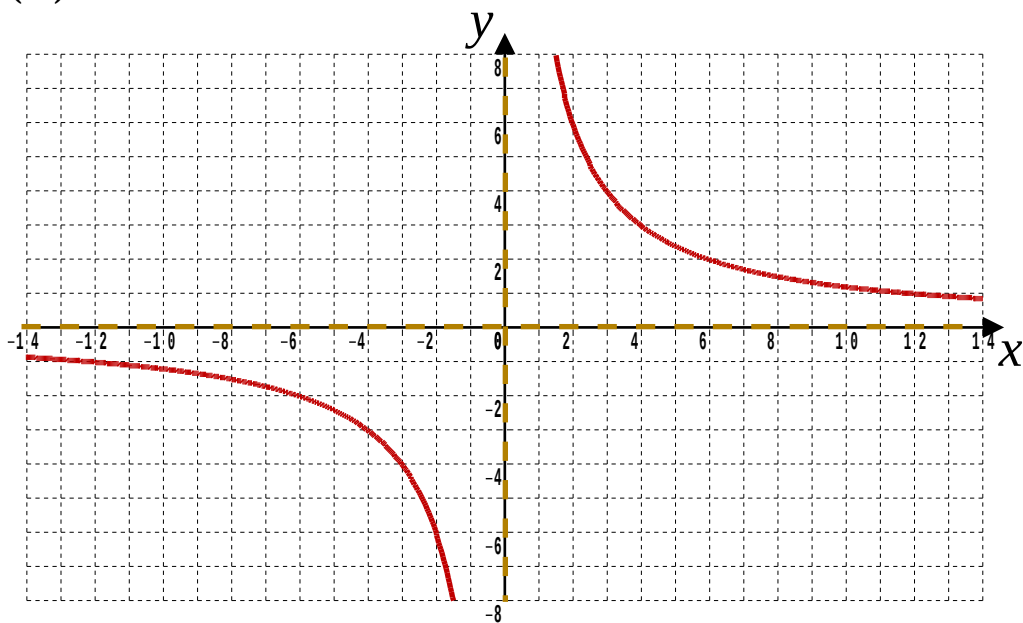


### 3.6 Answers

#### A-Level Pure Mathematics, Year 2 Functions II

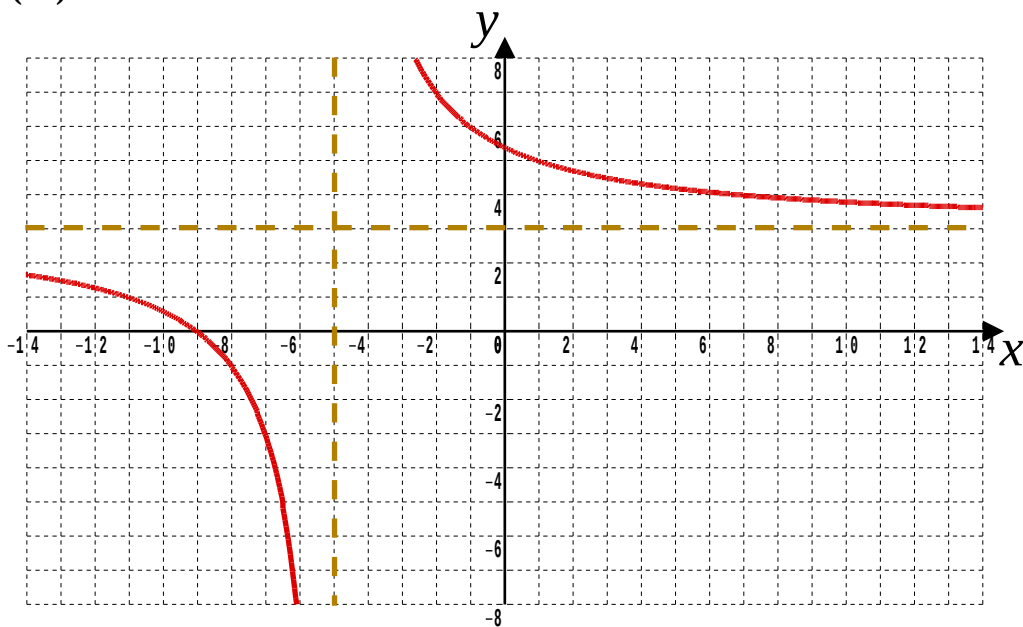
##### 3.6.1 Solution to 3.3 An Inverse Proportion Example

(a)



[ 2 marks ]

(b)



[ 2 marks ]

(c) (i) Domain:  $x \in \mathbb{R}, x \neq -5$

[ 1 mark ]

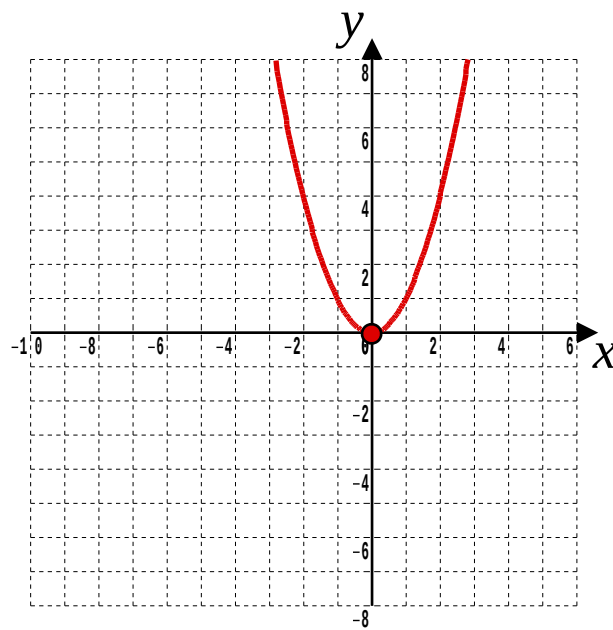
(ii) Range:  $g(x) \in \mathbb{R}, g(x) \neq 3$

[ 1 mark ]

### 3.6.2 Solution to 3.5 Exercise

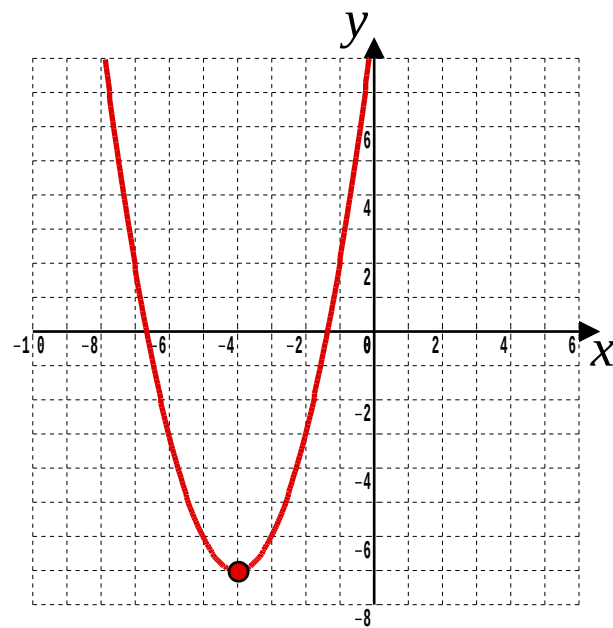
Answer 1

( a )



[ 1 mark ]

( b )



[ 2 marks ]

( c )    ( i )    Domain:  $x \in \mathbb{R}$

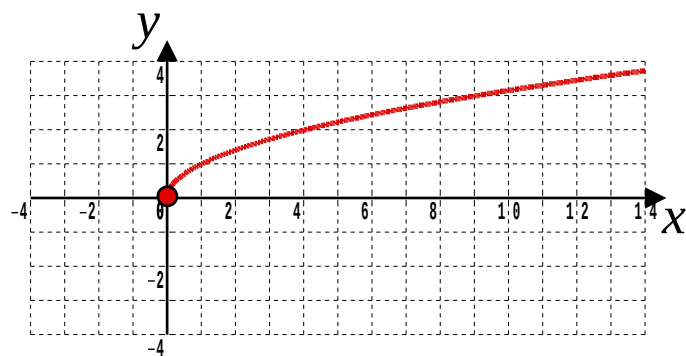
[ 1 mark ]

( ii )    Range:  $g(x) \in \mathbb{R}, g(x) \geq -7$

[ 1 mark ]

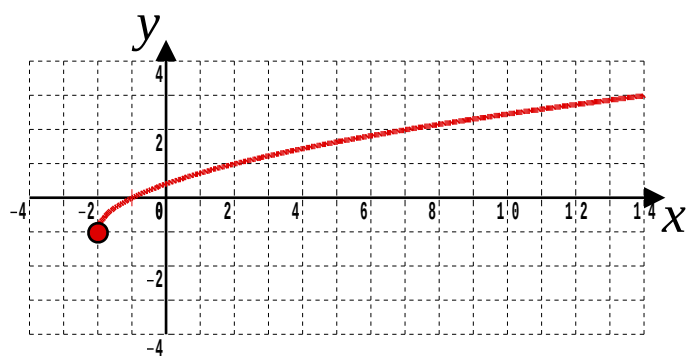
**Answer 2**

**( a )**



**[ 2 marks ]**

**( b )**



**[ 2 marks ]**

**( c )    ( i )    Domain:     $x \in \mathbb{R}, x \geq -2$**

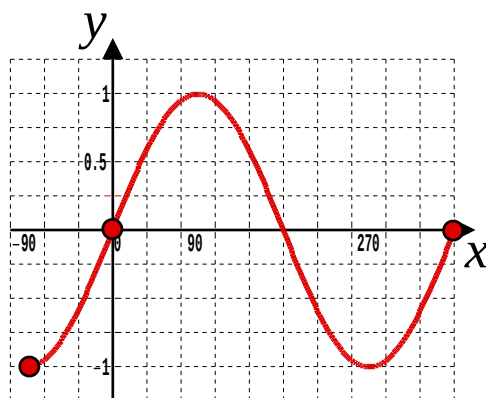
**[ 1 mark ]**

**( ii )    Range:  $g(x) \in \mathbb{R}, g(x) \geq -1$**

**[ 1 mark ]**

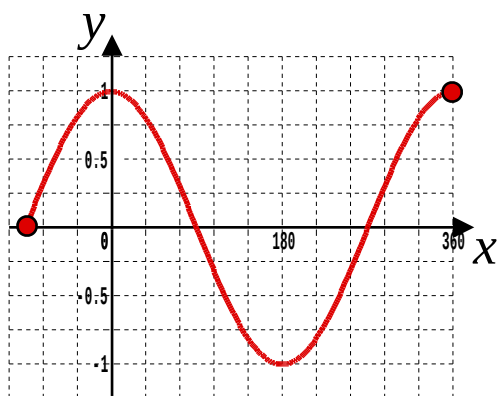
**Answer 3**

(i)



[ 1 mark ]

(ii)



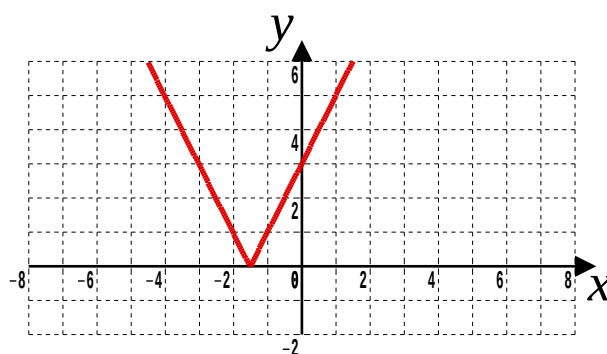
[ 2 marks ]

(iii)

$$\sin (x + 90^\circ) = \cos (x)$$

[ 1 mark ]

**Answer 4**



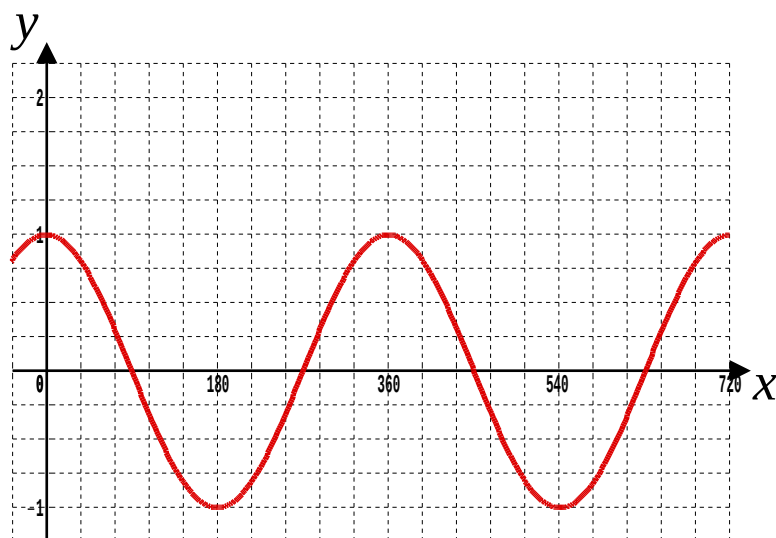
Contact points are,

$$\left(-\frac{3}{2}, 0\right), (0, 3)$$

[ 3 marks ]

**Answer 5**

**( i )**

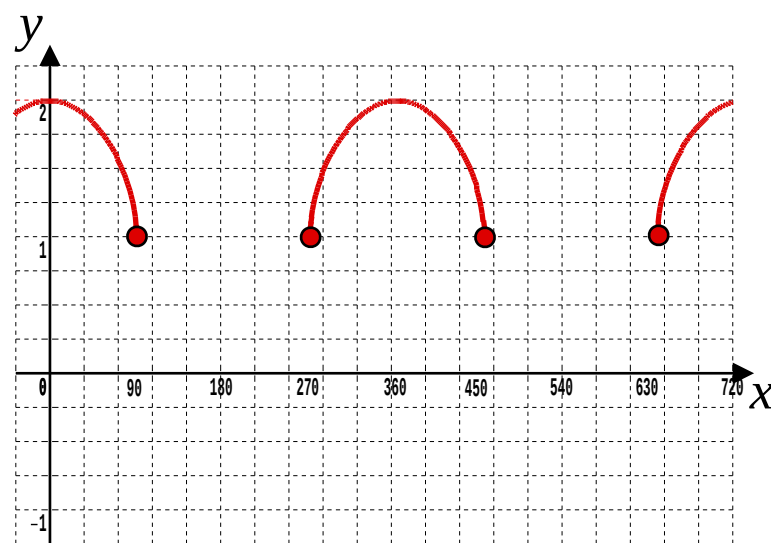


**[ 2 marks ]**

**( ii )** Negative for the intervals:  $90^\circ \leq x \leq 270^\circ$   $450^\circ \leq x \leq 630$

**[ 2 marks ]**

**( iii )**



**[ 2 marks ]**

**( iv )** It's said that if you scroll far enough to the right you'll get to the monster's head, and far enough to the left will arrive at Nessie's tail.  
In spite of some claiming to have seen these body parts there is no hard evidence of such sightings (see this topic's cover).

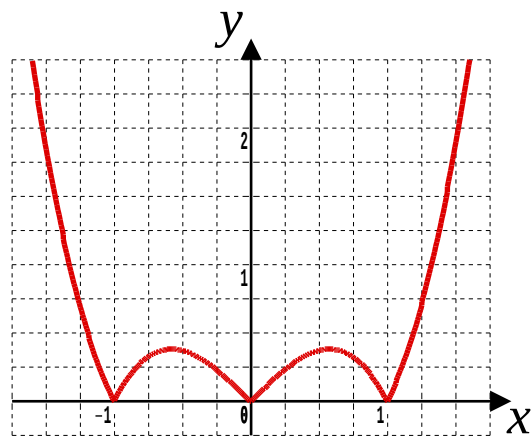
**[ 1 mark ]**

**Answer 6**

$$\begin{aligned} |2x - 4| &= x \\ -(2x - 4) &= x & + (2x - 4) &= x \\ -2x + 4 &= x & x &= 4 \\ 4 &= 3x \\ x &= \frac{4}{3} \\ x &\in \left\{ \frac{4}{3}, 4 \right\} \end{aligned}$$

[ 3 marks ]

**Answer 7**



[ 3 marks ]

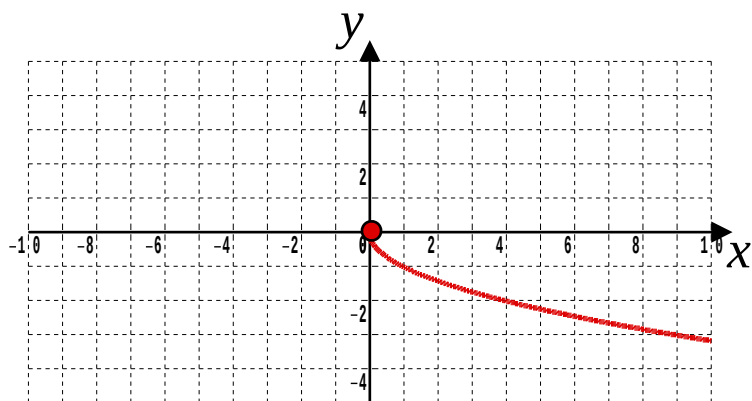
**Answer 8**

$$\begin{aligned} |3x + 2| &= 2 - x \\ -(3x + 2) &= 2 - x & + (3x + 2) &= 2 - x \\ -3x - 2 &= 2 - x & 4x &= 0 \\ -2x &= 4 & x &= 0 \\ x &= -2 \\ x &\in \{-2, 0\} \end{aligned}$$

[ 3 marks ]

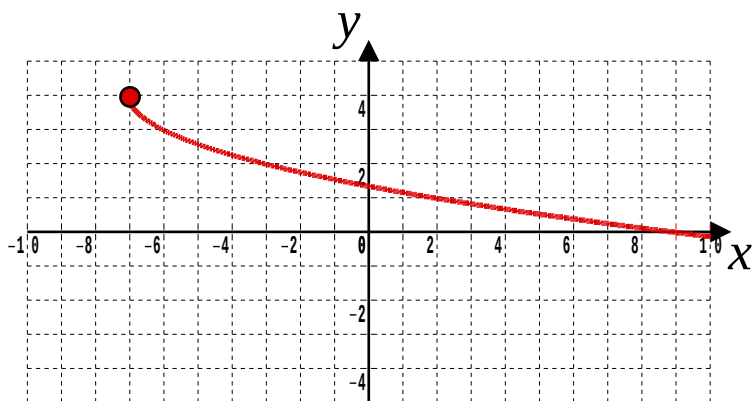
**Answer 9**

**( a )**



**[ 2 marks ]**

**( b )**



**[ 3 marks ]**

**( c )    ( i )    Domain:     $x \in \mathbb{R}, x \geq -7$**

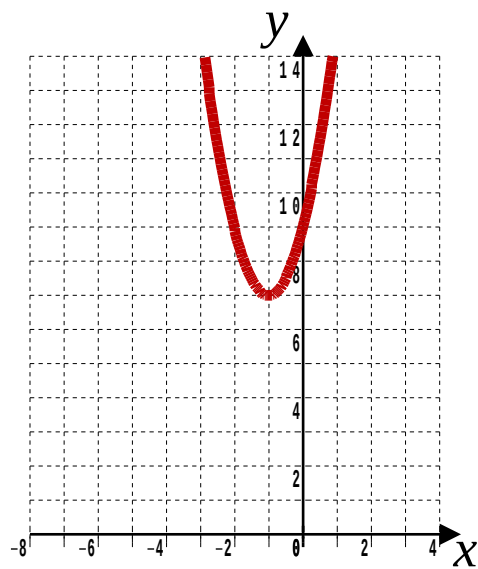
**[ 1 mark ]**

**( ii )    Range:     $g(x) \in \mathbb{R}, g(x) \leq 4$**

**[ 1 mark ]**

**Answer 10****( a )**

$$\begin{aligned}
 f(x) &= 2x^2 + 4x + 9 \\
 &= 2[x^2 + 2x] + 9 \\
 &= 2[(x + 1)^2 - 1] + 9 \\
 &= 2(x + 1)^2 + 7
 \end{aligned}$$

**[ 3 marks ]****( b )****[ 3 marks ]****( c ) ( i )**

$$\begin{aligned}
 g(x) &= 2(x - 2)^2 + 4x - 3 \\
 &= 2(x - 2)^2 + 4x - 8 + 8 - 3 \\
 &= 2(x - 2)^2 + 4(x - 2) + 5 \\
 &= 2(x - 2)^2 + 4(x - 2) + 9 - 9 + 5 \\
 &= 2(x - 2)^2 + 4(x - 2) + 9 - 4
 \end{aligned}$$

$$g(x) + 4 = 2(x - 2)^2 + 4(x - 2) + 9$$

Compare this with  $f(x) = 2x^2 + 4x + 9$

$\therefore f(x)$  translated  $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$  will give  $g(x)$

**[ 2 marks ]**

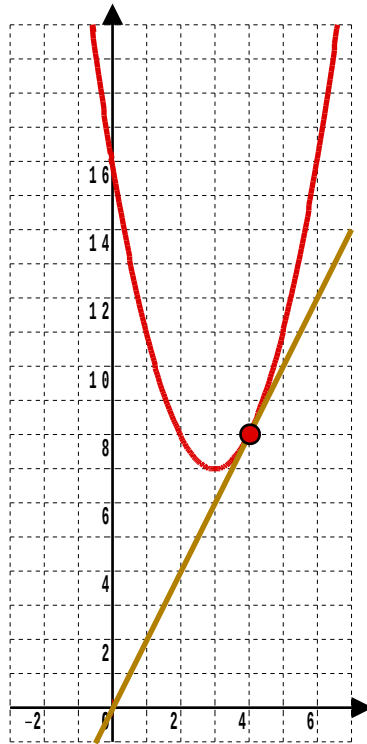
$$(ii) \quad h(x) = \frac{21}{2x^2 + 4x + 9} = \frac{21}{2(x + 1)^2 + 7}$$

$$\therefore h(x)_{MAX} = \frac{21}{7} = 3$$

$$\text{Range : } 0 < h(x) \leq 3$$

**[ 2 marks ]**



**Answer 11****( i )****[ 2 marks ]****( ii )** Range:  $f(x) \in \mathbb{R}, f(x) \geq 7$ **[ 1 mark ]****( iii )**

$$(x - 3)^2 + 7 = 2x$$

$$x^2 - 8x + 16 = 0$$

$$(x - 4)^2 = 0$$

$$x = 4 \text{ (twice)}$$

**[ 3 marks ]****( iv )** The line  $y = 2x$  is a tangent to the parabola at ( 4, 8 )**[ 3 marks ]**