

5.5 Answers

5.5.1 Solution to 5.2 Example (Teaching Video)

$$\begin{aligned}\text{LHS} &= \csc \theta (\cos \theta \cot \theta + \sin \theta) - 1 \\&= \frac{1}{\sin \theta} \left(\cos \theta \times \frac{\cos \theta}{\sin \theta} + \sin \theta \right) - 1 \\&= \frac{\cos^2 \theta}{\sin^2 \theta} + 1 - 1 \\&= \left(\frac{\cos \theta}{\sin \theta} \right)^2 \\&= \cot^2 \theta \\&= \text{RHS} \quad \square\end{aligned}$$

[3 marks]

5.5.2 Solutions to 5.4 Exercise

Answer 1

$$\begin{aligned}\text{LHS} &= \sin \theta (\csc \theta - \sin \theta) + \cos \theta (\sec \theta - \cos \theta) \\&= \sin \theta \left(\frac{1}{\sin \theta} - \sin \theta \right) + \cos \theta \left(\frac{1}{\cos \theta} - \cos \theta \right) \\&= 1 - \sin^2 \theta + 1 - \cos^2 \theta \\&= 1 + 1 - (\sin^2 \theta + \cos^2 \theta) \\&= 1 + 1 - 1 \\&= 1 \\&= \text{RHS} \quad \square\end{aligned}$$

[3 marks]

Answer 2

$$\begin{aligned}
\text{LHS} &= \frac{\sin \theta (\sin \theta - \csc \theta)}{\cos \theta (\cos \theta - \sec \theta)} \\
&= \frac{\sin \theta (\sin \theta - \frac{1}{\sin \theta})}{\cos \theta (\cos \theta - \frac{1}{\cos \theta})} \\
&= \frac{\sin^2 \theta - 1}{\cos^2 \theta - 1} \\
&= \frac{\sin^2 \theta - \cos^2 \theta - \sin^2 \theta}{\cos^2 \theta - \cos^2 \theta - \sin^2 \theta} \\
&= \frac{-\cos^2 \theta}{-\sin^2 \theta} \\
&= \cot^2 \theta \\
&= \text{RHS} \quad \square
\end{aligned}$$

[3 marks]**Answer 3**

$$\begin{aligned}
\text{LHS} &= \frac{(1 - \sin \theta)(1 + \sin \theta)}{(1 - \cos \theta)(1 + \cos \theta)} \\
&= \frac{1 - \sin^2 \theta}{1 - \cos^2 \theta} \\
&= \frac{\cos^2 \theta + \sin^2 \theta - \sin^2 \theta}{\cos^2 \theta + \sin^2 \theta - \cos^2 \theta} \\
&= \frac{\cos^2 \theta}{\sin^2 \theta} \\
&= \cot^2 \theta \\
&= \text{RHS} \quad \square
\end{aligned}$$

[3 marks]**Answer 4**

$$\begin{aligned}
\text{LHS} &= \sec^2 \theta - \tan^2 \theta + \csc^2 \theta - \cot^2 \theta \\
&= 1 + \tan^2 \theta - \tan^2 \theta + \cot^2 \theta + 1 - \cot^2 \theta \\
&= 1 + 1 \\
&= 2 \\
&= \text{RHS} \quad \square
\end{aligned}$$

[3 marks]

Answer 5

$$\begin{aligned}
\text{LHS} &= \sec \theta (\cos \theta + \sin \theta \tan \theta) \\
&= \frac{1}{\cos \theta} \left(\cos \theta + \sin \theta \times \frac{\sin \theta}{\cos \theta} \right) \\
&= 1 + \frac{\sin^2 \theta}{\cos^2 \theta} \\
&= 1 + \tan^2 \theta \\
&= \sec^2 \theta \\
&= \text{RHS} \quad \square
\end{aligned}$$

[3 marks]**Answer 6**

$$\begin{aligned}
\text{LHS} &= \frac{1}{\csc \theta (\cos \theta \cot \theta + \sin \theta)} \\
&= \frac{1}{\frac{1}{\sin \theta} (\cos \theta \times \frac{\cos \theta}{\sin \theta} + \sin \theta)} \\
&= \frac{1}{\frac{\cos^2 \theta}{\sin^2 \theta} + 1} \\
&= \frac{1}{\cot^2 \theta + 1} \\
&= \frac{1}{\csc^2 \theta} \\
&= \sin^2 \theta \\
&= \text{RHS} \quad \square
\end{aligned}$$

[3 marks]**Answer 7**

$$\begin{aligned}
\text{LHS} &= \frac{1}{(\tan^2 \theta + 1)} + \frac{1}{(\cot^2 \theta + 1)} \\
&= \frac{1}{\sec^2 \theta} + \frac{1}{\csc^2 \theta} \\
&= \cos^2 \theta + \sin^2 \theta \\
&= 1 \\
&= \text{RHS} \quad \square
\end{aligned}$$

[3 marks]

Answer 8

$$\begin{aligned}\text{LHS} &= (\sec^2 \theta - 1) (\csc^2 \theta - 1) \\&= (1 + \tan^2 \theta - 1) (\cot^2 \theta + 1 - 1) \\&= \tan^2 \theta \cot^2 \theta \\&= \tan^2 \theta \times \frac{1}{\tan^2 \theta} \\&= 1 \\&= \text{RHS} \quad \square\end{aligned}$$

[3 marks]**Answer 9**

$$\begin{aligned}\text{LHS} &= (\sec \theta + \tan \theta) (\sec \theta - \tan \theta) \\&= \sec^2 \theta - \tan^2 \theta \\&= 1 + \tan^2 \theta - \tan^2 \theta \\&= 1 \\&= \text{RHS} \quad \square\end{aligned}$$

[3 marks]**Answer 10**

$$\begin{aligned}\text{LHS} &= \frac{\cos \theta}{\sqrt{1 + \tan^2 \theta}} + \frac{\sin \theta}{\sqrt{1 + \cot^2 \theta}} \\&= \frac{\cos \theta}{\sqrt{\sec^2 \theta}} + \frac{\sin \theta}{\sqrt{\csc^2 \theta}} \\&= \frac{\cos \theta}{\sec \theta} + \frac{\sin \theta}{\csc \theta} \\&= \cos^2 \theta + \sin^2 \theta \\&= 1 \\&= \text{RHS} \quad \square\end{aligned}$$

[3 marks]

Answer 11

$$\begin{aligned}\text{LHS} &= \cot \theta - \tan \theta \\&= \frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta} \\&= \frac{\cos \theta}{\sin \theta} \times \frac{\cos \theta}{\cos \theta} - \frac{\sin \theta}{\cos \theta} \times \frac{\sin \theta}{\sin \theta} \\&= \frac{\cos^2 \theta - \sin^2 \theta}{\sin \theta \cos \theta} \times \frac{2}{2} \\&= \frac{2 (\cos^2 \theta - \sin^2 \theta)}{2 \sin \theta \cos \theta} \\&= \frac{2 \cos 2\theta}{\sin 2\theta} \\&= 2 \cot 2\theta \\&= \text{RHS} \quad \square\end{aligned}$$

[5 marks]**Answer 12**

$$\begin{aligned}\text{LHS} &= \cot \theta + \tan \theta \\&= \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} \\&= \frac{\cos \theta}{\sin \theta} \times \frac{\cos \theta}{\cos \theta} + \frac{\sin \theta}{\cos \theta} \times \frac{\sin \theta}{\sin \theta} \\&= \frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cos \theta} \times \frac{2}{2} \\&= \frac{2 (\cos^2 \theta + \sin^2 \theta)}{2 \sin \theta \cos \theta} \\&= \frac{2}{\sin 2\theta} \\&= 2 \csc 2\theta \\&= \text{RHS} \quad \square\end{aligned}$$

[5 marks]