3.7 Answers

3.7.1 Solution to 3.5 Example (Teaching Video)

$$
\begin{aligned}
\text{LHS} &= \frac{\cos 2A}{1 + \sin 2A} \\
&= \frac{\cos^2 A - \sin^2 A}{\cos^2 A + \sin^2 A + 2\sin A \cos A} \\
&= \frac{(\cos A - \sin A)(\cos A + \sin A)}{(\cos A + \sin A)^2} \\
&= \frac{(\cos A - \sin A)(\cos A + \sin A)}{(\cos A + \sin A)\ (\cos A + \sin A)} \\
&= \frac{(\cos A - \sin A)}{(\cos A + \sin A)} \\
&= \frac{(\cos A - \sin A)}{(\cos A + \sin A)} \times \frac{(\cos A - \sin A)}{(\cos A - \sin A)} \\
&= \frac{(\cos A - \sin A)^2}{\cos^2 A - \sin^2 A} \\
&= \frac{\cos^2 A - 2\sin A \cos A + \sin^2 A}{\cos 2A} \\
&= \frac{1 - \sin 2A}{\cos 2A} \\
&= \text{RHS} \qquad \square
\end{aligned}
$$

[6 marks]

As an end-of-lesson filler, here is another way of doing this question,

$$
\begin{aligned}
\text{LHS} &= \frac{\cos 2A}{1 + \sin 2A} \\
&= \frac{\cos 2A}{(1 + \sin 2A)} \times \frac{(1 - \sin 2A)}{(1 - \sin 2A)} \\
&= \frac{\cos 2A\ (1 - \sin 2A)}{1 - \sin^2 2A} \\
&= \frac{\cos 2A\ (1 - \sin 2A)}{\cos^2 2A + \sin^2 2A - \sin^2 2A} \\
&= \frac{\cos 2A\ (1 - \sin 2A)}{\cos^2 2A} \\
&= \frac{1 - \sin 2A}{\cos 2A} \\
&= \text{RHS} \qquad \square
\end{aligned}
$$

3.7.2 Solutions to 3.6 Exercise

Answer 1

$$\begin{aligned}
\text{LHS} &= \frac{\sin 2A}{1 + \cos 2A} \\
&= \frac{2 \sin A \cos A}{\cos^2 A + \sin^2 A + \cos^2 A - \sin^2 A} \\
&= \frac{2 \sin A \cos A}{2 \cos^2 A} \\
&= \frac{\sin A}{\cos A} \\
&= \tan A \\
&= \text{RHS} \qquad \square
\end{aligned}$$

[3 marks]

Answer 2

$$\begin{aligned}
\text{LHS} &= \sqrt{\frac{1 - \cos 2A}{1 + \cos 2A}} \\
&= \sqrt{\frac{\cos^2 A + \sin^2 A - (\cos^2 A - \sin^2 A)}{\cos^2 A + \sin^2 A + \cos^2 A - \sin^2 A}} \\
&= \sqrt{\frac{2 \sin^2 A}{2 \cos^2 A}} \\
&= \sqrt{\tan^2 A} \\
&= \tan A \\
&= \text{RHS} \qquad \square
\end{aligned}$$

[3 marks]

Answer 3

$$\begin{aligned}
\text{LHS} &= \frac{\cos 2A}{\cos A + \sin A} \\
&= \frac{\cos^2 A - \sin^2 A}{\cos A + \sin A} \\
&= \frac{(\cos A + \sin A)\ (\cos A - \sin A)}{\cos A + \sin A} \\
&= \cos A - \sin A \\
&= \text{RHS} \qquad \square
\end{aligned}$$

[3 marks]

Answer 4

$$\begin{aligned}
\text{LHS} &= \cos^4 A - \sin^4 A \\
&= \left(\cos^2 A\right)^2 - \left(\sin^2 A\right)^2 \\
&= \left(\cos^2 A - \sin^2 A\right)\left(\cos^2 A + \sin^2 A\right) \\
&= \left(\cos^2 A - \sin^2 A\right)(1) \\
&= \cos 2A \\
&= \text{RHS} \qquad \square
\end{aligned}$$

[3 marks]

Answer 5

$$\begin{aligned}
\text{LHS} &= \cos 3A \\
&= \cos\,(2A + A) \\
&= \cos 2A \cos A - \sin 2A \sin A \\
&= (\cos^2 A - \sin^2 A)\cos A - 2\sin A \cos A \sin A \\
&= \cos^3 A - \sin^2 A \cos A - 2\sin^2 A \cos A \\
&= \cos^3 A - 3\sin^2 A \cos A \\
&= \cos^3 A - 3\left(1 - \cos^2 A\right)\cos A \\
&= \cos^3 A - 3\cos A + 3\cos^3 A \\
&= 4\cos^3 A - 3\cos A \\
&= \text{RHS} \qquad \square
\end{aligned}$$

[6 marks]

Answer 6

(a)

$$\begin{aligned}
\text{LHS} &= 1 - 2\sin^2 x \\
&= \cos^2 x + \sin^2 x - 2\sin^2 x \\
&= \cos^2 x - \sin^2 x \\
&= \cos 2x \\
&= \text{RHS} \qquad \square
\end{aligned}$$

[2 marks]

(b) **(i)** $\sin x = \dfrac{BM}{1} \Rightarrow BM = \sin x$

[1 mark]

(ii) $BC = 2\,BM \Rightarrow BC = 2\sin x$

[1 mark]

(iii) $BC^2 = 1^2 + 1^2 - 2 \times 1 \times 1 \times \cos 2x$

$$BC^2 = 2 - 2\cos 2x$$

$$2\cos 2x = 2 - BC^2$$

$$\cos 2x = 1 - \frac{1}{2} BC^2$$

[2 marks]

(iv) $\cos 2x = 1 - \dfrac{1}{2} (2\sin x)^2$

$$= 1 - \frac{1}{2} \times 4 \times \sin^2 x$$

$$= 1 - 2\sin^2 x$$

[2 marks]

Answer 7

$$\text{LHS} = \tan 3A$$

$$= \tan(2A + A)$$

$$= \frac{\tan 2A + \tan A}{1 - \tan 2A \tan A}$$

$$= \frac{\frac{2\tan A}{1 - \tan^2 A} + \tan A}{1 - \frac{2\tan A}{1 - \tan^2 A} \times \tan A} \times \frac{(1 - \tan^2 x)}{(1 - \tan^2 x)}$$

$$= \frac{2\tan A + \tan A (1 - \tan^2 x)}{1 - \tan^2 x - 2\tan^2 A}$$

$$= \frac{2\tan A + \tan A - \tan^3 A}{1 - \tan^2 x - 2\tan^2 A}$$

$$= \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$$

$$= \text{RHS} \qquad \square$$

There is a connection with Pascal's Triangle, 1 3 3 1

[4 marks]