

3E.4 Answers

A-Level Pure Mathematics, Year 2 Functions II

3E.4.1 Solutions to 3E.3 Exercise

Answer 1

$$P(x) = 2x^2 - 6x - 1 : \deg(P(x)) = 2$$

$$Q(x) = x - 1 : \deg(Q(x)) = 1$$

Testing for horizontal asymptotes;

As $\deg(P(x)) > \deg(Q(x))$ there is no horizontal asymptote.

Testing for vertical asymptotes;

$$Q(x) = 0$$

$$x - 1 = 0$$

$$x = 1$$

There is a vertical asymptote with equation, $x = 1$

Testing for oblique asymptotes;

As $\deg(P(x)) - \deg(Q(x)) = 1$ there exists an oblique asymptote.

$$\begin{array}{r} \overline{2x - 4} \\ x - 1 \overline{) 2x^2 - 6x - 1} \\ \underline{2x^2 - 2x} \\ - 4x - 1 \\ \underline{- 4x - 4} \\ 3 \leftarrow \text{Ignore !} \end{array}$$

There is an oblique asymptote with equation $y = 2x - 4$

[6 marks]

Answer 2

$$P(x) = 7x^2 - 9 : \deg(P(x)) = 2$$

$$Q(x) = 3x^2 - 10 : \deg(Q(x)) = 2$$

Testing for horizontal asymptotes;

As $\deg(P(x)) = \deg(Q(x))$ there is a horizontal asymptote.

$$\text{It's equation is, } y = \frac{7}{3}$$

Testing for vertical asymptotes;

$$Q(x) = 0$$

$$3x^2 - 10 = 0$$

$$x^2 = \frac{10}{3}$$

$$x = \pm \sqrt{\frac{10}{3}}$$

There are two vertical asymptotes with equations, $x = \pm \sqrt{\frac{10}{3}}$

Testing for oblique asymptotes;

As $\deg(P(x)) - \deg(Q(x)) \neq 1$ there is no oblique asymptote.

[6 marks]

Answer 3

$$P(x) = x^3 - x^2 - 8 : \deg(P(x)) = 3$$

$$Q(x) = 2x^2 - 8 : \deg(Q(x)) = 2$$

Testing for horizontal asymptotes;

As $\deg(P(x)) > \deg(Q(x))$ there is no horizontal asymptote.

Testing for vertical asymptotes;

$$Q(x) = 0$$

$$2x^2 - 8 = 0$$

$$x^2 = 4$$

There are two vertical asymptotes with equations, $x = \pm 2$

Testing for oblique asymptotes;

As $\deg(P(x)) - \deg(Q(x)) = 1$ there exists an oblique asymptote.

$$\begin{array}{r}
 \frac{1}{2}x - \frac{1}{2} \\
 2x^2 + 0x - 8 \overline{) \begin{array}{l} x^3 - x^2 + 0x - 8 \\ x^3 - 0x^2 - 4x \\ \hline - x^2 + 4x - 8 \\ - x^2 + 0x + 4 \\ \hline 4x - 12 \end{array} } \leftarrow \text{Ignore!}
 \end{array}$$

There is an oblique asymptote with equation $y = \frac{1}{2}x - \frac{1}{2}$

[6 marks]