

6.5 Answers

6.5.1 Solution to 6.2 Example (Teaching Video)

$$2 \cot^2 3\theta = 7 \csc 3\theta - 5$$

$$2 (\csc^2 3\theta - 1) = 7 \csc 3\theta - 5$$

$$2 \csc^2 3\theta - 2 - 7 \csc 3\theta + 5 = 0$$

$$2 \csc^2 3\theta - 7 \csc 3\theta + 3 = 0$$

$$(2 \csc 3\theta - 1)(\csc 3\theta - 3) = 0$$

$$\text{Either } 2 \csc 3\theta - 1 = 0 \quad \text{or} \quad \csc 3\theta - 3 = 0$$

$$\csc 3\theta = \frac{1}{2}$$

$$\csc 3\theta = 3$$

$$\frac{1}{\sin 3\theta} = \frac{1}{2}$$

$$\frac{1}{\sin 3\theta} = \frac{3}{1}$$

$$\sin 3\theta = 2$$

$$\sin 3\theta = \frac{1}{3}$$

No solutions

$$3\theta = 19.5, 160.5, 379.5, 520.5$$

$$\theta = 6.5^\circ, 53.5^\circ, 126.5^\circ, 173.5^\circ$$

[10 marks]

6.5.2 Solutions to 6.3 Exercise

Answer 1

$$2 \cos 2\theta = 1 - 2 \sin \theta$$

$$2 (\cos^2 \theta - \sin^2 \theta) = 1 - 2 \sin \theta$$

$$2 (1 - \sin^2 \theta - \sin^2 \theta) = 1 - 2 \sin \theta$$

$$2 - 4 \sin^2 \theta = 1 - 2 \sin \theta$$

$$4 \sin^2 \theta - 2 \sin \theta - 1 = 0$$

$$\sin \theta = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 4 \times (-1)}}{2 \times 4}$$

$$= \frac{2 \pm \sqrt{20}}{8}$$

$$= \frac{1 \pm \sqrt{5}}{4}$$

$$\theta = 54^\circ, 126^\circ, 198^\circ, 342^\circ$$

[6 marks]

Answer 2

$$\csc^2 2x - \cot 2x = 1$$

$$\cot^2 2x + 1 - \cot 2x = 1$$

$$\cot^2 2x - \cot 2x = 0$$

$$\cot 2x (\cot 2x - 1) = 0$$

$$\text{Either } \cot 2x = 0 \text{ or } \cot 2x = 1$$

$$\cot 2x = 0 \text{ when } \tan 2x \text{ is undefined; } 2x = 90, 270$$

$$x = 45, 135$$

$$\cot 2x = 1 \text{ when } \frac{1}{\tan 2x} = 1, \therefore \text{ when } \tan 2x = 1; 2x = 45, 225$$

$$x = 22.5, 112.5$$

$$\text{Solutions are } 22.5^\circ, 45^\circ, 112.5^\circ, 135^\circ$$

[7 marks]**Answer 3****(a)**

$$\begin{aligned} \text{LHS} &= \frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} \\ &= \frac{(\cos^2 \theta + \sin^2 \theta) - (\cos^2 \theta - \sin^2 \theta)}{2 \sin \theta \cos \theta} \\ &= \frac{2 \sin^2 \theta}{2 \sin \theta \cos \theta} \\ &= \frac{\sin \theta}{\cos \theta} \\ &= \tan \theta \\ &= \text{RHS} \quad \square \end{aligned}$$

[4 marks]**(b) (i)**

$$\begin{aligned} \tan 15^\circ &= \frac{1}{\sin 30^\circ} - \frac{\cos 30^\circ}{\sin 30^\circ} \\ &= 2 - \sqrt{3} \end{aligned}$$

[3 marks]

$$\text{(ii) } \operatorname{cosec} 4\theta - \cot 4\theta = 1$$

$$\frac{1}{\sin 4\theta} - \frac{\cos 4\theta}{\sin 4\theta} = 1$$

$$\tan 2\theta = 1$$

$$2\theta = 45, 225, 405, 585$$

$$\theta = 22.5^\circ, 112.5^\circ, 202.5^\circ, 292.5^\circ$$

[5 marks]

Answer 4**(a)**

$$\sin^2 \theta + \cos^2 \theta = 1$$

Divide through by $\sin^2 \theta$

$$\frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta}$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\therefore \csc^2 \theta - \cot^2 \theta = 1 \quad \square$$

[2 marks]**(b)**

$$\text{LHS} = \csc^4 \theta - \cot^4 \theta$$

$$= (\csc^2 \theta)^2 - (\cot^2 \theta)^2$$

$$= (\csc^2 \theta + \cot^2 \theta) (\csc^2 \theta - \cot^2 \theta)$$

$$= (\csc^2 \theta + \cot^2 \theta) \times 1$$

$$= \csc^2 \theta + \cot^2 \theta$$

$$= \text{RHS} \quad \square$$

[2 marks]**(c)**

$$\operatorname{cosec}^4 \theta - \cot^4 \theta = 2 - \cot \theta$$

$$\csc^2 \theta + \cot^2 \theta = 2 - \cot \theta$$

$$\cot^2 \theta + 1 + \cot^2 \theta = 2 - \cot \theta$$

$$2 \cot^2 \theta + \cot \theta - 1 = 0$$

$$(2 \cot \theta - 1)(\cot \theta + 1) = 0$$

$$\text{Either } \cot \theta = \frac{1}{2} \quad \text{or} \quad \cot \theta = -1$$

$$\tan \theta = 2 \quad \tan \theta = -1$$

$$\theta = 135^\circ$$

[6 marks]

Answer 5**(a)**

$$\sin^2 \theta + \cos^2 \theta = 1$$

Divide through by $\sin^2 \theta$

$$\frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta}$$

$$1 + \cot^2 \theta = \csc^2 \theta \quad \square$$

[2 marks]**(b)**

$$2 \cot^2 \theta - 9 \csc \theta = 3$$

$$2 (\csc^2 \theta - 1) - 9 \csc \theta = 3$$

$$2 \csc^2 \theta - 9 \csc \theta - 5 = 0$$

$$(2 \csc \theta + 1)(\csc \theta - 5) = 0$$

$$\text{Either } \csc \theta = -\frac{1}{2} \quad \text{or} \quad \csc \theta = 5$$

$$\sin \theta = -2 \quad \sin \theta = \frac{1}{5}$$

$$\text{No solutions} \quad \theta = 11.5^\circ, 168.5^\circ$$

[6 marks]**Answer 6****(a)**

$$\begin{aligned} \text{LHS} &= \frac{\sin 2\theta}{1 + \cos 2\theta} \\ &= \frac{2 \sin \theta \cos \theta}{\cos^2 \theta + \sin^2 \theta + \cos^2 \theta - \sin^2 \theta} \\ &= \frac{2 \sin \theta \cos \theta}{2 \cos^2 \theta} \\ &= \frac{\sin \theta}{\cos \theta} \\ &= \tan \theta \\ &= \text{RHS} \quad \square \end{aligned}$$

[2 marks]**(b)**

$$\frac{2 \sin 2\theta}{1 + \cos 2\theta} = 1$$

$$\tan \theta = \frac{1}{2}$$

$$\theta = -153.4^\circ, 26.6^\circ$$

[3 marks]

Answer 7**(a)**

$$\begin{aligned}
\text{LHS} &= \sin 3\theta \\
&= \sin (2\theta + \theta) \\
&= \sin 2\theta \cos \theta + \cos 2\theta \sin \theta \\
&= 2 \sin \theta \cos \theta \cos \theta + (\cos^2 \theta - \sin^2 \theta) \sin \theta \\
&= 2 \sin \theta \cos^2 \theta + \cos^2 \theta \sin \theta - \sin^3 \theta \\
&= 3 \sin \theta \cos^2 \theta - \sin^3 \theta \\
&= 3 \sin \theta (1 - \sin^2 \theta) - \sin^3 \theta \\
&= 3 \sin \theta - 3 \sin^3 \theta - \sin^3 \theta \\
&= 3 \sin \theta - 4 \sin^3 \theta \\
&= \text{RHS} \quad \square
\end{aligned}$$

[5 marks]**(b)**

$$\begin{aligned}
\sin 3\theta &= 3\left(\frac{\sqrt{3}}{4}\right) - 4\left(\frac{\sqrt{3}}{4}\right)^3 \\
&= \frac{3\sqrt{3}}{4} - \frac{4 \times 3\sqrt{3}}{4 \times 16} \\
&= \frac{12\sqrt{3}}{16} - \frac{3\sqrt{3}}{16} \\
&= \frac{9\sqrt{3}}{16}
\end{aligned}$$

[2 marks]