4.5 Answers

4.5.1 Solution to 4.1.1 Example

All sorts of guesses can be made, most of which will have other effects !
The logical way to proceed is as follows.

There will exist a modulus sketching rule similar to rule 1;

Modulus Sketching Rule 1b

To sketch $x = |f(y)|$ **Bounce negative *y***

◇ Sketch $x = f(y)$ using a dashed line for points left of the *y*-axis.

◇ Reflect any part of the curve left of the *y*-axis in the *y*-axis.

$$y = x^2 - 4x + 3$$

$$y = (x - 2)^2 + 1$$

$$(x - 2)^2 = y - 1$$

Square root both sides

$$x - 2 = \pm\sqrt{y - 1}$$

$$x = 2 \pm \sqrt{y - 1}$$

$\therefore$ To plot curve C use $x = \left|2 \pm \sqrt{y - 1}\right|$

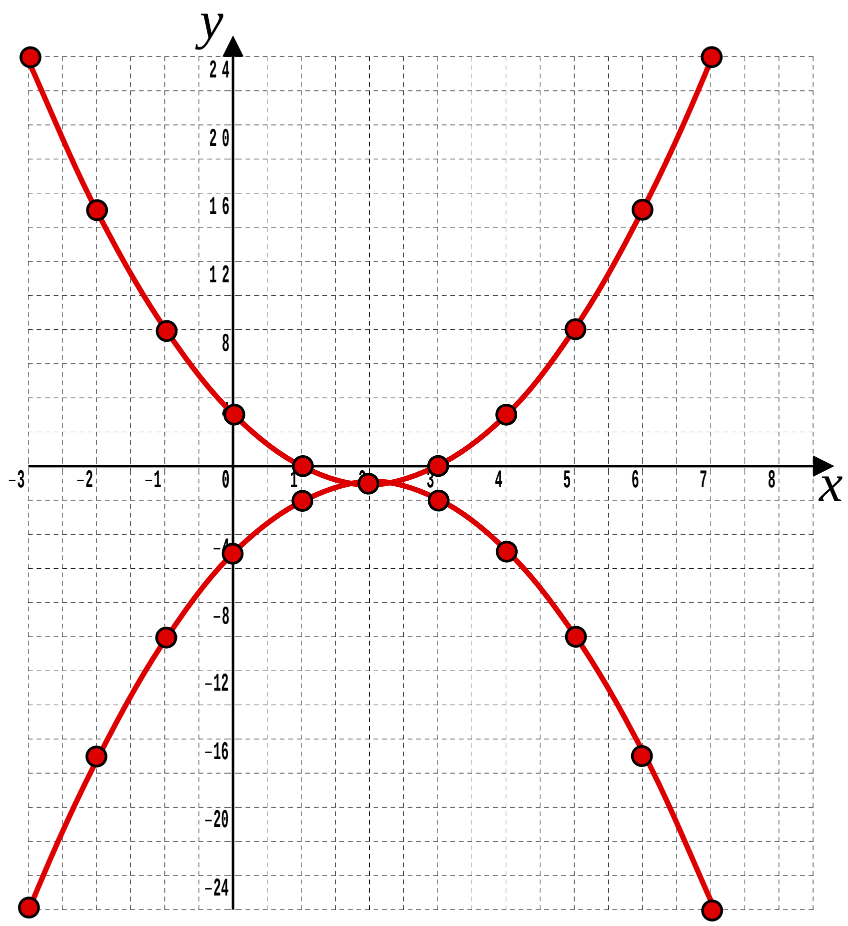
To plot this (depending on the sophistication of your plotter) you may have to plot the two curves separately, $x = \left|2 + \sqrt{y - 1}\right|$, $x = \left|2 - \sqrt{y - 1}\right|$

[6 marks]

4.5.2 Solution to 4.2.1 Example

$$|y + 1| = |(x - 2)^2|$$

x	-3	-2	-1	0	1	2	3	4	5	6	7
y	24 -26	15 -17	8 -10	3 -5	0 -2	-1 -1	0 -2	3 -5	8 -10	15 -17	24 -26



Geometrically, this is both the “original” curve and its reflection in the line, $y = -1$

[6 marks]

4.5.3 Solution to 4.3.1 Example

Previously...

$$y = x^2 - 4x + 3 \Leftrightarrow x = 2 \pm \sqrt{y - 1}$$

This time each individual y needs to be modulised,

$$\therefore \text{ To plot curve } C \text{ use } x = 2 \pm \sqrt{|y| - 1}$$

Modulus Sketching Rule 2b

To sketch $x = f(|y|)$ **Overwrite negative y**

◇ Sketch the curve of $x = f(y)$ for $y \geqslant 0$

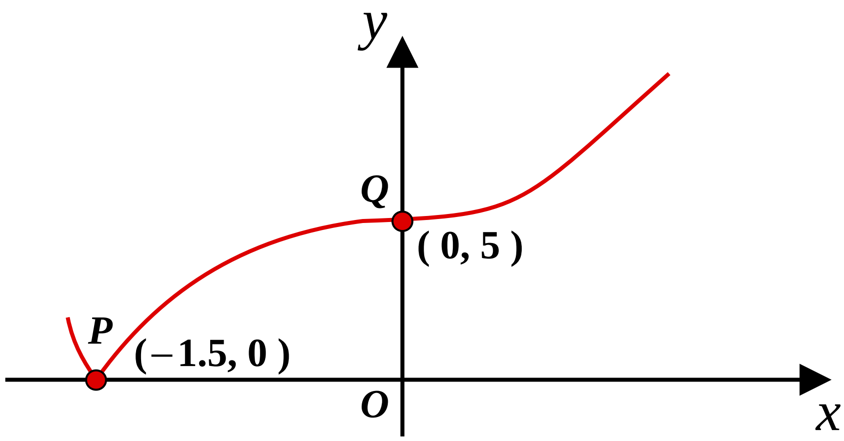
◇ Reflect the parts of the curve above the x-axis in the x-axis.

[2 marks]

4.5.4 Solution to 4.4 Exercise

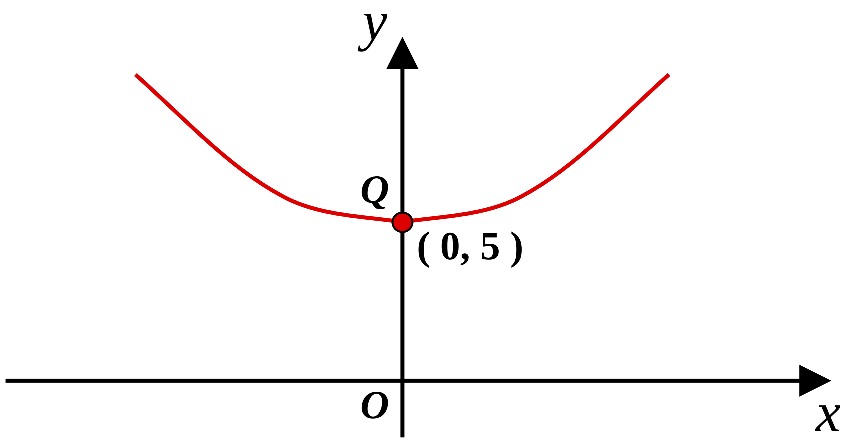
Answer 1

(a)



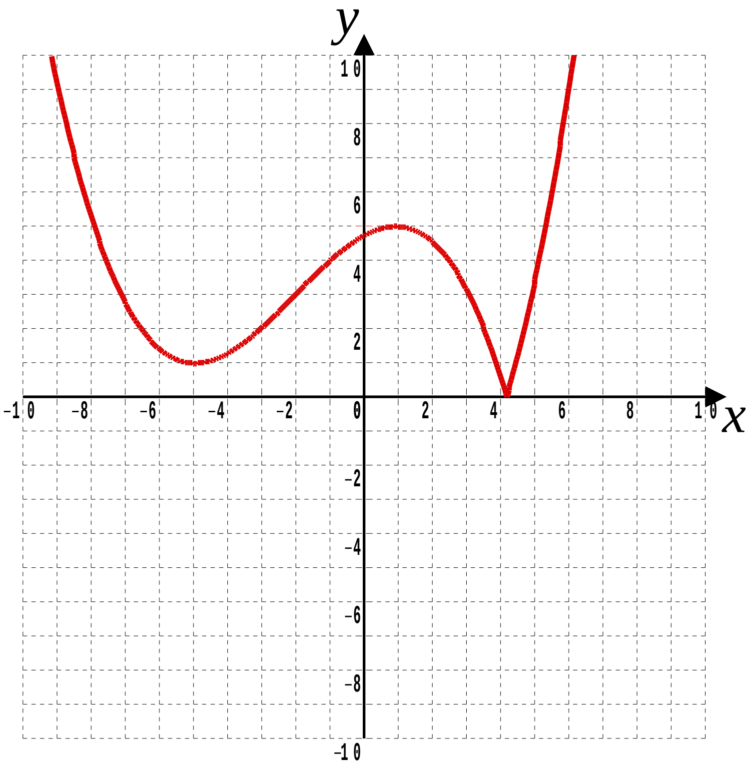
[2 marks]

(b)



[2 marks]

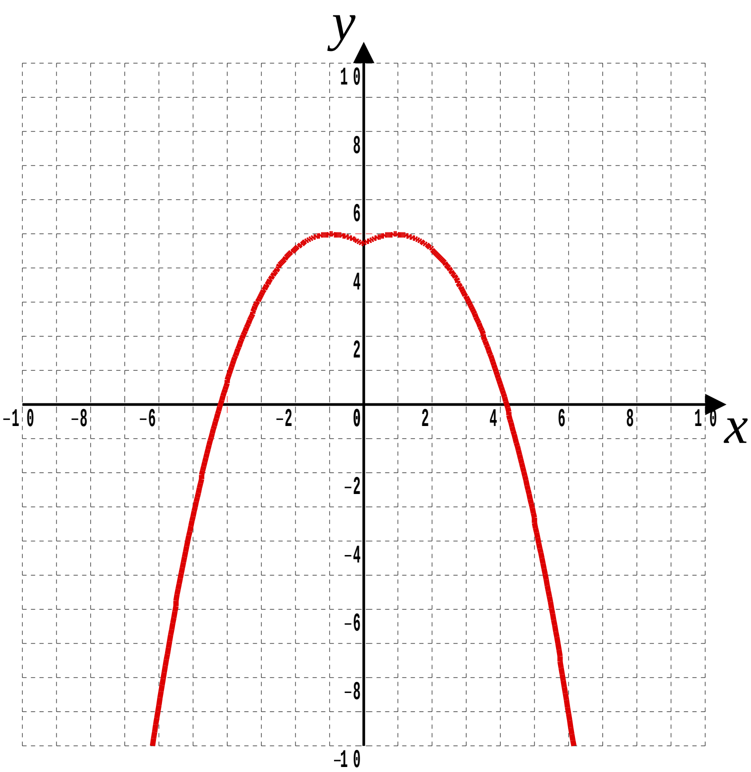
Answer 2
(i)



(1, 5) is a maximum and (– 5, 1) is a minimum turning point.

[2 marks]

(ii)



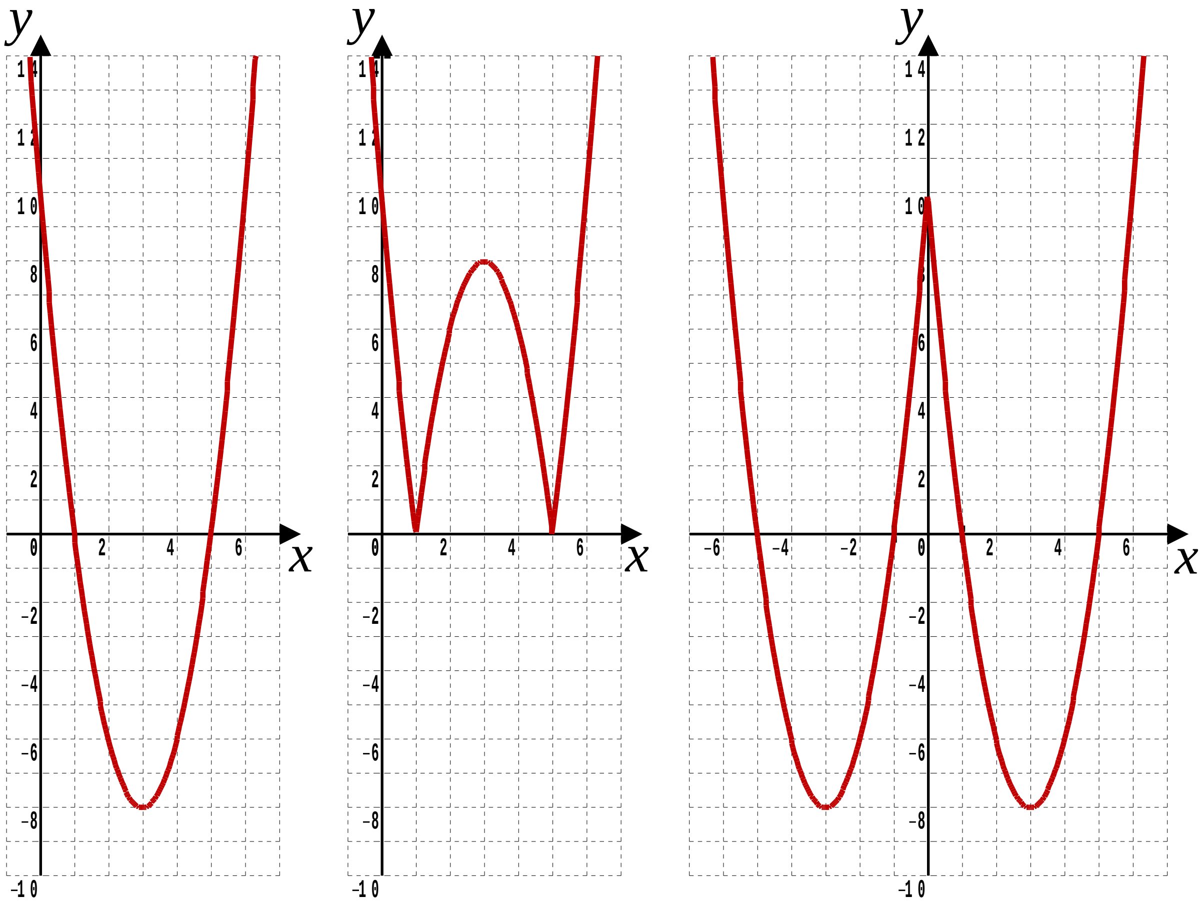
Two maximum turning points: One at (1, 5), the other at (– 1, 5)

[3 marks]

Answer 3

(a)

(i) **(ii)** **(iii)**



To work out x-axis intercepts of original equation:

$$2(x-3)^2-8=0$$

$$(x-3)^2-4=0$$

$$(x-3)^2=4$$

$$x-3=\pm 2$$

$$x=1 \text{ or } x=5$$

[3, 3, 3 marks]

(b) $-8<k<10$

[2 marks]

Answer 4

(a) Finding the line equations of the sides of the triangle, (other than the *x*-axis);

$$y = 7 - (3x - 5) \qquad y = 7 + (3x - 5)$$

$$y = 7 - 3x + 5 \qquad y = 7 + 3x - 5$$

$$y = -3x + 12 \qquad y = 3x + 2$$

Finding where each intersects the *x*-axis;

$$-3x + 12 = 0 \qquad 3x + 2 = 0$$

$$x = 4 \qquad x = -\frac{2}{3}$$

So the base of the triangle is of length $\frac{14}{3}$

Finding the apex of triangle, where lines intersect,

$$3x + 2 = -3x + 12$$

$$6x = 10$$

$$x = \frac{5}{3} \qquad \therefore\ y = 3 \times \frac{5}{3} + 2$$

$$= 7$$

Finding the area *R;*

$$Area\ \triangle = \frac{1}{2} \times \frac{14}{3} \times 7$$

$$= \frac{49}{3} \qquad \left(= 16\frac{1}{3}\right)$$

[4 marks]

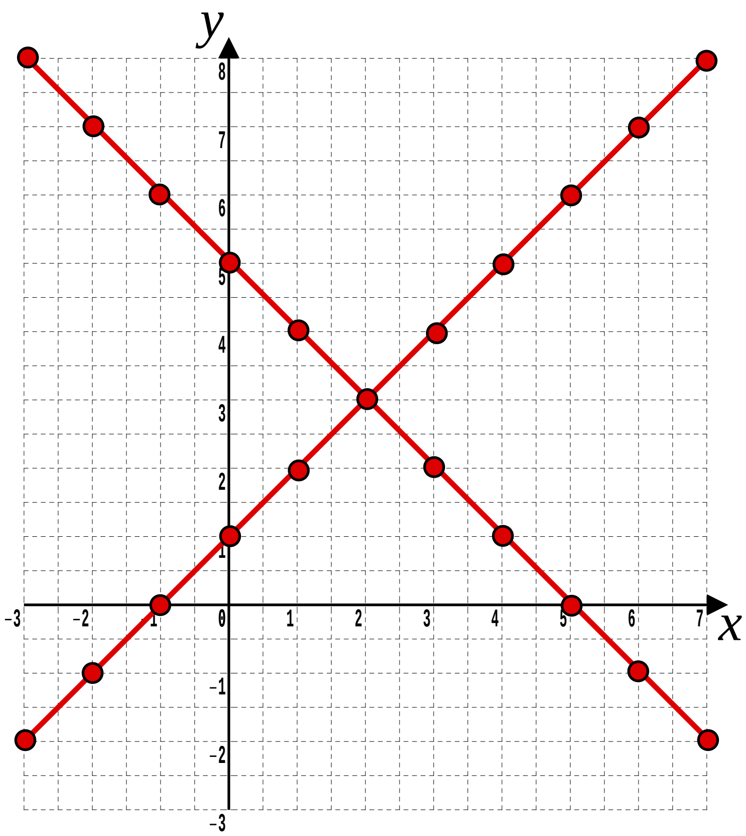
(b) $k < 7$

[1 mark]

Answer 5

(i) $$|y - 3| = |x - 2|$$

x	-3	-2	-1	0	1	2	3	4	5	6	7
y	8 -2	7 -1	6 0	5 1	4 2	3 3	2 4	1 5	0 6	-1 7	-2 8



[5 marks]

(ii) $k = 3$

[1 mark]

(iii) $c = \dfrac{5}{2}$

[1 mark]

Answer 6

(a) **(i)** True **(ii)** True **(iii)** True

[3 marks]

(b)

$$f(x) = |x|^2 - 4|x| + 1$$

$$= x^2 - 4|x| + 1 \quad \text{(Making use of part (a))}$$

Case 1 : $y = x^2 - 4x + 1$ for $x \geqslant 0$

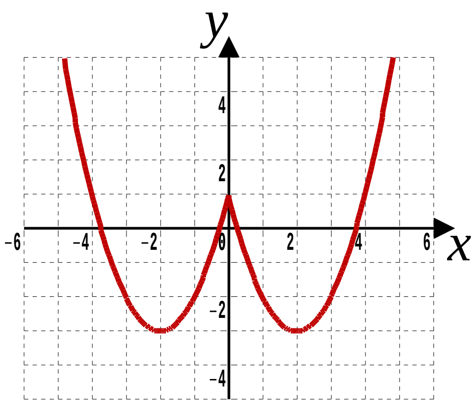
$$= (x - 2)^2 - 4 + 1$$

$$= (x - 2)^2 - 3 \qquad \therefore \text{ Minimum at } (2, -3)$$

Case 2 : $y = x^2 + 4x + 1$ for $x < 0$

$$= (x + 2)^2 - 4 + 1$$

$$= (x + 2)^2 - 3 \qquad \therefore \text{ Minimum at } (-2, -3)$$



[3 marks]

(c)

$$f(x) = -2$$

$$x^2 - 4|x| + 1 = -2 \quad \text{(Making use of part(a))}$$

Case 1 : $x^2 - 4x + 1 = -2$ for $x \geqslant 0$

$$x^2 - 4x + 3 = 0$$

$$(x - 1)(x - 3) = 0 \qquad \therefore x = 1 \text{ or } x = 3$$

Case 2 : $x^2 + 4x + 1 = -2$ for $x < 0$

$$x^2 + 4x + 3 = 0$$

$$(x + 1)(x + 3) = 0 \qquad \therefore x = -1 \text{ or } x = -3$$

Overall, $x \in \{\pm 1, \pm 3\}$

[3 marks]

(d) $k > 1$ or $k = -3$

[3 marks]

Answer 7

$$f(x) = 2|3 - x| + 5$$

(a) Range : $f(x) \in \mathbb{R},\ x \geqslant 5$

[1 mark]

(b)

$$2|3 - x| + 5 = \frac{1}{2}x + 30$$

$$-2(3 - x) + 5 = \frac{1}{2}x + 30 \qquad\qquad 2(3 - x) + 5 = \frac{1}{2}x + 30$$

$$-6 + 2x + 5 = \frac{1}{2}x + 30 \qquad\qquad 6 - 2x + 5 = \frac{1}{2}x + 30$$

$$\frac{3}{2}x = 31 \qquad\qquad -\frac{5}{2}x = 19$$

$$x = \frac{62}{3} \qquad\qquad x = -\frac{38}{5}$$

However, $-\dfrac{38}{5}$ is an extraneous solution.

$\therefore$ The only solution is $\dfrac{62}{3}$

[3 marks]

(c) $5 < k \leqslant 11$

[2 marks]

Answer 8
(i)
Modulus Sketching Rule 2b

To sketch $\boldsymbol{x = f(\,|\,y\,|\,)}$ **Overwrite negative** $\boldsymbol{y}$

◇ Sketch the curve of $x = f(y)$ for $y \geqslant 0$
◇ Reflect the parts of the curve above the x-axis in the x-axis

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$$\therefore\ x = 3 \pm \sqrt{25 - (\,|y| - 2\,)^2}$$

[3 marks]

(ii)
Modulus Sketching Rule 1b

To sketch $\boldsymbol{x = |\,f(y)\,|}$ **Bounce negative** $\boldsymbol{y}$

◇ Sketch $x = f(\mathrm{y})$ using a dashed line for points left of the y-axis.
◇ Reflect any part of the curve left of the y-axis in the y-axis.

$$\therefore\ x = \left|3 \pm \sqrt{25 - (\,y - 2\,)^2}\,\right|$$

[3 marks]

(iii)
Modulus Sketching Rule 1

To sketch $\boldsymbol{y = |\,f(x)\,|}$ **Bounce negative** $\boldsymbol{x}$

◇ Sketch $y = f(x)$ using a dashed line for points below the x-axis.
◇ Reflect any part of the curve below the x-axis in the x-axis.

$$y = \left|2 \pm \sqrt{25 - (\,x - 3\,)^2}\,\right|$$

[3 marks]

(iv)
Modulus Sketching Rule 2

To sketch $\boldsymbol{y = f(\,|\,x\,|\,)}$ **Overwrite negative** $\boldsymbol{x}$

◇ Sketch the curve of $y = f(x)$ for $x \geqslant 0$
◇ Reflect the parts of the curve to the right of the y-axis in the y-axis.

$$y = 2 \pm \sqrt{25 - (\,|x| - 3\,)^2}$$

[3 marks]

Answer 9

(a)

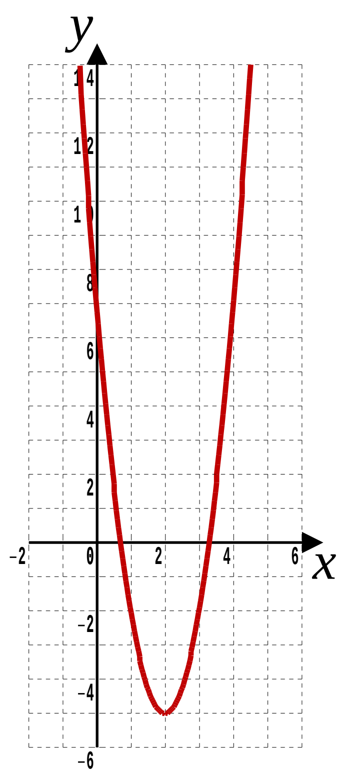
$$\begin{aligned} f(x) &= 3x^2 - 12x + 7 \\ &= 3\left[x^2 - 4x\right] + 7 \\ &= 3\left[(x-2)^2 - 4\right] + 7 \\ &= 3(x-2)^2 - 12 + 7 \\ &= 3(x-2)^2 - 5 \end{aligned}$$

[3 marks]

(b) To obtain the x-axis crossing points,

$$\begin{aligned} 3(x-2)^2 - 5 &= 0 \\ 3(x-2)^2 &= 5 \\ (x-2)^2 &= \frac{5}{3} \\ x - 2 &= \pm\sqrt{\frac{5}{3}} \\ x &= 2 \pm \frac{\sqrt{15}}{3} \end{aligned}$$

(Approx: $x = 0.709$ $x = 3.291$)



[3 marks]

(c) **(i)**

$$g(x) = 3(x-1)^2 - 12x + 9$$
$$= 3(x-1)^2 - 12x + 12 - 12 + 9$$
$$= 3(x-1)^2 - 12(x-1) - 3$$
$$= 3(x-1)^2 - 12(x-1) + 7 - 7 - 3$$
$$= 3(x-1)^2 - 12(x-1) + 7 - 10$$
$$g(x) + 10 = 3(x-1)^2 - 12(x-1) + 7$$

Compare this with $f(x) = 3x^2 - 12x + 7$

$\therefore\ f(x)$ translated $\begin{pmatrix} 1 \\ -10 \end{pmatrix}$ will give $g(x)$

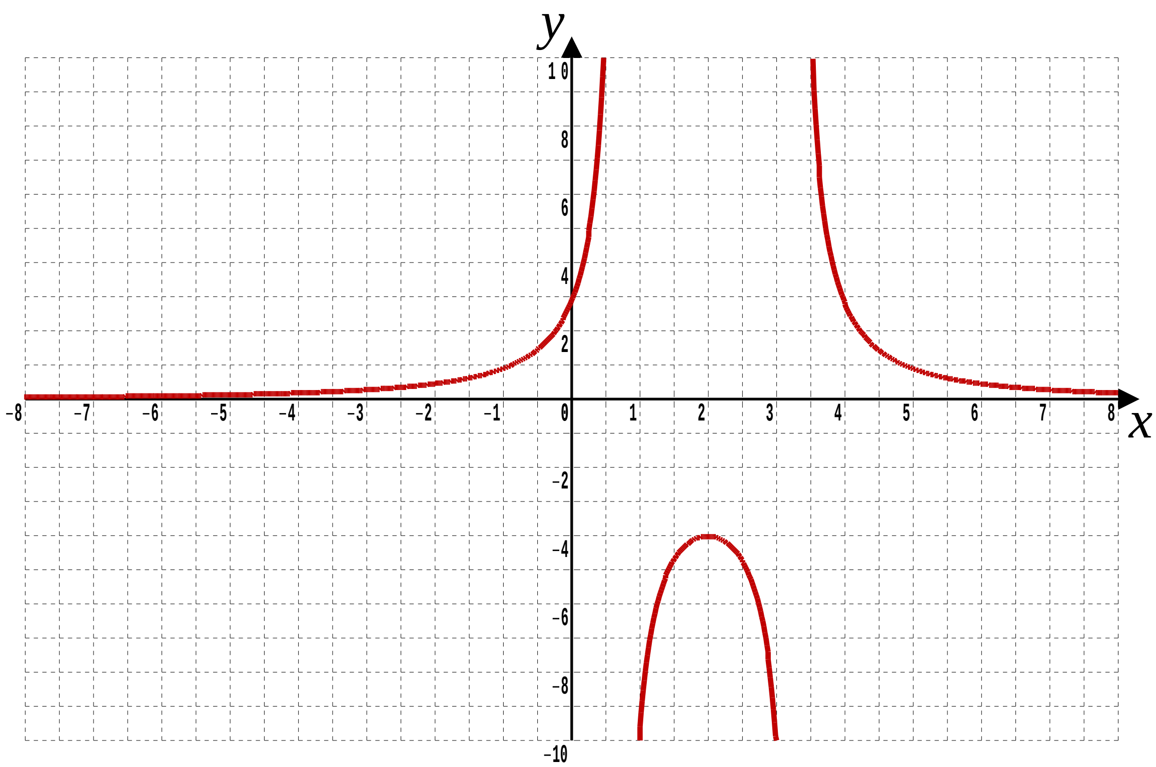
[2 marks]

(ii) This is tricky but the graph below helps with the understanding;

$$h(x) = \frac{20}{3x^2 - 12x + 7} = \frac{20}{3(x-2)^2 - 5}$$

$$h(2) = \frac{20}{-5} = -4 \quad \text{(A local maximum)}$$

Range : $h(x) < -4,\ h(x) > 0$

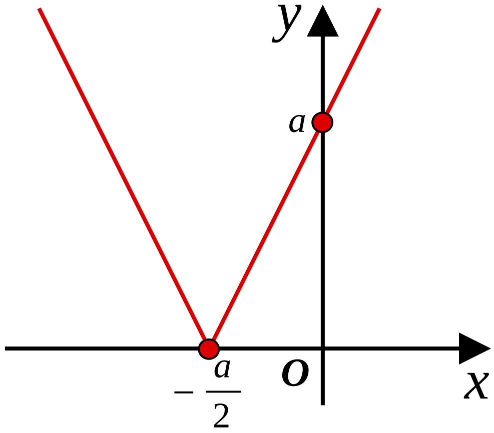


[2 marks]

Answer 10

(a) **(i)** Working out the x-axis "touch";

$$|2x + a| = 0$$
$$2x + a = 0$$
$$x = -\frac{a}{2}$$



[2 marks]

(ii) Working out the x-axis crossing points;

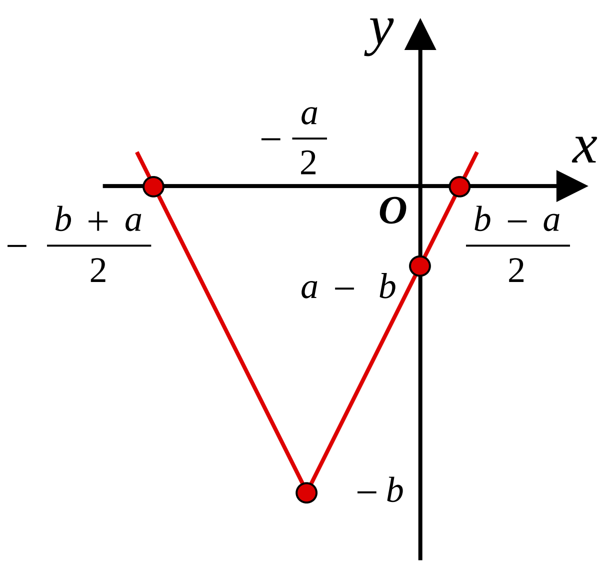
$$|2x + a| - b = 0$$
$$|2x + a| = b$$

Case 1: $2x + a = b$

$$2x = b - a$$
$$x = \frac{b - a}{2}$$

Case 2: $-2x - a = b$

$$2x + a = -b$$
$$2x = -b - a$$
$$x = -\frac{b + a}{2}$$



[4 marks]

(b)

$$|2x + a| - b = \frac{1}{3}x$$

$$\text{Case 1: } 2x + a - b = \frac{1}{3}x$$

$$\frac{6x}{3} - \frac{x}{3} = b - a$$

$$5x = 3(b - a)$$

$$x = \frac{3}{5}(b - a)$$

$$\text{Case 2: } -2x - a - b = \frac{1}{3}x$$

$$-a - b = \frac{x}{3} + \frac{6x}{3}$$

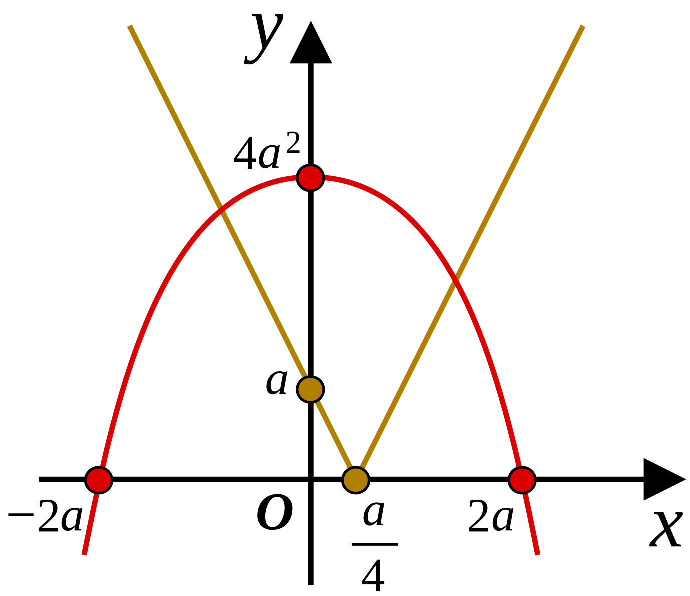
$$7x = -3(a + b)$$

$$x = -\frac{3}{7}(a + b)$$

[4 marks]

Answer 11

(i)



Red: $f(x) = 4a^2 - x^2$ Gold: $g(x) = |4x - a|$

[6 marks]

(ii)

$$4 - x^2 = |4x - 1|$$

$$4 - x^2 = 4x - 1 \qquad\qquad 4 - x^2 = -4x + 1$$

$$x^2 + 4x - 5 = 0 \qquad\qquad x^2 - 4x - 3 = 0$$

$$(x + 5)(x - 1) = 0 \qquad\qquad (x - 2)^2 - 7 = 0$$

$$x = -5 \text{ or } x = 1 \qquad\qquad x = 2 \pm \sqrt{7}$$

Two of these proposed solutions are extraneous.

The only valid solutions are, $x = 1$ or $2 + \sqrt{7}$

[4 marks]

Answer 12

$$f(x) = \frac{\pi}{180} x - 5 + \sin x \qquad 300° \leqslant x \leqslant 600°$$

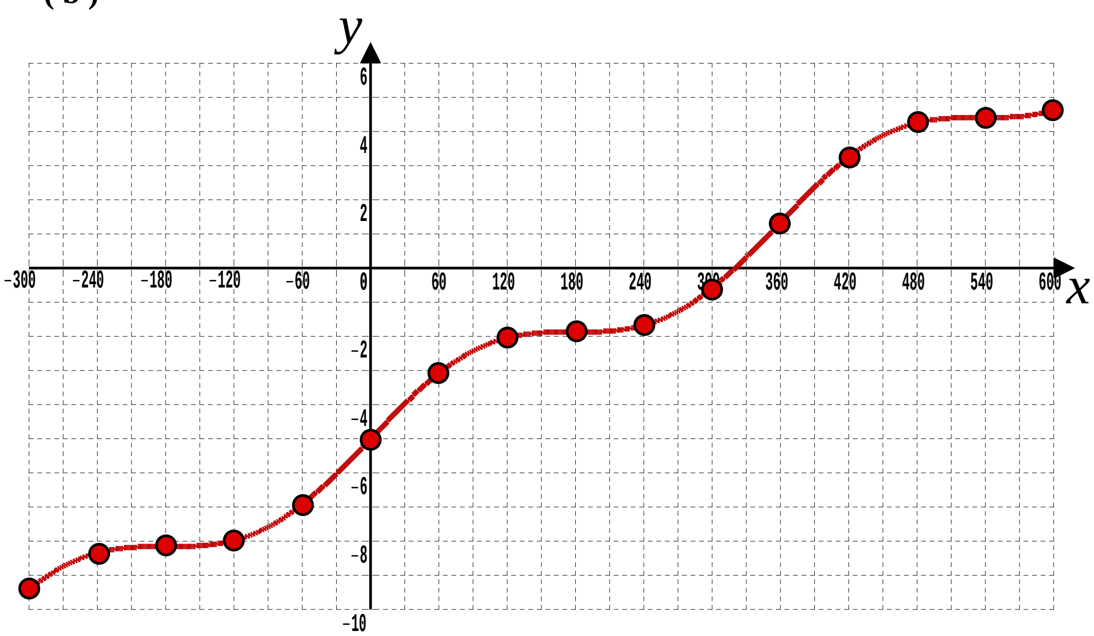
(a)

x	– 300	– 240	– 180	– 120	– 60	0	60	120
y	– 9.37	– 8.32	– 8.14	– 7.96	– 6.91	– 5	– 3.09	– 2.04

x	180	240	300	360	420	480	540	600
y	– 1.86	– 1.68	– 0.630	1.28	3.20	4.24	4.42	4.61

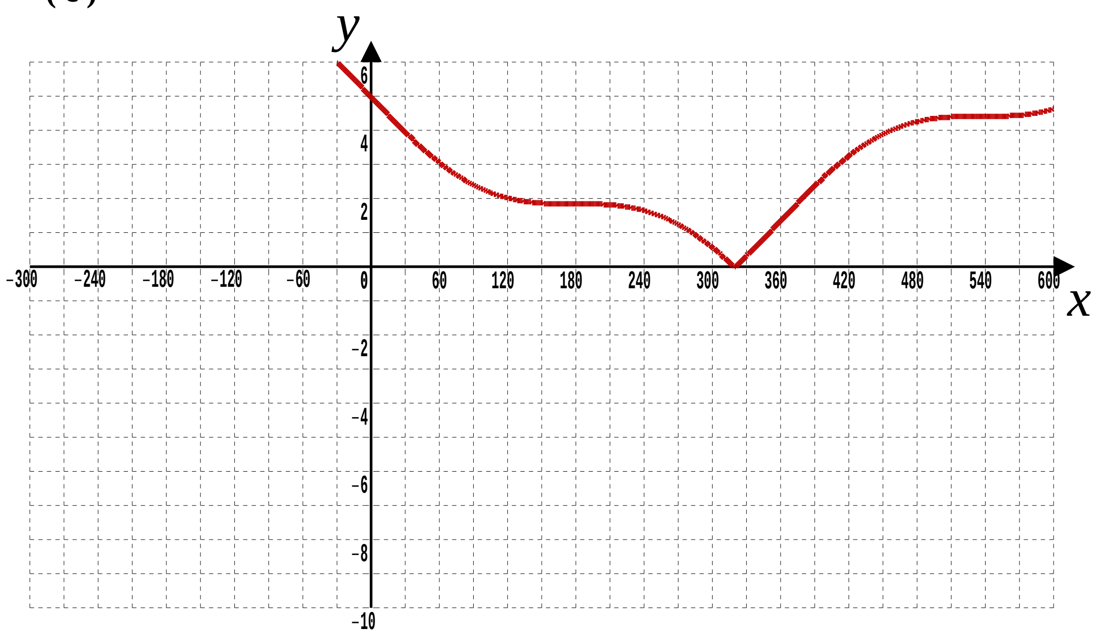
[3 marks]

(b)



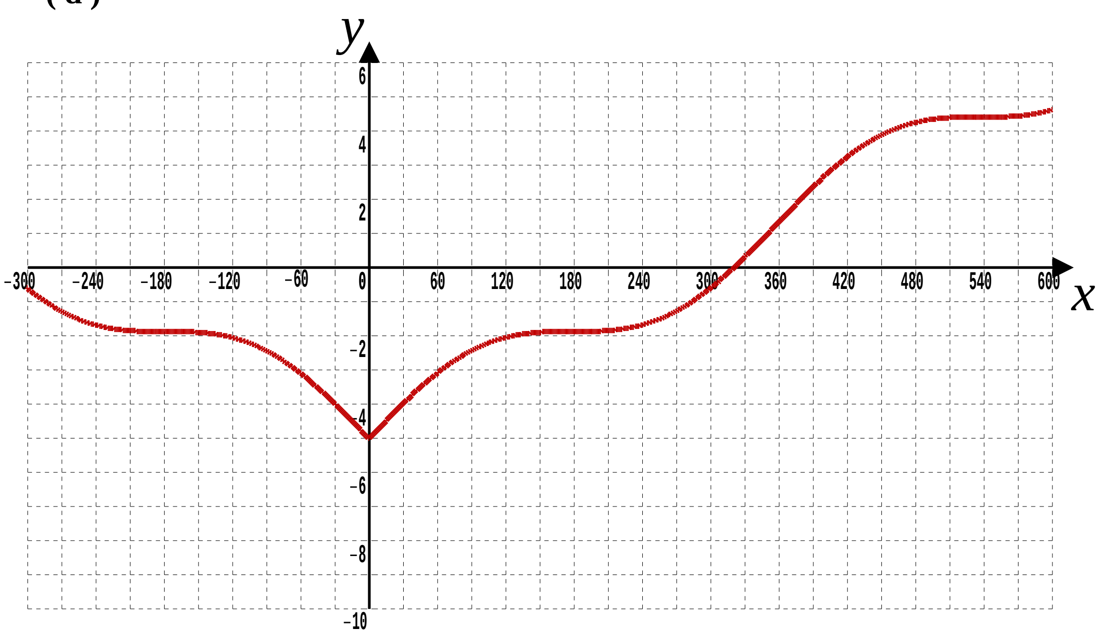
[2 marks]

(c)



[2 marks]

(d)



[2 marks]