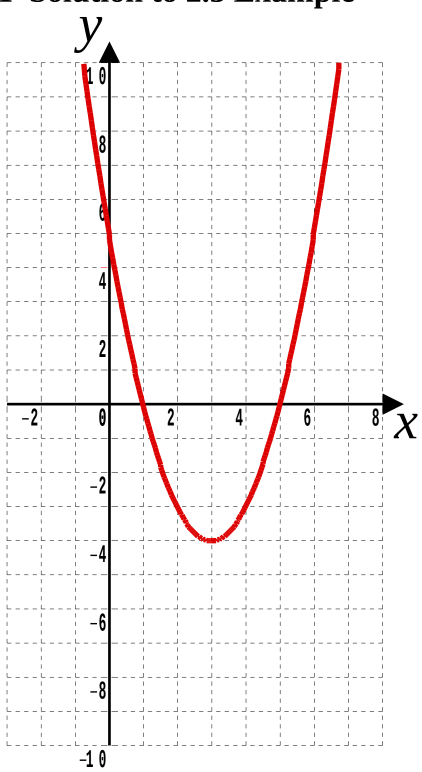
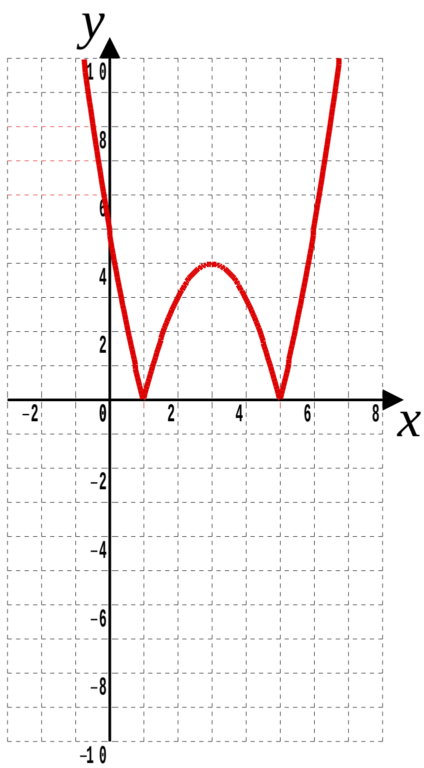
2.7 Answers

2.7.1 Solution to 2.3 Example



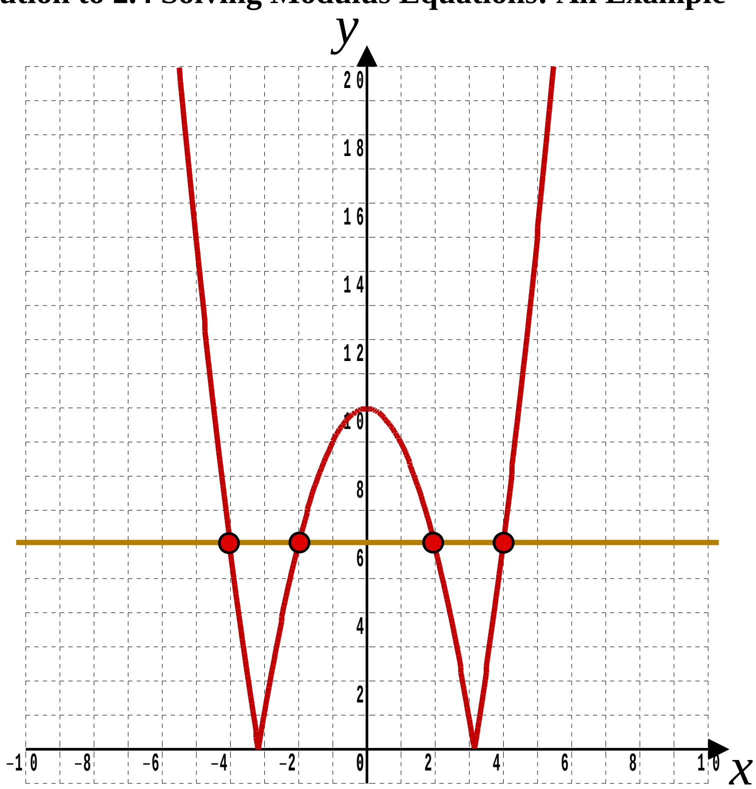
$y = x^2 - 4$



$y = |x^2 - 4|$

[2, 2 marks]

2.7.2 Solution to 2.4 Solving Modulus Equations: An Example



(i) From the graph provided (red) with the addition of the line $y = 6$ (gold) the solutions are $x = \pm 4,\ x = \pm 2$

[2 marks]

(ii) Strategy: Replace the modulus with ordinary brackets and consider the two possible signs in front of those brackets.

$$+\left(x^2 - 10\right) = 6 \qquad\qquad -\left(x^2 - 10\right) = 6$$

$$x^2 = 16 \qquad\qquad -x^2 = -4$$

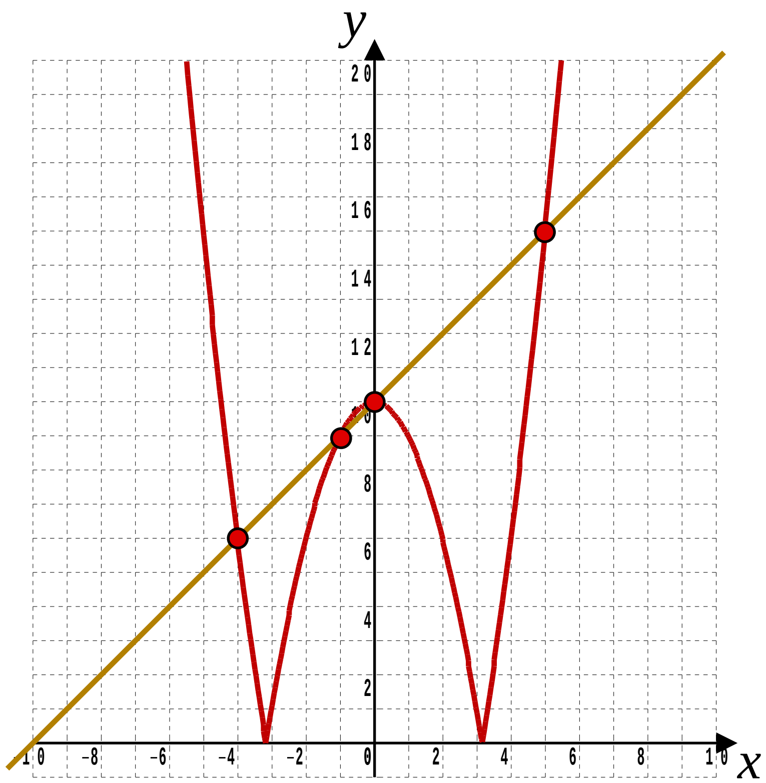
$$x = \pm 4 \qquad\qquad x^2 = 4$$

$$x = \pm 2$$

$$x \in \{\pm 2, \pm 4\}$$

[4 marks]

(iii)



[1 mark]

(iv) Part (iii) graph gives, $x = -4,\ x = -1,\ x = 0,\ x = 5$

[2 marks]

(v)

$$\left|x^2 - 10\right| = x + 10$$

$$+\left(x^2 - 10\right) = x + 10 \qquad\qquad -\left(x^2 - 10\right) = x + 10$$

$$x^2 - x - 20 = 0 \qquad\qquad x^2 + x = 0$$

$$(x + 4)(x - 5) = 0 \qquad\qquad x(x + 1) = 0$$

$$x = -4 \text{ or } x = 5 \qquad \text{or} \qquad x = 0 \text{ or } x = -1$$

[4 marks]

2.7.2 Solution to 2.6 Exercise

Answer 1

(i) $|x - 4| = 3, \quad x \in \mathbb{R}$

$-(x-4) = 3 \qquad +(x-4) = 3$

$-x + 4 = 3 \qquad x = 7$

$x = 1$

$x \in \{1,\ 7\}$

[2 marks]

(ii) $|x - 7| = 15, \quad x \in \mathbb{R}$

$-(x-7) = 15 \qquad +(x-7) = 15$

$-x + 7 = 15 \qquad x = 22$

$x = -8$

$x \in \{-8,\ 22\}$

[2 marks]

(iii) $|9 - x| = 2, \quad x \in \mathbb{R}$

$-(9-x) = 2 \qquad +(9-x) = 2$

$-9 + x = 2 \qquad -x = -7$

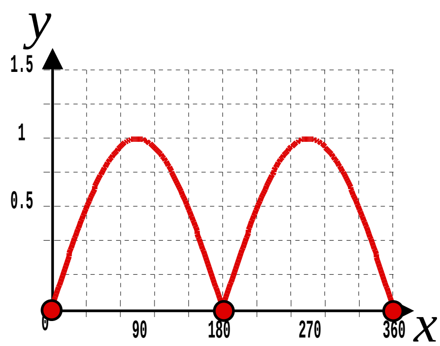
$x = 11 \qquad x = 7$

$x \in \{7,\ 11\}$

[2 marks]

Answer 2

(i)



[2 marks]

(ii) $-(\sin x) = 0.5 \qquad +(\sin x) = 0.5$

$\sin x = -0.5 \qquad x = 30,\ 150$

$x = 210,\ 330$

$x \in \{30,\ 150,\ 210,\ 330\}$

[2 marks]

Answer 3

$$|x - 2| - 3 = 1$$

$$-(x - 2) - 3 = 1 \qquad\qquad +(x - 2) - 3 = 1$$

$$-x + 2 - 3 = 1 \qquad\qquad x = 6$$

$$x = -2$$

$$x \in \{-2, 6\}$$

[3 marks]

Answer 4

$$\left|\frac{2}{x - 3}\right| = 3$$

$$-\left(\frac{2}{x - 3}\right) = 3 \qquad\qquad +\left(\frac{2}{x - 3}\right) = 3$$

$$-2 = 3(x - 3) \qquad\qquad 2 = 3(x - 3)$$

$$-2 = 3x - 9 \qquad\qquad 2 = 3x - 9$$

$$3x = 7 \qquad\qquad 3x = 11$$

$$x = \frac{7}{3} \qquad\qquad x = \frac{11}{3}$$

$$x \in \left\{\frac{7}{3}, \frac{11}{3}\right\}$$

[3 marks]

Answer 5

$$2 - |x + 1| = \frac{1}{2}x$$

$$2 - (x + 1) = \frac{1}{2}x \qquad\qquad 2 + (x + 1) = \frac{1}{2}x$$

$$4 - 2x - 2 = x \qquad\qquad 4 + 2x + 2 = x$$

$$2 = 3x \qquad\qquad x = -6$$

$$x = \frac{2}{3}$$

$$x \in \left\{-6, \frac{2}{3}\right\}$$

[3 marks]

Answer 6

$$4 - x^2 = |2x - 1|$$

Case 1 :

$$4 - x^2 = -(2x - 1)$$

$$x^2 - 2x - 3 = 0$$

$$(x + 1)(x - 3) = 0$$

$$x = -1 \text{ or } x = 3$$

Case 2 :

$$4 - x^2 = +(2x - 1)$$

$$x^2 + 2x - 5 = 0$$

$$x = \frac{-2 \pm \sqrt{4 - 4 \times 1 \times (-5)}}{2}$$

$$= \frac{-2 \pm \sqrt{24}}{2}$$

$= -1 \pm \sqrt{6}$ exact solutions asked for, but,

as approximations, $x = -3.449$ or $x = 1.449$

Combining cases,

$$x \in \{-1 \pm \sqrt{6}, -1, 3\}$$

[5 marks]

Answer 7

$$||x - 4| - 2| = 1$$

Case 1 :

$$-(|x - 4| - 2) = 1$$

$$-|x - 4| + 2 = 1$$

$$|x - 4| = 1$$

$$x = 3 \text{ or } x = 5$$

Case 2 :

$$(|x - 4| - 2) = 1$$

$$|x - 4| = 3$$

$$x = 1 \text{ or } x = 7$$

Combining cases,

$$x \in \{1, 3, 5, 7\}$$

[5 marks]

Answer 8

(i) Let a be a given positive value of x, in which case,

$$f \mid -a \mid \; = \; f \mid a \mid$$

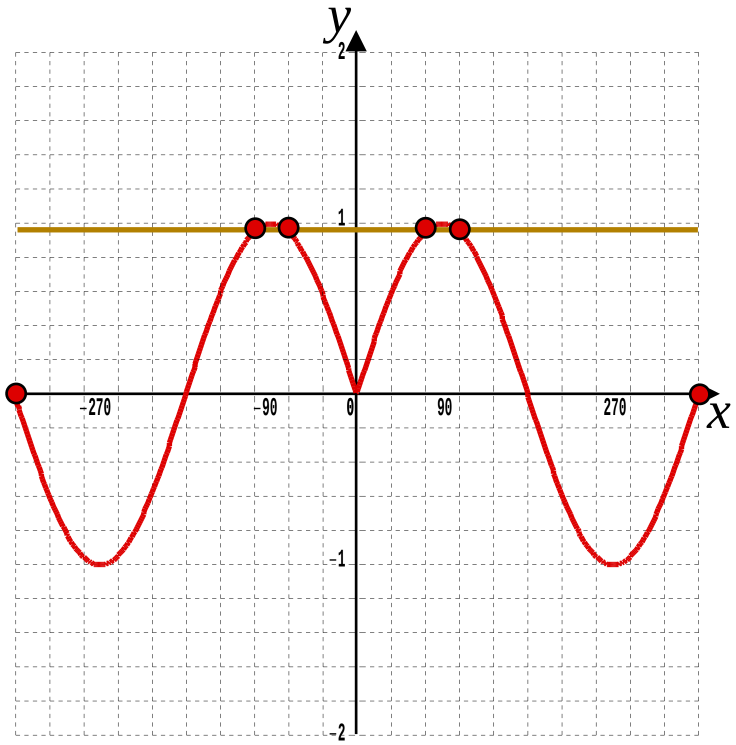
which has mirror symmetry in the y-axis.

An Even Function

In general if $f\,(-\,a) \; = \; f\,(a)$
then the function is described as being "even"
and has mirror symmetry in the *y-axis*

[2 marks]

(ii)



[2 marks]

(iii) $x \; \in \; \{-\,120, \; -\,60, \; 60, \; 120 \;\}$

[2 marks]

Extra :

An Odd Function

In general if $f\,(-\,a) \; = \; -\,f\,(a)$
then the function is described as being "odd"
and has half-turn rotational symmetry about the origin

Answer 9

$$\left| x^2 - \pi \right| = x + \pi$$

Case 1 :

$$-\left(x^2 - \pi \right) = x + \pi$$
$$-x^2 + \pi = x + \pi$$
$$x^2 + x = 0$$
$$x\,(x + 1) = 0$$
$$x = -1 \text{ or } x = 0$$

Case 2 :

$$+\left(x^2 - \pi \right) = x + \pi$$
$$x^2 - x - 2\pi = 0$$
$$x = \frac{1 \pm \sqrt{1 - 4 \times 1 \times (-2\pi)}}{2}$$
$$= \frac{1 \pm \sqrt{1 + 8\pi}}{2}$$
$$= -2.056 \text{ or } 3.056$$

Combining cases,

$$x \in \{-2.056, -1, 0, 3.056\}$$

[5 marks]