

6.4 Answers

A-Level Pure Mathematics, Year 2 Functions II

6.4.1 Solutions to 6.2 Example

(i) 49 (ii) 79

[1, 1 marks]

(iii) $(x + 1)^2 - 2$ (iv) $((x - 2) + 1)^2$

[1, 1 marks]

(v)

$$\begin{aligned}(x + 1)^2 - 2 &= ((x - 2) + 1)^2 \\ x^2 + 2x + 1 - 2 &= (x - 1)^2 \\ x^2 + 2x - 1 &= x^2 - 2x + 1 \\ 4x &= 2 \\ x &= \frac{1}{2}\end{aligned}$$

[2 marks]

6.4.2 Solutions to 6.4 Exercise

Answer 1

$$gf(x) = 15x + 6$$

Answer 2

$$fg(x) = 65 - 7x$$

[2, 2 marks]

Answer 3

(i) $x^2 + 7x + 12$

(ii) $x \in \{-3, -4\}$

[2, 2 marks]

Answer 4

(i) 11

(ii) 17

[1, 1 marks]

(iii) 7

(iv) 50

[2, 2 marks]

(v) $4x^2 + 3$

(vi) $fg(x) \in \mathbb{R}, fg(x) \geq 3$

[3, 1 marks]

Answer 5

$$\frac{2}{x}$$

[2 marks]

Answer 6

$$\begin{array}{llll}
 \text{(a)} & \text{(i)} & 30 & \text{(ii)} & 42 & \text{(iii)} & -5 \\
 & \text{(iv)} & 108 & \text{(v)} & 2 & \text{(vi)} & -2
 \end{array}$$

[3 marks]

$$\text{(b)} \quad x^2 - 3x + 2 \quad \text{(c)} \quad x \in \{1, 2\}$$

[2, 2 marks]

Answer 7

$$\text{(i)} \quad fg(x) = \sqrt{x+2} + 3$$

[2 marks]

$$\text{(ii)} \quad x \in \mathbb{R}, x \geq -2$$

[1 mark]

$$\text{(iii)} \quad fg(x) \in \mathbb{R}, fg(x) \geq 3$$

[1 mark]

Answer 8

$$ff(x) = 0$$

$$(x^2 - 1)^2 - 1 = 0$$

$$x^4 - 2x^2 = 0$$

$$x^2(x^2 - 2) = 0$$

$$x \in \{0, \pm\sqrt{2}\}$$

[4 marks]

Answer 9

$$fg(x) = gf(x)$$

$$f(2x - 5) = g(x^2 + 8)$$

$$(2x - 5)^2 + 8 = 2(x^2 + 8) - 5$$

$$4x^2 - 20x + 25 + 8 = 2x^2 + 16 - 5$$

$$2x^2 - 20x + 22 = 0$$

$$x^2 - 10x + 11 = 0$$

$$x = \frac{10 \pm \sqrt{10^2 - 4 \times 1 \times 11}}{2}$$

$$= \frac{10 \pm \sqrt{56}}{2}$$

$$= 5 \pm \sqrt{14}$$

[5 marks]

Answer 10

(i) 92

(ii) 21

[1, 1 marks]

(iii) 961

(iv) 91

[1, 1 marks]

(v) $mn(x) = 991 - 10\sqrt{x}$

(vi) $100 - \sqrt{10x - 9}$

[2, 2 marks]

(vii) $m(0) = -9$

 $nm(0) = n(-9)$ but then are attempting to square root a negative number**[2 marks]**

(viii) $n(9820.81) = 0.9$ and $m(0.9) = 0$

If $x > 9820.81$ then input into m is ≤ 0.9 That is, outside domain of m **[2 marks]****Answer 11**

(i) 4

(ii) 58

[2, 2 marks]

(iii)

$$\begin{aligned}
 mn(z^2 + 7) &= m(\sqrt{z^2 + 7} - 7) \\
 &= m(z) \\
 &= 9z - 5
 \end{aligned}$$

[2 marks]

(iv)

$$\begin{aligned}
 mn(4z^2 + 7) &= m(\sqrt{4z^2 + 7} - 7) \\
 &= m(2z) \\
 &= 18z - 5
 \end{aligned}$$

[2 marks]