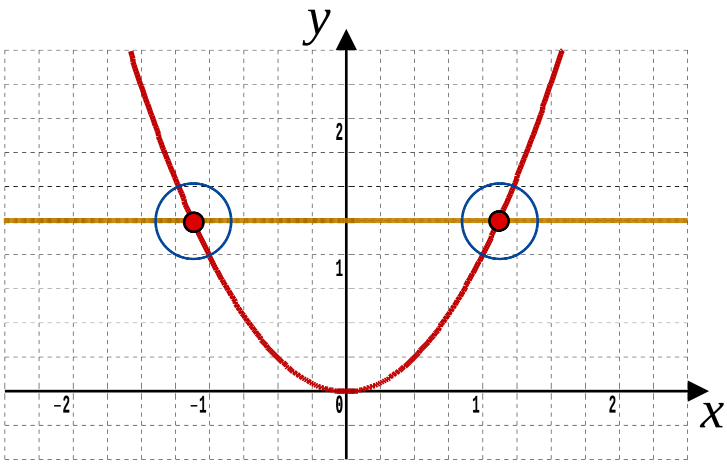
1.10 Answers

1.10.1 Solutions to Exercise 1.9

Answer 1
(i), (ii)



$y = f(x)$ (red) with a horizontal line (gold) cutting $f(x)$ more than once

[2, 1 marks]

(iii) Range: $f(x) \in \mathbb{R},\ f(x) \geqslant 0$
"All real numbers that are non-negative"

[2 marks]

Answer 2
Every vertical line that can be drawn that cuts the hyperbola more than once.
To be a function, every vertical line (within the function's domain) cuts the graph of the function precisely once.

[2 marks]

Answer 3

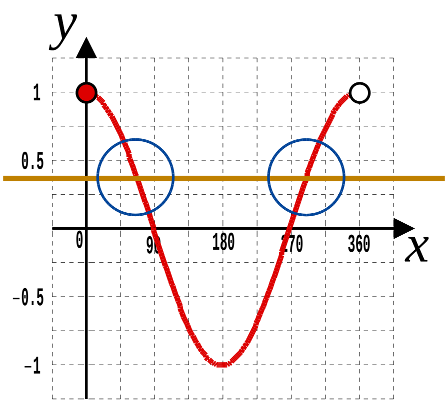
Excluded from the domain: $x = -1$

Reason : If included it would cause a division by zero.

[2 marks]

Answer 4

(i), (ii)



$y = f(x)$ (red) with a horizontal line (gold) cutting $f(x)$ more than once

[3, 1 marks]

(iii) Range: $f(x) \in \mathbb{R}, \ -1 \leqslant f(x) \leqslant 1$

"All real numbers between – 1 and 1 inclusive"

[2 marks]

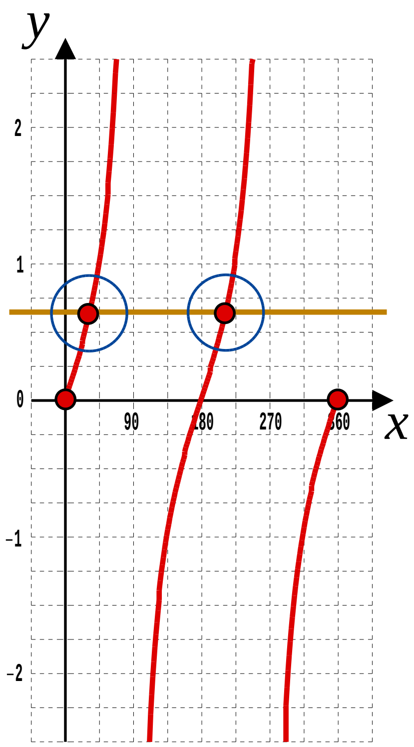
Answer 5

Domain: $x \in \mathbb{R}, \ 0 \leqslant x < 5$

[2 marks]

Answer 6

(i), (ii)



$y = f(x)$ (red) with a horizontal line (gold) cutting $f(x)$ more than once

[2, 1 marks]

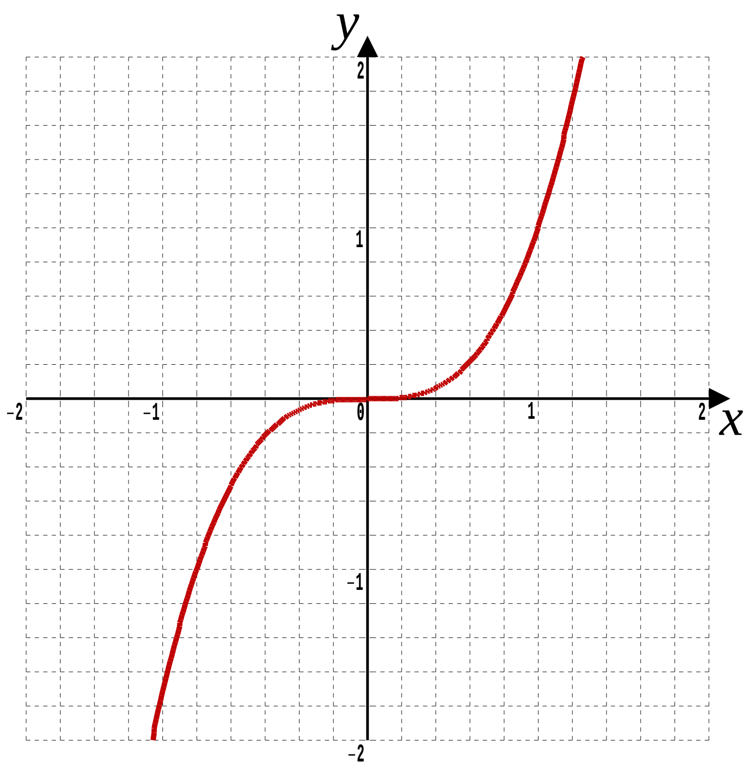
(iii) Domain: $x \in \mathbb{R},\ 0 \leqslant x \leqslant 360°,\ x \neq 90°,\ 270°$

$\tan x = \dfrac{\sin x}{\cos x}$ but $\cos x = 0$ when $x = 90°$ or $x = 270°$

The need to exclude 90° and 270° from the domain of *tan x* is thus, once again, arising from the need to avoid division by zero.

[3 marks]

Answer 7
(i)



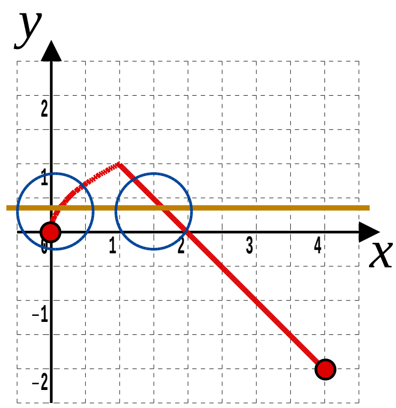
$y \;=\; f\,(x)$ (red). No horizontal line cuts $f\,(x)$ more than once

[2 marks]

(ii) The cubing function is one-to-one.

[2 marks]

Answer 8
(i), (ii)



$y \;=\; f\,(x)$ (red) with a horizontal line (gold) cutting $f\,(x)$ more than once

[3, 1 marks]

(iii) Range: $f\,(x) \in \mathbb{R}, \quad -2 \leqslant f\,(x) \leqslant 1$

[3 marks]

Answer 9

Range: $f(x) \in \mathbb{R},\ 2 \leqslant f(x) \leqslant 4$

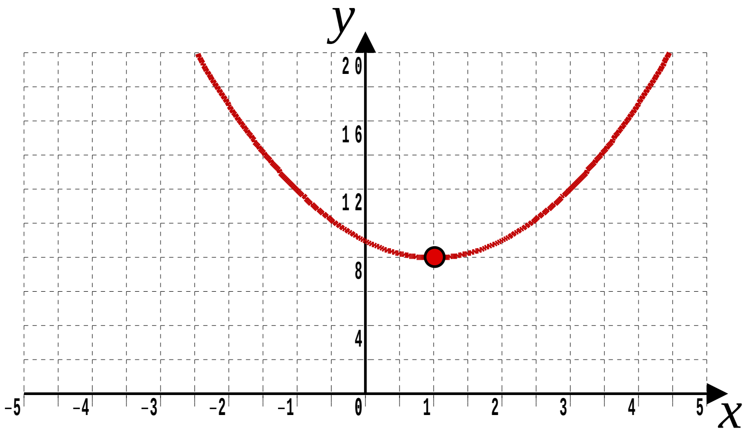
[3 marks]

Answer 10

(i) $f(x) = (x - 1)^2 + 8$ That is, $a = 1$ and $b = 8$

[2 marks]

(ii)

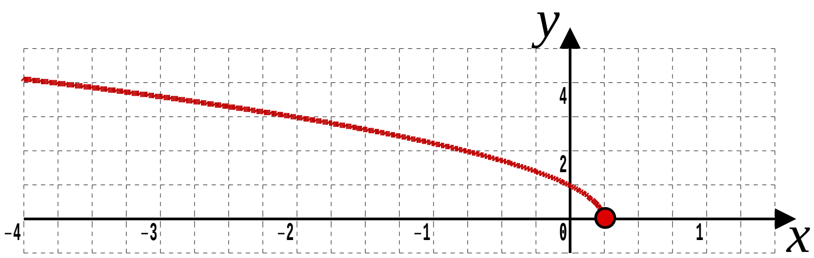


[2 marks]

(iii) Range: $f(x) \in \mathbb{R},\ f(x) \geqslant 8$

[2 marks]

Answer 11



(i) Domain: $x \in \mathbb{R},\ x \leqslant \dfrac{1}{4}$

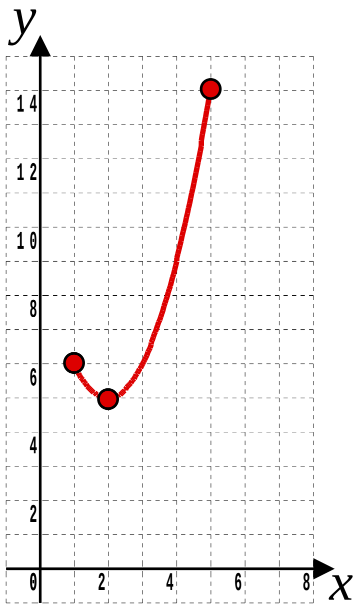
[2 marks]

(ii) Range: $f(x) \in \mathbb{R},\ f(x) \geqslant 0$

[2 marks]

Answer 12

(i)



[3 marks]

(ii) Range: $f(x) \in \mathbb{R}, \quad 5 \leqslant f(x) \leqslant 14$

[2 marks]

(iii)

$$f(x) = 9$$

$$(x - 2)^2 + 5 = 9$$

$$x^2 - 4x + 4 + 5 = 9$$

$$x^2 - 4x = 0$$

$$x(x - 4) = 0$$

Either $x = 0$ (rejected) or $x = 4$ (the only solution).

[2 marks]

Answer 13
Here are two methods of finding the local minimum point.

Method 1 : Completing The Square

$$\begin{aligned} f(x) &= 2x^2 + 3kx + k^2 \\ &= \left[\, 2x^2 + 3kx \,\right] + k^2 \\ &= 2\left[\, x^2 + \frac{3k}{2}x \,\right] + k^2 \\ &= 2\left[\, \left(x + \frac{3k}{4} \right)^2 - \frac{9k^2}{16} \,\right] + k^2 \\ &= 2\left(x + \frac{3k}{4} \right)^2 - \frac{9k^2}{8} + \frac{8k^2}{8} \\ &= 2\left(x + \frac{3k}{4} \right)^2 - \frac{k^2}{8} \end{aligned}$$

$$\therefore \text{ Minimum is } \left(-\frac{3k}{4}, -\frac{k^2}{8} \right)$$

Method 2 : Differentiation

$$\begin{aligned} f(x) &= 2x^2 + 3kx + k^2 \\ f'(x) &= 4x + 3k \\ \text{At the minimum point, } f'(x) &= 0 \\ 4x + 3k &= 0 \\ x &= -\frac{3k}{4} \\ f\left(-\frac{3k}{4} \right) &= 2\left(-\frac{3k}{4} \right)^2 + 3k\left(-\frac{3k}{4} \right) + k^2 \\ &= \frac{9k^2}{8} - \frac{9k^2}{4} + k^2 \\ &= \frac{9k^2 - 18k^2 + 8k^2}{8} \\ &= -\frac{k^2}{8} \end{aligned}$$

$$\therefore \text{ Minimum is } \left(-\frac{3k}{4}, -\frac{k^2}{8} \right)$$

As $-\dfrac{3k}{4}$ lies in the domain $(-4k \leqslant x \leqslant 0)$ the $-\dfrac{k^2}{8}$ is the minimum value.

Each endpoint of the segment of curve now needs to be determined to see which is the maximum value of the function.

For the endpoint with an x value of $-4k$,

$$\begin{aligned} f(-4k) &= 2(-4k)^2 + 3k(-4k) + k^2 \\ &= 32k^2 - 12k^2 + k^2 \\ &= 21k^2 \end{aligned}$$

For the endpoint with an x value of 0,

$$\begin{aligned} f(0) &= 2 \times 0^2 + 3k \times 0 + k^2 \\ &= k^2 \end{aligned}$$

So the maximum value is $21k^2$

The range is thus $f(x) \in \mathbb{R}, \ -\dfrac{k^2}{8} \leqslant f(x) \leqslant 21k^2$

[6 marks]