2.7 Answers

2.7.1 Solution to 2.4 Example (Teaching Video)

The Addition Formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\begin{aligned} \text{LHS} &= \cos(90 - \theta) \\ &= \cos 90^\circ \cos\theta + \sin 90^\circ \sin\theta \\ &= 0 \times \cos\theta + 1 \times \sin\theta \\ &= \sin\theta \\ &= \text{RHS} \qquad \square \end{aligned}$$

[3 marks]

2.7.2 Solutions to 2.5 Exercise

Answer 1

$$\begin{aligned} \text{LHS} &= \sin(90^\circ + \theta) \\ &= \sin 90 \cos\theta + \cos 90 \sin\theta \\ &= 1 \times \cos\theta + 0 \times \sin\theta \\ &= \cos\theta \\ &= \text{RHS} \qquad \square \end{aligned}$$

[3 marks]

Answer 2

$$\begin{aligned} \text{LHS} &= \sin(180^\circ - \theta) \\ &= \sin 180 \cos\theta - \cos 180 \sin\theta \\ &= 0 \times \cos\theta - (-1) \times \sin\theta \\ &= \sin\theta \\ &= \text{RHS} \qquad \square \end{aligned}$$

[3 marks]

Answer 3

$$\begin{aligned} \text{LHS} &= sin\,(A + B) + sin\,(A - B) \\ &= sin\,A\,cos\,B + cos\,A\,sin\,B + sin\,A\,cos\,B - cos\,A\,sin\,B \\ &= 2\,sin\,A\,cos\,B \\ &= \text{RHS} \qquad \square \end{aligned}$$

[3 marks]

Answer 4

$$\begin{aligned} \text{LHS} &= cos\,(A - B) - cos\,(A + B) \\ &= cos\,A\,cos\,B + sin\,A\,sin\,B - \left[cos\,A\,cos\,B - sin\,A\,sin\,B \right] \\ &= cos\,A\,cos\,B + sin\,A\,sin\,B - cos\,A\,cos\,B + sin\,A\,sin\,B \\ &= 2\,sin\,A\,sin\,B \\ &= \text{RHS} \qquad \square \end{aligned}$$

[3 marks]

Answer 5

$$\begin{aligned} \text{LHS} &= \frac{sin\,(A + B)}{cos\,A\,cos\,B} \\ &= \frac{sin\,A\,cos\,B + cos\,A\,sin\,B}{cos\,A\,cos\,B} \\ &= \frac{sin\,A\,cos\,B}{cos\,A\,cos\,B} + \frac{cos\,A\,sin\,B}{cos\,A\,cos\,B} \\ &= tan\,A + tan\,B \\ &= \text{RHS} \qquad \square \end{aligned}$$

[3 marks]

Answer 6

$$\begin{aligned} \text{LHS} &= sin\,2A \\ &= sin\,(A + A) \\ &= sin\,A\,cos\,A + cos\,A\,sin\,A \\ &= 2\,sin\,A\,cos\,A \\ &= \text{RHS} \qquad \square \end{aligned}$$

[3 marks]

Answer 7

$$\begin{aligned} \text{LHS} &= \cos 2A \\ &= \cos\,(A + A) \\ &= \cos A \cos A - \sin A \sin A \\ &= \cos^2 A - \sin^2 A \\ &= \text{RHS} \qquad \square \end{aligned}$$

[3 marks]

Answer 8

$$\begin{aligned} \text{LHS} &= \tan 2A \\ &= \tan\,(A + A) \\ &= \frac{\tan A + \tan A}{1 - \tan A \tan A} \\ &= \frac{2 \tan A}{1 - \tan^2 A} \\ &= \text{RHS} \qquad \square \end{aligned}$$

[3 marks]

Answer 9

The Double Angle Formulae

$$\begin{aligned} \sin 2A &= 2\sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ \tan 2A &= \frac{2 \tan A}{1 - \tan^2 A} \end{aligned}$$

[3 marks]
[1 Bonus]

Answer 10
A trigonometric identity is a statement that is true for all values of θ
(Sometimes with a technical exclusion)
A trigonometric equation is a statement that is only true for a certain special values of θ that, typically, are to be found.

Plus an example of each.

[4 marks]

Answer 11

$$2\,sin\,x = cos\,(x + 60°)$$

$$2\,sin\,x = cos\,x\,cos\,60 - sin\,x\,sin\,60$$

$$2\,sin\,x = \frac{1}{2}\,cos\,x - \frac{\sqrt{3}}{2}\,sin\,x$$

multiply through by 2

$$4\,sin\,x = cos\,x - \sqrt{3}\,sin\,x$$

$$sin\,x\,\left(4 + \sqrt{3}\right) = cos\,x$$

$$\frac{sin\,x}{cos\,x} = \frac{1}{4 + \sqrt{3}} \times \frac{4 - \sqrt{3}}{4 - \sqrt{3}}$$

$$tan\,x = \frac{4 - \sqrt{3}}{16 - 3}$$

$$tan\,x = \frac{4 - \sqrt{3}}{13}$$

$$tan\,x = 0.1745$$

$$x = 9.9°,\ 189.9°$$

[6 marks]