7.5 Answers

7.5.1 Solution to 7.3 Example (Teaching Video)

$$R \sin(\theta + \alpha) = R \sin\theta \cos\alpha + R \cos\theta \sin\alpha$$

which is required to be $8 \sin\theta + 15 \cos\theta$

$$\therefore R \cos\alpha = 8 \quad \text{and} \quad R \sin\alpha = 15$$

Solving these two equations simultaneously by division,

$$\frac{R \sin\alpha}{R \cos\alpha} = \frac{15}{8}$$

$$\alpha = \arctan\left(\frac{15}{8}\right)$$

$$\alpha = 61.9°$$

$$R = \sqrt{8^2 + 15^2}$$

$$= 17$$

Conclusion :

$$8 \sin\theta + 15 \cos\theta = 17 \sin(\theta + 61.9°)$$

[4 marks]

7.5.2 Solutions to 7.4 Exercise

Answer 1

(i)

$$R \sin(\theta + \alpha) = R \sin\theta \cos\alpha + R \cos\theta \sin\alpha$$

which is required to be $2 \sin\theta + \sqrt{5} \cos\theta$

$$\therefore R \cos\alpha = 2 \quad \text{and} \quad R \sin\alpha = \sqrt{5}$$

Solving these two equations simultaneously by division,

$$\frac{R \sin\alpha}{R \cos\alpha} = \frac{\sqrt{5}}{2}$$

$$\alpha = \arctan\left(\frac{\sqrt{5}}{2}\right)$$

$$\alpha = 48.2°$$

$$R = \sqrt{2^2 + \left(\sqrt{5}\right)^2}$$

$$= 3$$

Conclusion :

$$2 \sin\theta + \sqrt{5} \cos\theta = 3 \sin(\theta + 48.2°)$$

[4 marks]

(ii) 3

[1 mark]

(iii) (-3)

[1 mark]

(iv)

$$2 \sin\theta + \sqrt{5} \cos\theta = 1.5$$

$$3 \sin(\theta + 48.2) = 1.5$$

$$\sin(\theta + 48.2) = \frac{1}{2}$$

$$\theta + 48.2 = 30, \; 150, \; 390, \; \ldots$$

$$\theta = 101.8°, \; 341.8°$$

[4 marks]

Answer 2

(i)

$$R \sin(\theta + \alpha) = R \sin\theta \cos\alpha + R \cos\theta \sin\alpha$$

which is required to be $\sqrt{3} \sin\theta + \cos\theta$

$$\therefore R \cos\alpha = \sqrt{3} \quad \text{and} \quad R \sin\alpha = 1$$

Solving these two equations simultaneously by division,

$$\frac{R \sin\alpha}{R \cos\alpha} = \frac{1}{\sqrt{3}}$$

$$\alpha = \arctan\left(\frac{1}{\sqrt{3}}\right)$$

$$\alpha = 30°$$

$$R = \sqrt{(\sqrt{3})^2 + 1^2}$$

$$= 2$$

Conclusion :

$$\sqrt{3} \sin\theta + \cos\theta = 2 \sin(\theta + 30°)$$

[4 marks]

(ii) (-2)

[1 mark]

(iii) $3 \sin\theta + \sqrt{3} \cos\theta = \sqrt{3}\left(\sqrt{3} \sin\theta + \cos\theta\right)$

$$= \sqrt{3} \times 2 \sin(\theta + 30)$$

which has a maximum value of $2\sqrt{3}$

[1 mark]

(iv)

$$\sqrt{3} \sin\theta + \cos\theta = \sqrt{2}$$

$$2 \sin(\theta + 30) = \sqrt{2}$$

$$\sin(\theta + 30) = \frac{\sqrt{2}}{2}$$

$$\theta + 30 = 45, \ 135, \ 405, \ \ldots$$

$$\theta = 15°, \ 105°$$

[4 marks]

Answer 3

(i)

$$R \sin(\theta + \alpha) = R \sin\theta \cos\alpha + R \cos\theta \sin\alpha$$

which is required to be $\sin\theta + \cos\theta$

$$\therefore R\cos\alpha = 1 \quad \text{and} \quad R\sin\alpha = 1$$

Solving these two equations simultaneously by division,

$$\frac{R\sin\alpha}{R\cos\alpha} = \frac{1}{1}$$

$$\alpha = \arctan(1)$$

$$\alpha = 45°$$

$$R = \sqrt{1^2 + 1^2}$$

$$= \sqrt{2}$$

Conclusion :

$$\sin\theta + \cos\theta = \sqrt{2}\sin(\theta + 45°)$$

[3 marks]

(ii) $\left(-\sqrt{2}\right)$

[1 mark]

(iii) $\frac{1}{\sin\theta + \cos\theta + 3} = \frac{1}{\sqrt{2}\sin(\theta + 45) + 3}$

which has a maximum value of $\frac{1}{-\sqrt{2} + 3} \times \frac{3 + \sqrt{2}}{3 + \sqrt{2}}$

$$= \frac{3 + \sqrt{2}}{7}$$

[2 mark]

(iv)

$$\sin\theta + \cos\theta = 1$$

$$\sqrt{2}\sin(\theta + 45) = 1$$

$$\sin(\theta + 45) = \frac{1}{\sqrt{2}}$$

$$\theta + 45 = 45,\ 135,\ 405,\ \ldots$$

$$\theta = 0°,\ 90°,\ 360°$$

[4 marks]

Answer 4

$$R\,sin\,(\,\theta\,+\,\alpha\,)\,=\,R\,sin\,\theta\,cos\,\alpha\,+\,R\,cos\,\theta\,sin\,\alpha$$

which is required to be $A\,sin\,\theta\,+\,B\,cos\,\theta$

$$\therefore\,R\,cos\,\alpha\,=\,A\quad\text{and}\quad R\,sin\,\alpha\,=\,B$$

Solving these two equations simultaneously by division,

$$\frac{R\,sin\,\alpha}{R\,cos\,\alpha}\,=\,\frac{B}{A}$$

$$\alpha\,=\,arctan\left(\frac{B}{A}\right)$$

[3 marks]

Answer 5

(i) $cos\,(\,\theta\,+\,\alpha\,)\,=\,cos\,\theta\,cos\,\alpha\,-\,sin\,\theta\,sin\,\alpha$

[1 mark]

(ii)

$$R\,cos\,(\,\theta\,+\,\alpha\,)\,=\,R\,cos\,\theta\,cos\,\alpha\,-\,R\,sin\,\theta\,sin\,\alpha$$

which is required to be $9\,cos\,\theta\,-\,12\,sin\,\theta$

$$\therefore\,R\,cos\,\alpha\,=\,9\quad\text{and}\quad R\,sin\,\alpha\,=\,12$$

Solving these two equations simultaneously by division,

$$\frac{R\,sin\,\alpha}{R\,cos\,\alpha}\,=\,\frac{12}{9}$$

$$\alpha\,=\,arctan\left(\frac{4}{3}\right)$$

$$\alpha\,=\,53.1^\circ$$

$$R\,=\,\sqrt{9^2\,+\,12^2}$$

$$=\,15$$

Conclusion :

$$9\,cos\,\theta\,-\,12\,sin\,\theta\,=\,15\,cos\,(\,\theta\,+\,53.1^\circ\,)$$

[3 marks]

(iii)

$$9\,cos\,\theta\,-\,12\,sin\,\theta\,=\,15$$

$$15\,cos\,(\,\theta\,+\,53.1\,)\,=\,15$$

$$cos\,(\,\theta\,+\,53.1\,)\,=\,1$$

$$\theta\,+\,53.1\,=\,0,\;360,\;...$$

$$\theta\,=\,306.9^\circ$$

[3 marks]

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