8.4 Answers

8.4.1 Solution to 8.2 Example (Teaching Video)

$$R \cos(\theta - \alpha) = R \cos\theta \cos\alpha + R \sin\theta \sin\alpha$$

which is required to be $4 \cos\theta + 3 \sin\theta$

$$\therefore R \cos\alpha = 4 \quad \text{and} \quad R \sin\alpha = 3$$

Solving these two equations simultaneously by division,

$$\frac{R \sin\alpha}{R \cos\alpha} = \frac{3}{4}$$

$$\alpha = arctan\left(\frac{3}{4}\right)$$

$$\alpha = 36.9°$$

$$R = \sqrt{4^2 + 3^2}$$

$$= 5$$

Conclusion :

$$4 \cos\theta + 3 \sin\theta = 5 \cos(\theta - 36.9°)$$

[4 marks]

8.4.2 Solutions to 8.3 Exercise

Answer 1

(i)

$$R\,cos\,(\,\theta\,-\,\alpha\,)\;=\;R\,cos\,\theta\,cos\,\alpha\,+\,R\,sin\,\theta\,sin\,\alpha$$

which is required to be $15\,cos\,\theta\,+\,36\,sin\,\theta$

$$\therefore\;R\,cos\,\alpha\;=\;15\quad\text{and}\quad R\,sin\,\alpha\;=\;36$$

Solving these two equations simultaneously by division,

$$\frac{R\,sin\,\alpha}{R\,cos\,\alpha}\;=\;\frac{36}{15}$$

$$\alpha\;=\;arctan\left(\frac{12}{5}\right)$$

$$\alpha\;=\;67.4°$$

$$R\;=\;\sqrt{15^2\,+\,36^2}$$

$$=\;39$$

Conclusion :

$$15\,cos\,\theta\,+\,36\,sin\,\theta\;=\;39\,cos\,(\,\theta\,-\,67.2°\,)$$

[3 marks]

(ii)

$$15\,cos\,\theta\,+\,36\,sin\,\theta\;=\;13$$

$$39\,cos\,(\,\theta\,-\,67.4°\,)\;=\;13$$

$$cos\,(\,\theta\,-\,67.4°\,)\;=\;\frac{1}{3}$$

$$\theta\,-\,67.4\;=\;70.5,\;\;289.5$$

$$\theta\;=\;137.9°,\;\;356.9°$$

[3 marks]

Answer 2

(a)

$$R\cos(\theta + \alpha) = R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$$

which is required to be $12\cos\theta - 4\sin\theta$

$$\therefore R\cos\alpha = 12 \quad \text{and} \quad R\sin\alpha = 4$$

Solving these two equations simultaneously by division,

$$\frac{R\sin\alpha}{R\cos\alpha} = \frac{4}{12}$$

$$\alpha = arctan\left(\frac{1}{3}\right)$$

$$\alpha = 18.43°$$

$$R = \sqrt{12^2 + 4^2}$$

$$= 4\sqrt{10} \quad \text{(As a decimal, 12.6)}$$

Conclusion :

$$12\cos\theta - 4\sin\theta = 4\sqrt{10}\cos(\theta + 18.43°)$$

[4 marks]

(b)

$$12\cos\theta - 4\sin\theta = 7$$

$$4\sqrt{10}\cos(\theta + 18.43°) = 7$$

$$\cos(\theta + 18.43°) = 0.5534$$

$$\theta + 18.43 = 56.40, \ 303.60$$

$$\theta = 38°, \ 285.2°$$

[5 marks]

(c) **(i)** $-4\sqrt{10}$

[1 mark]

(ii)

$$\cos(\theta + 18.43) = -1$$

$$\theta + 18.43 = 180$$

$$\theta = 161.57°$$

[2 marks]

Answer 3

(a)

$$R\,cos\,(\,\theta\;-\;\alpha\,)\;=\;R\,cos\,\theta\;cos\,\alpha\;+\;R\,sin\,\theta\;sin\,\alpha$$

which is required to be $3\,cos\,\theta\;+\;4\,sin\,\theta$

$$\therefore\;R\,cos\,\alpha\;=\;3\quad\text{and}\quad R\,sin\,\alpha\;=\;4$$

Solving these two equations simultaneously by division,

$$\frac{R\,sin\,\alpha}{R\,cos\,\alpha}\;=\;\frac{4}{3}$$

$$\alpha\;=\;arctan\left(\frac{4}{3}\right)$$

$$\alpha\;=\;53.13°$$

$$R\;=\;\sqrt{3^2\;+\;4^2}$$

$$=\;5$$

Conclusion :

$$3\,cos\,\theta\;+\;4\,sin\,\theta\;=\;5\,cos\,(\,\theta\;-\;53.13°\,)$$

[4 marks]

(b) Maximum value is 5 which occurs when,

$$cos\,(\,\theta\;-\;53.13\,)\;=\;1$$

$$\theta\;-\;53.13\;=\;0$$

$$\theta\;=\;53.13°$$

[3 marks]

(c) $f\,(t)\;=\;10\;+\;3\,cos\;(15t)\;+\;4\,sin\;(15t)$

$$=\;10\;+\;5\,cos\,(\,15t\;-\;53.13\,)$$

$\therefore$ The minimum temperature of the warehouse is 5 (degrees Centigrade, perhaps)

[2 marks]

(d) This minimum will occur when, $cos\,(\,15t\;-\;53.13\,)\;=\;-1$

$$15t\;-\;53.13\;=\;180$$

$$15t\;=\;233.13$$

$$t\;=\;15.54$$

This is 15 hours and 32 minutes after midday which is 3.32 am the following morning

[3 marks]

Answer 4

(a) The immediate problem is the different number of $sin^2\,\theta$ and $cos^2\,\theta$

They can be evened up as follows,

$$5\,sin^2\,\theta \;-\; 3\,cos^2\,\theta \;+\; 6\,sin\,\theta\,cos\,\theta$$

$$=\; 4\,sin^2\,\theta \;+\; sin^2\,\theta \;-\; 3\,cos^2\,\theta \;+\; 6\,sin\,\theta\,cos\,\theta$$

$$=\; 4\,sin^2\,\theta \;+\; (\,1 \;-\; cos^2\,\theta\,) \;-\; 3\,cos^2\,\theta \;+\; 6\,sin\,\theta\,cos\,\theta$$

$$=\; (-\,4)\,(\,cos^2\,\theta \;-\; sin^2\,\theta\,) \;+\; 1 \;+\; 3\;(\,2\,sin\,\theta\,cos\,\theta\,)$$

$$=\; (-\,4)\;cos\,2\theta \;+\; 1 \;+\; 3 \;\times\; sin\,2\theta$$

$$=\; 3\,sin\,2\theta \;-\; 4\,cos\,2\theta \;+\; 1$$

i.e. $a \;=\; 3,\;\; b \;=\; -\,4$ and $c \;=\; 1$

[3 marks]

(b)

$$R\,sin\,(\,2\theta \;-\; \alpha\,) \;=\; R\,sin\,2\theta\,cos\,\alpha \;-\; R\,cos\,2\theta\,sin\,\alpha$$

which is required to be $3\,sin\,2\theta \;-\; 4\,cos\,2\theta$

$$\therefore\; R\,cos\,\alpha \;=\; 3 \quad \text{and} \quad R\,sin\,\alpha \;=\; 4$$

Solving these two equations simultaneously by division,

$$\frac{R\,sin\,\alpha}{R\,cos\,\alpha} \;=\; \frac{4}{3}$$

$$\alpha \;=\; arctan\left(\frac{4}{3}\right)$$

$$\alpha \;=\; 53.13°$$

$$R \;=\; \sqrt{3^2 \;+\; 4^2}$$

$$=\; 5$$

Conclusion :

$$3\,sin\,2\theta \;-\; 4\,cos\,2\theta \;=\; 5\,sin\,(\,2\theta \;-\; 53.13°\,)$$

$$5\,sin^2\,\theta \;-\; 3\,cos^2\,\theta \;+\; 6\,sin\,\theta\,cos\,\theta \;=\; 3\,sin\,2\theta \;-\; 4\,cos\,2\theta \;+\; 1$$

$$=\; 5\,sin\,(\,2\theta \;-\; 53.13°\,) \;+\; 1$$

which will have a maximum value of 6 and a minimum value of $(-\,4)$

[3 marks]

(c)

$$\begin{aligned}
5\,sin^2\,\theta \;-\; 3\,cos^2\,\theta \;+\; 6\,sin\,\theta\,cos\,\theta &= -1 \\
3\,sin\,2\theta \;-\; 4\,cos\,2\theta \;+\; 1 &= -1 \\
5\,sin\,(\,2\theta \;-\; 53.13°\,) \;+\; 1 &= -1 \\
5\,sin\,(\,2\theta \;-\; 53.13°\,) &= -2 \\
sin\,(\,2\theta \;-\; 53.13°\,) &= -\frac{2}{5} \\
2\theta \;-\; 53.13 &= -23.58,\;\; 203.58 \\
2\theta &= 29.55,\;\; 256.71 \\
\theta &= 14.8°,\;\; 128.4°
\end{aligned}$$

[4 marks]