9.2 Answers

Answer 1

(i) $60°, 120°, 420°, 480°$

[4 marks]

(ii) $2\theta + 10 = 60,\ 120,\ 420,\ 480$

$2\theta = 50,\ 110,\ 410,\ 470$

$\theta = 25°,\ 55°,\ 205°,\ 235°$

[2 marks]

Answer 2

$$\begin{aligned} \text{LHS} &= (\cos\theta + \sin\theta)(\cos\theta - \sin\theta) + 1 \\ &= \cos^2\theta - \sin^2\theta + 1 \\ &= \cos^2\theta - (1 - \cos^2\theta) + 1 \\ &= \cos^2\theta - 1 + \cos^2\theta + 1 \\ &= 2\cos^2\theta \\ &= \text{RHS} \qquad \square \end{aligned}$$

[5 marks]

Answer 3

$$\begin{aligned} \text{LHS} &= \tan 75 \\ &= \tan(45 + 30) \\ &= \frac{\tan 45 + \tan 30}{1 - \tan 45 \tan 30} \\ &= \frac{1 + \frac{\sqrt{3}}{3}}{1 - 1 \times \frac{\sqrt{3}}{3}} \times \frac{3}{3} \\ &= \frac{3 + \sqrt{3}}{3 - \sqrt{3}} \qquad \square \\ &= \frac{3 + \sqrt{3}}{3 - \sqrt{3}} \times \frac{3 + \sqrt{3}}{3 + \sqrt{3}} \\ &= \frac{9 + 6\sqrt{3} + 3}{9 - 3} \\ &= \frac{12 + 6\sqrt{3}}{6} \\ &= 2 + \sqrt{3} \end{aligned}$$

i.e. $a = 2$ and $b = 1$

[6 marks]

Answer 4

$$\begin{aligned} \text{LHS} &= \csc^2\theta + \sec^2\theta \\ &= \frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta} \\ &= \frac{\cos^2\theta + \sin^2\theta}{\sin^2\theta\cos^2\theta} \\ &= \frac{1}{\sin^2\theta\cos^2\theta} \\ &= \frac{1}{\sin^2\theta} \times \frac{1}{\cos^2\theta} \\ &= \csc^2\theta\sec^2\theta \\ &= \text{RHS} \qquad \square \end{aligned}$$

[5 marks]

Answer 5

(i) $R\sin(\theta + \alpha) = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$

which is required to be $5\sin\theta + 12\cos\theta$

$\therefore R\cos\alpha = 5$ and $R\sin\alpha = 12$

Solving these two equations simultaneously by division,

$$\frac{R\sin\alpha}{R\cos\alpha} = \frac{12}{5}$$

$$\alpha = \arctan\left(\frac{12}{5}\right)$$

$$\alpha = 67.4°$$

$$R = \sqrt{5^2 + 12^2}$$

$$= 13$$

Conclusion :

$$5\sin\theta + 12\cos\theta = 13\sin(\theta + 67.4°)$$

[4 marks]

(ii) $5\sin\theta + 12\cos\theta = 9$

$$13\sin(\theta + 67.4) = 9$$

$$\sin(\theta + 67.4) = 0.6923$$

$$\theta + 67.4 = 43.81,\ 136.19,\ 403.81$$

$$\theta = 68.79°,\ 336.41°$$

[4 marks]

Answer 6

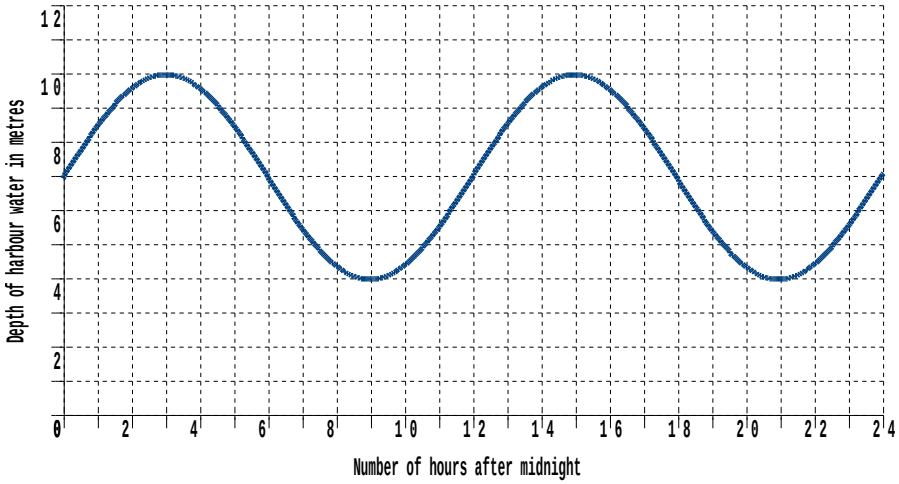
(a) For D to be a maximum, $sin\,(30t) = 1$

$$30t = 90,\ 450,\ \ldots$$

$$t = 3,\ 15,\ \ldots$$

i.e. High tide at 3 am and 3 pm

[2 marks]



(b) To find the critical times,

$$7 + 3\,sin\,(30t) = 6$$

$$3\,sin\,(30t) = -1$$

$$sin\,(30t) = -\frac{1}{3}$$

$$30t = 199.5,\ 340.5,\ 559.5,\ 700.5$$

$$t = 6.65,\ 11.35,\ 18.65,\ 23.35 \text{ hours}$$

The boat must leave before 18 hours, 39 minutes

i.e. Before 6.39 pm

[8 marks]

Answer 7

(i)

$$\begin{aligned}
\text{LHS} &= \cos 3\theta \\
&= \cos\,(2\theta + \theta) \\
&= \cos 2\theta \cos\theta - \sin 2\theta \sin\theta \\
&= (\cos^2\theta - \sin^2\theta)\cos\theta - 2\sin\theta\cos\theta\sin\theta \\
&= \cos^3\theta - \sin^2\theta\cos\theta - 2\sin^2\theta\cos\theta \\
&= \cos^3\theta - 3\sin^2\theta\cos\theta \\
&= \cos^3\theta - 3\left(1 - \cos^2\theta\right)\cos\theta \\
&= \cos^3\theta - 3\cos\theta + 3\cos^3\theta \\
&= 4\cos^3\theta - 3\cos\theta \\
&= \text{RHS} \qquad \square
\end{aligned}$$

[5 marks]

(ii)

$$\begin{aligned}
\cos 3\theta + \cos\theta &= 0 \\
4\cos^3\theta - 3\cos\theta + \cos\theta &= 0 \\
4\cos^3\theta - 2\cos\theta &= 0 \\
2\cos\theta\left(2\cos^2\theta - 1\right) &= 0
\end{aligned}$$

Either $2\cos\theta = 0$ or $2\cos^2\theta - 1 = 0$

$\theta = 90°,\ 270°$ $\qquad\qquad$ $\cos^2\theta = \frac{1}{2}$

$$\cos\theta = \pm\frac{\sqrt{2}}{2}$$

$$\theta = 45°,\ 135°,\ 225°,\ 315°$$

$\therefore$ Solutions are $45°,\ 90°,\ 135°,\ 225°,\ 270°,\ 315°$

[5 marks]