

### 6.5.3 Solutions to 6.4 Homework

#### Answer 1

( a )     At 6.30 am,  $D = 5 + 2 \sin (30 \times 6.5)$   
 $= 4.5$  metres

[ 1 mark ]

( b )     Solve,  $5 + 2 \sin (30t) = 3.8$

$$2 \sin (30t) = -1.2$$

$$\sin (30t) = -0.6$$

$$30t = 216.9, 323.1 + 360n$$

$$t = 7.23, 10.77$$

The boat is not ready to leave at 7.14 am when the tide falls below the depth needed and so has to wait until 10.46 am on the rising tide.

[ 4 marks ]

#### Answer 2

$$\begin{aligned} \text{LHS} &= 1 - \cos 2\theta \\ &= (\cos^2 \theta + \sin^2 \theta) - (\cos^2 \theta - \sin^2 \theta) \\ &= 2 \sin^2 \theta \\ &= 2 \sin^2 \theta \times \frac{\cos \theta}{\cos \theta} \\ &= \frac{\sin \theta}{\cos \theta} \times 2 \sin \theta \cos \theta \\ &= \tan \theta \sin 2\theta \\ &= \text{RHS} \quad \square \end{aligned}$$

[ 3 marks ]

**Answer 3****( a )**

$$\begin{aligned}
\text{LHS} &= \sin 2x - \tan x \\
&= \tan x \left( \sin 2x \times \frac{1}{\tan x} - 1 \right) \\
&= \tan x \left( 2 \sin x \cos x \times \frac{\cos x}{\sin x} - 1 \right) \\
&= \tan x ( 2 \cos^2 x - 1 ) \\
&= \tan x ( 2 \cos^2 x - ( \cos^2 x + \sin^2 x ) ) \\
&= \tan x ( \cos^2 x - \sin^2 x ) \\
&= \tan x \cos 2x \\
&= \text{RHS} \quad \square
\end{aligned}$$

**[ 4 marks ]****( b )**

$$\sin 2x - \tan x = 3 \tan x \sin x$$

$$\tan x \cos 2x = 3 \tan x \sin x$$

$$\tan x \cos 2x - 3 \tan x \sin x = 0$$

$$\tan x ( \cos 2x - 3 \sin x ) = 0$$

$$\text{Either } \tan x = 0 \text{ or } \cos 2x - 3 \sin x = 0$$

$$\text{No solutions as } x = 0^\circ, 180^\circ \quad \text{or} \quad \cos^2 x - \sin^2 x - 3 \sin x = 0$$

$$1 - 2 \sin^2 x - 3 \sin x = 0$$

$$2 \sin^2 x + 3 \sin x - 1 = 0$$

$$\begin{aligned}
\sin x &= \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-1)}}{2 \times 2} \\
&= \frac{-3 \pm \sqrt{17}}{4} \\
&= -1.7808, 0.2808
\end{aligned}$$

$$x = 16.3^\circ, 163.7^\circ$$

$$\text{Solutions are, } 0^\circ, 16.3^\circ, 163.7^\circ, 180^\circ$$

**[ 5 marks ]**

**Answer 4****( a )**

$$5 \sin 2\theta = 9 \tan \theta$$

$$10 \sin \theta \cos \theta - 9 \times \frac{\sin \theta}{\cos \theta} = 0$$

Multiply through by  $\cos \theta$ 

$$10 \sin \theta \cos^2 \theta - 9 \sin \theta = 0$$

$$\sin \theta (10 \cos^2 \theta - 9) = 0$$

$$\text{Either } \sin \theta = 0 \text{ or } 10 \cos^2 \theta - 9 = 0$$

$$\theta = -180, 0, 180 \quad \text{or} \quad \cos^2 \theta = \frac{9}{10}$$

$$\begin{aligned} \cos \theta &= \pm \sqrt{\frac{9}{10}} \\ &= \pm \frac{3\sqrt{10}}{10} \\ &= \pm 0.9487 \end{aligned}$$

$$\theta = \pm 18.4$$

Solutions,  $-180^\circ, -18.4^\circ, 0^\circ, 18.4^\circ, 180^\circ$ **[ 6 marks ]****( b )**

$$5 \sin (2x - 50^\circ) = 9 \tan (x - 25^\circ)$$

$$\Rightarrow 5 \sin 2(x - 25^\circ) = 9 \tan (x - 25^\circ)$$

 $\therefore$  Smallest positive solution is,  $x - 25 = -18.4$ 

$$x = 6.6^\circ$$

**[ 2 marks ]**

**Answer 5**

( a )

$$\begin{aligned}\text{LHS} &= \frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} \\&= \frac{\cos 3\theta \cos \theta + \sin 3\theta \sin \theta}{\sin \theta \cos \theta} \times \frac{2}{2} \\&= \frac{\cos (3\theta - \theta)}{2 \sin \theta \cos \theta} \times 2 \\&= \frac{\cos 2\theta}{\sin 2\theta} \times 2 \\&= 2 \cot 2\theta \\&= \text{RHS} \quad \square\end{aligned}$$

[ 4 marks ]

( b )

$$\begin{aligned}\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} &= 4 \\2 \cot 2\theta &= 4 \\\cot 2\theta &= 2 \\\tan 2\theta &= \frac{1}{2} \\2\theta &= 26.6, 206.6, 386.6, \dots \\\theta &= 103.3^\circ\end{aligned}$$

[ 3 marks ]