + \sin 60° \neq \sin 90°$$

$$\therefore \ \sin A + \sin B \neq \sin (A + B) \text{ in general}$$

[2 marks]

(ii) With $A = 30°$ and $B = 60°$ the claim is that,

$$\sin (30 + 60) = \sin 30 \cos 60 + \cos 30 \sin 60$$

LHS $= \sin (30 + 60)$

$= \sin 90$

$= 1$

RHS $= \sin 30 \cos 60 + \cos 30 \sin 60$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4}$$

$$= 1$$

As LHS = RHS it is true that,

$$\sin (A + B) = \sin A \cos B + \cos A \sin B$$

when $A = 30°$ and $B = 60°$

[2 marks]

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